

The Transformation Matrix in Terms of Euler's Angles

The rotation of one rectangular set of axes into another set can involve only three independent variables, although there are an infinite number of ways to select the variables. Three such variables are the Euler angles often used to describe motion of rigid bodies in three dimensional space. These angles are sometimes defined as:

A1 = a positive rotation about the z axis

A2 = a positive rotation about the new x axis

A3 = a positive rotation about the new z axis

In terms of these angles, the transformation matrix, C_{ij} has the following components:

$$C(1,1) = +\cos(A3)\cos(A1) - \cos(A2)\sin(A1)\sin(A3)$$

$$C(1,2) = -\sin(A3)\cos(A1) - \cos(A2)\sin(A1)\cos(A3)$$

$$C(1,3) = +\sin(A2)\sin(A1)$$

$$C(2,1) = +\cos(A3)\sin(A1) + \cos(A2)\cos(A1)\sin(A3)$$

$$C(2,2) = -\sin(A3)\sin(A1) + \cos(A2)\cos(A1)\cos(A3)$$

$$C(2,3) = -\sin(A2)\cos(A1)$$

$$C(3,1) = +\sin(A2)\sin(A3)$$

$$C(3,2) = +\sin(A2)\cos(A3)$$

$$C(3,3) = +\cos(A2)$$

where $\sigma'_{ij} = C_{im}C_{jn}\sigma_{mn}$