Knowledge Integration Pre-work 1

**Basic Objective of KI-1:** The knowledge integration module 1 or KI-1 is a vehicle to help you better grasp the commonality and connections between concepts covered in ECE 311, 331 and 341 during LSM 1 and 2. In the KI, we will consider the illustrative example of a general radio system e.g., a cellphone to see how the materials covered in the courses ECE 311, 331, and 341 inform engineering and design.

**Pre-work:** To be completed and turned into the BC Infill on Monday, September 23rd by 8 am.

Consider the system in Figure 1 and its two (sub)-systems: the rectifier in green and the RC circuit in blue. Assume that all components are ideal (unless otherwise stated) and the characteristics of the components do not change over time.

![Figure 1: Power Supply Circuit](image)

1) Linearity and time-invariance:

   c. If the rectifier is removed from Figure 1, can a DC output voltage be generated from an AC source with the RC circuit alone? Justify your answer.
   d. Find a mathematical expression for the impulse response $h(t)$ of the RC circuit.

2) In diodes, the drift current signifies that the velocity of charge carriers is proportional to the applied electric field $E$. This is only true for moderate magnitudes of the field $E$. The velocity is no longer proportional for very high magnitudes of the field $E$. Why? Feel free to use Google and/or any other electronic resource to explain why this is a problem in MOS transistors.

3) Let us revisit the depletion region in a diode as shown in Figure 2. If the electric field in this region is in the longitudinal ($\rightarrow$) direction as shown, we all know this field retards diffusion current but aids drift current. Now, if we add a new electric field in the transverse ($\uparrow$) direction as well, what will happen to the magnitude of the drift and diffusion current? Can you name a
device where such a transverse electric field occurs (hint: you will definitely cover this device in ECE 331 in LSM 3)? Feel free to use Google and/or any other electronic tools information sources for your answer.

![Depletion Region](image)

**Figure 2: Depletion Region**

4) **Convolution:** Consider the entire system in Figure 1. Suppose \( v_{in}(t) = A\sin(\omega_0 t) \).

   a. Using linearity and time-invariance, explain whether or not you expect the output \( v_{out}(t) \) of the system to be periodic.

   b. Using the convolution integral, find a mathematical expression for \( v_{out}(t) \).

   *Hint:*
   
   \[
   \int \sin(\omega_0 \tau) e^{a\tau} d\tau = \frac{1}{\omega_0^2 + a^2} e^{a\tau} (a\sin(\omega_0 \tau) - \omega_0 \cos(\omega_0 \tau))
   \]

5) Sketched in Figure 3 is a \( pn \) junction between two semiconducting half-spaces, doped \( p \) type and \( n \) type, respectively. The volume charge distribution in the semiconductor can be approximated by the following function: \( \rho(x) = \rho_0 \left( \frac{x}{a} \right) \exp(-|x|/a) \), where \( \rho_0 \) and \( a \) are positive constants. The permittivity of the semiconductor is \( \varepsilon \). Find: (a) the electric field intensity vector in the semiconductor, (b) the electric scalar potential in the semiconductor, and (c) the built-in voltage of the \( pn \) junction (diode), namely, the voltage between the ends of the semiconductor, from the end on the \( n \)-type side to the end on the \( p \)-type side of the junction.

![Model of a pn junction with a linear-exponential charge distribution](image)

**Figure 3: Model of a pn junction with a linear-exponential charge distribution**