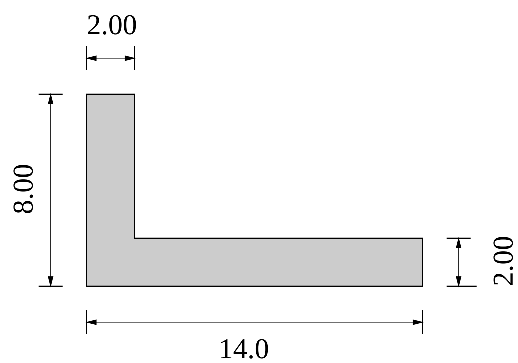
CE560 Advanced Mechanics of Solids

Non-symmetric Pure Bending

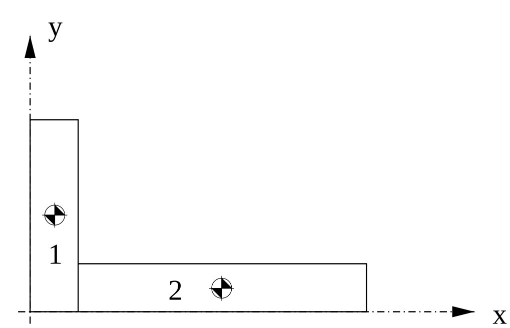
The flexure formula is valid oly when bending is about a principal axis. When this is not the case, then the moment can be divided into components in the direction of principal axes, and their results summed using the principle of superposition.

EXAMPLE:

Given a beam with the cross-section shown and subjected to a pure bending moment about a vertical axis. Determine the maximum and minimum axial stresses. (Note: units of length and force can be in any consitant set of units).



Step 1: Determine the centroid.

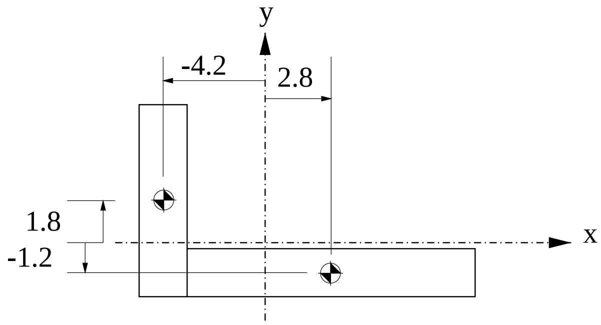


Section	Area	X	Y	X*Area	Y*Area
1	16	1	4	16	64
2	24	8	1	192	24
SUM	40			208	88

$$X_c = \frac{208}{40} = 5.2$$

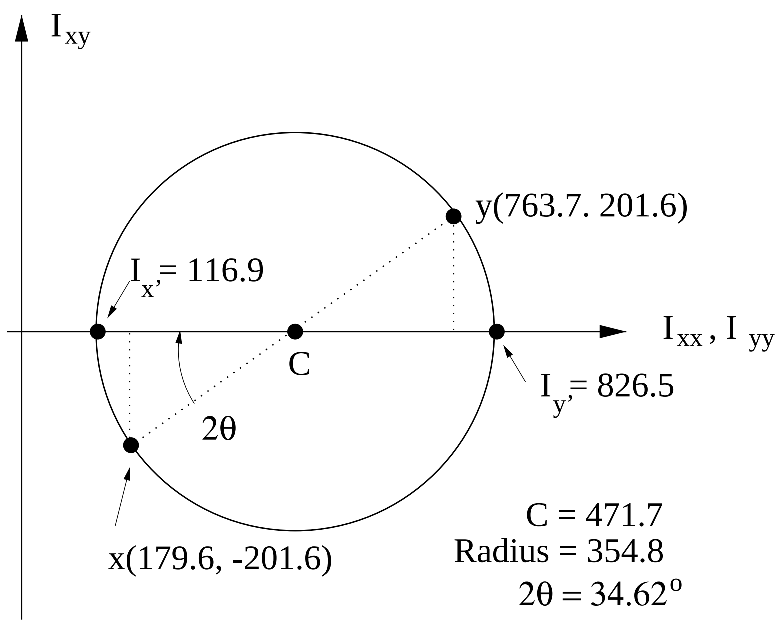
$$Y_c = \frac{88}{40} = 2.2$$

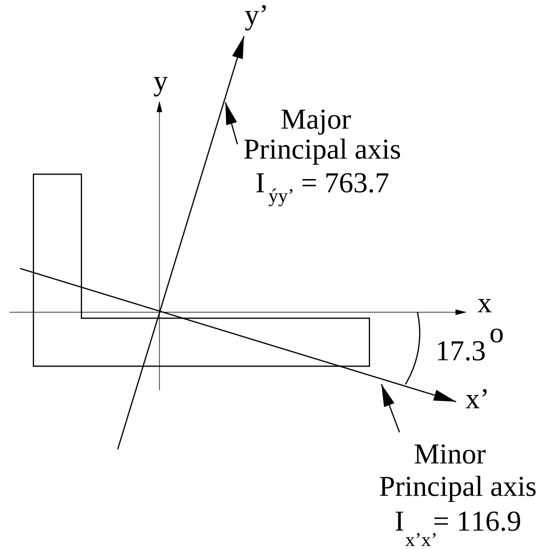
Step 2: Determine Moments of Inertia.



