Climate, Soil, and Vegetation

PETER S. EAGLESON

Reprinted from
WATER RESOURCES RESEARCH
VOL. 14, NO. 5 OCTOBER 1978

Climate, Soil, and Vegetation
1. Introduction to Water Balance Dynamics

PETER S. EAGLESON

Department of Civil Engineering, Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

A statistical dynamic formulation of the vertical water budget at a land-atmosphere interface is outlined. Physically based dynamic and conservation equations express the infiltration, exfiltration, transpiration, percolation to groundwater and capillary rise from the water table during rainstorms and interstorm periods in terms of independent variables representing the precipitation, potential evapotranspiration, soil and vegetal properties, and water table elevation. Uncertainty is introduced into these equations through the probability density functions of the independent climatic variables and yields derived probability distributions of the dependent water balance elements: surface runoff, evapotranspiration, and groundwater runoff. The mean values of these quantities give a long-term average water balance which, to the first order, defines the annual water yield and water loss in terms of the annual precipitation and potential evapotranspiration and in terms of physical parameters of the soil, vegetation, climate, and water table. This analytical framework provides physical insight into the dynamic coupling of climate-soil-vegetation systems. Details are presented in a series of subsequent papers.

INTRODUCTION

It has long been recognized that the atmospheric and soil-vegetation systems are dynamically coupled through the physical processes which produce transport of thermal energy and water mass across the land surface.

At land surfaces the soil column responds dynamically to the climatic sequence of precipitation and evapotranspiration events and accepts part of the moisture during periods of precipitation, pumps some of this back to the surface during evaporative periods, and rejects the remainder to the water table more or less continuously. This surface moisture exchange and thus the surface heat exchange, to a large degree, depend critically upon the physical properties of the soil and vegetation as well as upon the weather conditions during the alternate periods of precipitation and evapotranspiration. The quantitative relation among the long-term averages of this partition of precipitation is called the 'water balance.'

The land surface alone returns to the base of the atmosphere by evapotranspiration about 12% of the entire earth's precipitation and through latent heat about 17% of all its absorbed solar radiation [*Budyko et al.*, 1962; *Houghton*, 1954]. It is reasonable to conclude therefore that large-scale patterns of climate may be influenced in significant measure by regional variability of these surface processes. We seek here to establish this climate-land surface coupling in a form which provides insight into the physical basis for man-induced change in both climate and water balance.

Such insight can come only through retention in our model of the underlying (and well-known) physical determinism. At the same time, however, uncertainty plays such a large role in nature that our approach must deal with probability distributions. This uncertainty exists in the configuration of the physical system, including the values of its parameters, in the values of the independent (i.e., input) variables to be expected, and, of course, in the information (i.e., observations) with which we evaluate the system behavior. It is seldom practical to deal with all sources of uncertainty simultaneously.

Putting aside the question of observational error, we can often deal successfully with spatially complex dynamic natural systems in the manner illustrated schematically in Figure 1. In this approach the spatial complexity is replaced by a highly idealized deterministic configuration of physical elements performing in such a way as to preserve the essential relationships among characteristic times, volumes, rates, etc. The relationships are obtained from the process physics, and their parameters may be either fixed or uncertain. Of course, the greater the physical idealization in a particular dimension, the more difficult is the determination of the appropriate effective values of the respective parameters.

The input variables are considered to be stochastic, and their probability distributions are transformed into the probability distribution of the output variable by using the deterministic physical process.

From the viewpoint of applied statistics this is classified as a problem in 'derived distributions,' while in applied mechanics it falls in the realm of 'statistical mechanics.' By this formulation we have traded off some fidelity in physical behavior for the ability to include in the derived output statistics an explicit representation of both the essential system dynamics and the input statistics. This has great utility in environmental impact analysis and in other situations where insight into uncertain behavior takes precedence over quantitative accuracy.

An example of this approach which illustrates its power is the derivation of flood frequency from rainfall statistics [*Eagleson*, 1972] and from snowmelt statistics [*Carlson and Fox*, 1976] by using the kinematic wave model of surface runoff. We shall use the same technique here to look at the annual water balance.

Our objective in this work is to gain insight into the physical basis for uncertainty (and for changes in uncertainty) in the elements of the annual water balance as a first step toward understanding man-induced alterations in the hydrologic cycle. In so doing, we will strive for analytical as opposed to numerical solutions, for it is our personal belief that where it is feasible this approach holds greater promise for realization of these objectives.