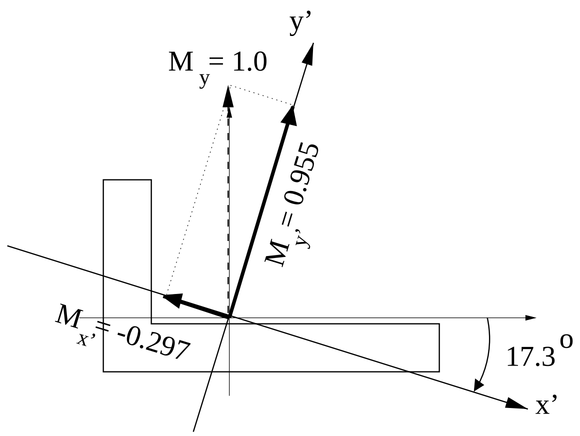
Section	Area (A)	I_{xx}	I_{yy}	I_{xy}	X	Y	I_{xx} $+AY^2$	I_{yy} AX^2	I_{xy} $+AXY$
1	16	85.33	5.33	0.0	-4.2	1.8	287.57	137.17	-121.0
2	24	8.00	288.	0.0	2.8	-1.2	476.16	42.567	-80.64
SUM							763.8	179.7	-201.6

Step 3: Determine Principal Moments of Inertia by Mohr's Circle





Step 4: Determine Components of Moment in Principal Directions.



Step 5: Determine Total Axial Stress

The total axial stress is obtained by applying the flexure formula to each of the above moments. Care must be taken to make sure the signs are consistant with the direction shown for the moments. If these moments represent the resultant of the stresses, then, when y' is positive, the moment shown about the x' axis must create a compressive stress. When x' is positive, the the moment shown about the y' must create a compressive stress. Hence, the equation for total stress is:

$$\begin{aligned}\sigma_z &= -\frac{0.297y'}{116.9} - \frac{0.955x'}{826.5} \\ &= -2.541(10^{-3})y' - 1.155(10^{-3})x'\end{aligned}$$

Step 6: Determine the Orientation of the Neutral Axis.

The neutral axis is the locus of points for which the axial stress is zero. It is the axis about which the cross-section rotates. We find its equation by setting the axial stress equal to zero. Hence:

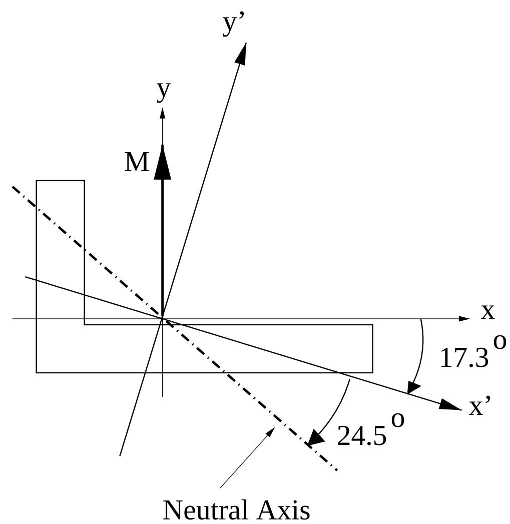
$$-2.541(10^{-3})y' - 1.155(10^{-3})x' = 0$$

or simply,

$$y' = -.455x'$$

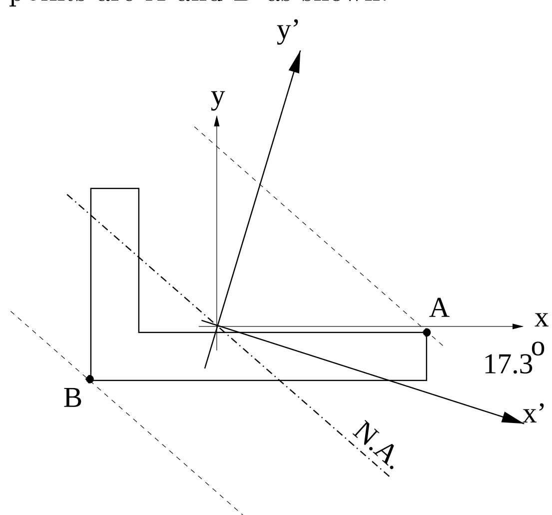
The angle which this line makes with the x' axis is:

$$\beta = \text{atan}(-.455) = -24.5\,\text{deg}$$



Step 7: Determine Maximum Tensile and Compressive Stress

The maximum values of tensile and compress stress occur at the farthest points each side of neutral axis. These points are A and B as shown.



The x and y coordinates of these points are:

Point	x	y
A	8.8	−0.2
B	−5.2	−2.2

and by using the matrix for coordinate rotations, the x' and y' coordinates are:

$$\begin{Bmatrix} x'_A \\ y'_A \end{Bmatrix} = \begin{bmatrix} \cos(17.3) & -\sin(17.3) \\ \sin(17.3) & \cos(17.3) \end{bmatrix} \begin{Bmatrix} 8.8 \\ -0.2 \end{Bmatrix} = \begin{Bmatrix} 8.46 \\ 2.43 \end{Bmatrix}$$

and

$$\begin{Bmatrix} x'_B \\ y'_B \end{Bmatrix} = \begin{bmatrix} \cos(17.3) & -\sin(17.3) \\ \sin(17.3) & \cos(17.3) \end{bmatrix} \begin{Bmatrix} -5.2 \\ -2.2 \end{Bmatrix} = \begin{Bmatrix} -4.31 \\ -3.65 \end{Bmatrix}$$

Thus, the axial stresses at these points are:

$$\begin{aligned} \sigma_A &= -2.541(10^{-3})(2.43) - 1.155(10^{-3})(8.46) \\ &= -15.9 \quad \text{units of stress in Compression} \end{aligned}$$

and

$$\begin{aligned} \sigma_B &= -2.541(10^{-3})(-3.65) - 1.155(10^{-3})(-4.31) \\ &= +14.3 \quad \text{units of stress in Tensions} \end{aligned}$$