Only an outline is presented here. Details appear in a series of subsequent papers [*Eagleson*, 1978*a, b, c, d, e, f*].

PROBLEM FORMULATION

The history of fluid mechanics in general and of hydraulic engineering in particular is replete with examples of the advances in understanding that dimensionless formulation can

Paper number 8W0184.
0043-1397/78/058W-0184$01.00

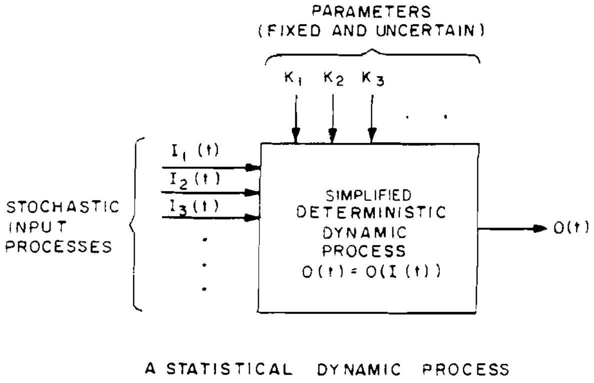

Fig. 1. A simple statistical dynamic process.

bring to bafflingly complex physical problems. Perhaps because it involves the dynamic interaction of atmosphere, surface water, soil moisture, vegetation, and groundwater, the overall hydrologic cycle has not yielded to such generalized analysis. Solutions are usually obtained to specific problems by using specific parametric values and the simulation methods used do not lend themselves readily to the extraction of general conclusions or to the generation of physical insight.

It is our hope in this work to produce a dimensionless analytical representation of the one-dimensional annual water balance which is based upon physically realistic dynamic models of hydrological subprocesses yet which is simple enough to allow analytical derivation of the probability distributions of the critical hydrologic variables from parameterized distributions of the climatic input variables.

It is realized at the outset that the achievement of this goal will require many simplifications of the true system (some apparently 'gross'). Although there is always the danger of destroying model validity through such simplifications, we believe that the potential yield of understanding is worth the risk.

One-dimensional representation. We will consider the dynamics of only those processes which operate in the vertical direction, namely, precipitation, infiltration, and evapotranspiration. They will act at the upper surface of a vertically sided control volume illustrated by the dashed lines in Figure 2*a*. In this figure, $i(t)$ is the precipitation intensity, $e_T(t)$ is the evapotranspiration rate, $v_{ss}(t)$ is the rate of capture of precipitation in surface storage, $r_s(t)$ is the surface runoff rate, $r_g(t)$ is the groundwater runoff rate, $y(t)$ is the yield rate, $V_{ss}(t)$ is the volume of storage on the surface, and $V_{sg}(t)$ is the volume of storage in the ground. Snow, ice, and the movement of soil moisture in its vapor state will not be considered.

The volumetric water balance per unit of surface area over time t is given by

$$\int_0^t \{i(t) - e_T(t) - \frac{\partial}{\partial t}[V_{ss}(t) + V_{sg}(t)]\}\, dt$$
$$= \int_0^t [r_s(t) + r_g(t)]\, dt = \int_0^t y(t)\, dt \qquad (1)$$

The dependence of all terms (except $i(t)$ and $V_{ss}(t)$) upon the hard to assess soil moisture level and distribution makes (1) very difficult to evaluate for an arbitrary period of time. To do so, one must develop a soil moisture accounting scheme and then find $y(t)$ through (1) by simulation [*Crawford and Linsley*, 1966]. For a particular time period the result is critically dependent upon the elusive storage terms. To circumvent this problem, we will first assume the system to be stationary in the mean. If one lets the integration interval be a full year and takes the expected value of (1) term by term, the change of storage terms must then vanish and give the average annual water balance equation:

$$E[P_A] - E[E_{T_A}] = E[R_{s_A}] + E[R_{g_A}] = E[Y_A] \qquad (2)$$

where

- P_A annual total precipitation, equal to $\int_0^{1\,yr} i(t)\, dt$;
- E_{T_A} annual total evapotranspiration, equal to $\int_0^{1\,yr} e_T(t)\, dt$;
- R_{s_A} annual total surface runoff, equal to $\int_0^{1\,yr} r_s(t)\, dt$;
- R_{g_A} annual total groundwater runoff, equal to $\int_0^{1\,yr} r_g(t)\, dt$;
- Y_A annual total yield, equal to $\int_0^{1\,yr} y(t)\, dt$;
- $E[\ ]$ expected value of [ ].

Equation (2) is illustrated by the fluxes external to the steady state control volume of Figure 3.

Looking at the instantaneous partition of precipitation as shown in Figure 2*b*, where $f_i(t)$ is the infiltration rate, we can write the auxiliary conservation equation

$$\int_0^t i(t)\, dt - \int_0^t v_{ss}(t)\, dt - \int_0^t f_i(t)\, dt = \int_0^t r_s(t)\, dt \qquad (3)$$

If we introduce the crude approximation that evaporation losses from surface runoff are negligible, then averaging over a

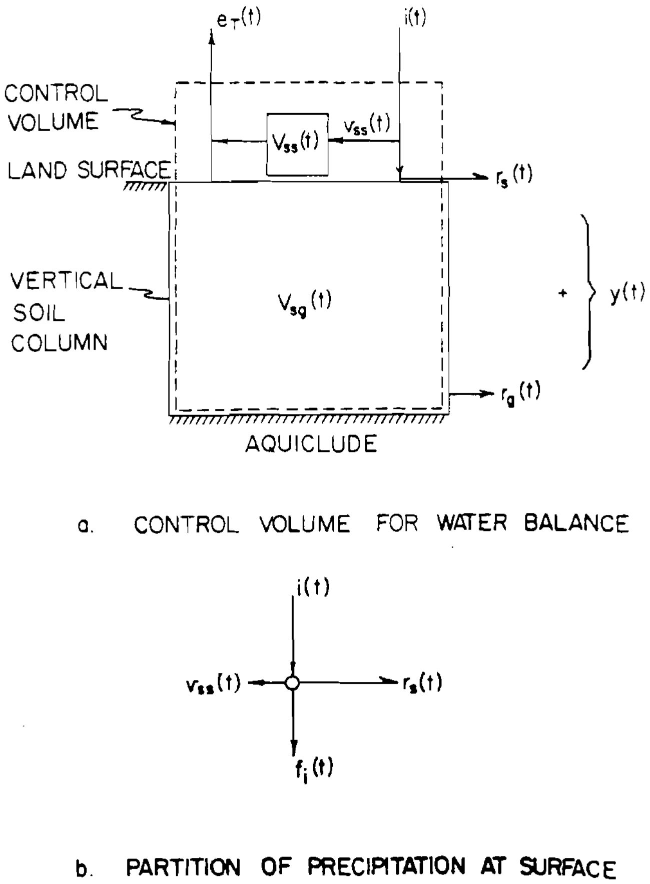

Fig. 2. Instantaneous water balance.

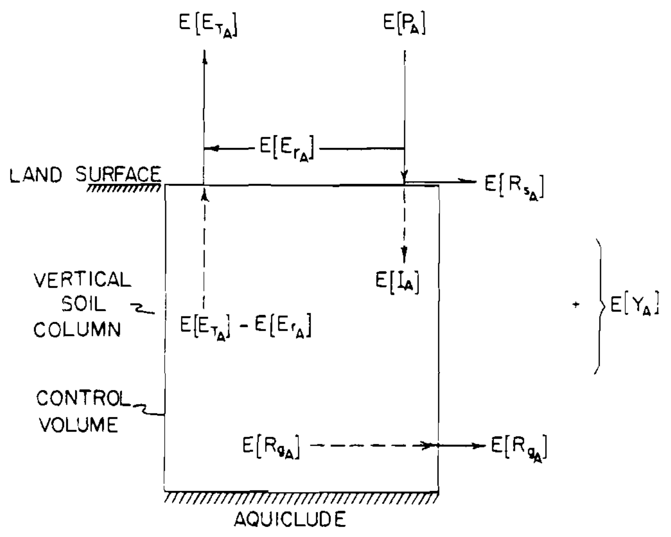

Fig. 3. Average annual water balance.

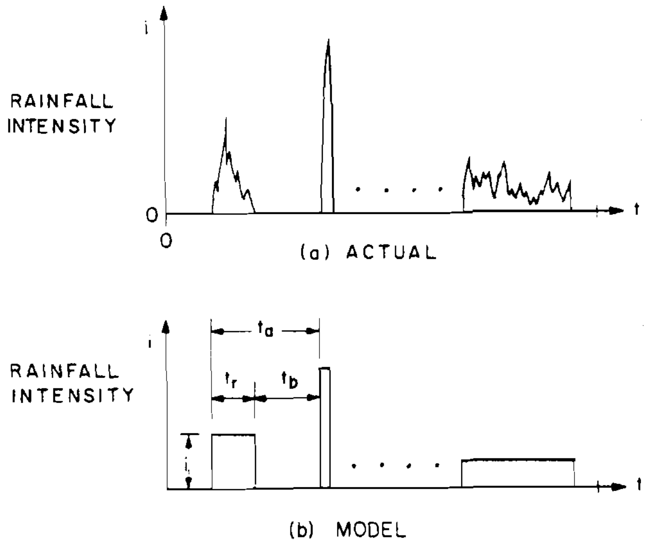

Fig. 5. Model of precipitation event series.

large number of water years gives for a stationary system,

$$E[P_A] - E[I_A] = E[R_{s_A}] + E[E_{r_A}] \equiv E[R_{s_A}{}^*] \qquad (4)$$

where

I_A annual total infiltration, equal to $\int_0^{1\,yr} f_i(t)\,dt$;

E_{r_A} annual total evaporative loss from surface retention, equal to $\int_0^{1\,yr} v_{ss}(t)\,dt$;

$R_{s_A}{}^*$ annual total rainfall excess, equal to $R_{s_A} + E_{r_A}$.

When evaluating this relationship it must be remembered that we are considering processes that operate in the vertical dimension only. Thus evapotranspiration and infiltration are being neglected during lateral movement of the quantity called surface runoff here. The surface retention losses, which are included here, are normally small but may nevertheless be significant in arid climates.

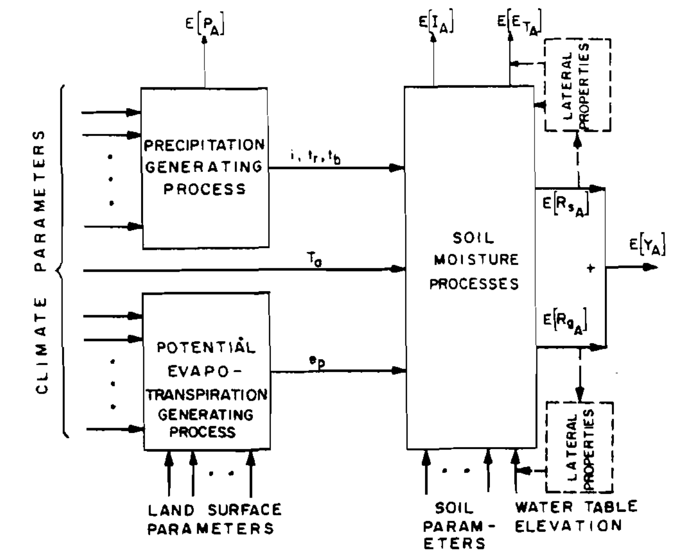

Fig. 4. Computational flow chart for annual water balance.

Subtracting (2) and (4) gives the useful average annual water balance for soil moisture as

$$E[I_A] = E[E_{T_A}] - E[E_{r_A}] + E[R_{g_A}] \qquad (5)$$

which is illustrated by the fluxes internal to the steady state control volume of Figure 3.

System structure. We represent the physical processes by the schematic flow chart shown in Figure 4. In this representation the climate provides a set of independent random variables which are shown as inputs to the soil moisture processes. The output of these processes is the set of average annual water balance components.

In the true three-dimensional system the lateral properties such as surface physiography and medium transmissivity will provide a coupling between the groundwater flow and the water table elevation. A similar feedback results from the evaporation and infiltration of surface runoff as it is conveyed away from its point of generation. These feedback links are shown by the dashed lines in Figure 4 but are not included in the present model.

Precipitation is generated phenomenologically by Poisson arrivals of independent and identically distributed rectangular pulses such as are shown in Figure 5. This model is chosen for the following reasons:

1. It has the ability to represent the distribution of total precipitation in any time period (e.g., a year) in terms of a few parameters of the storm process [*Todorovic*, 1968]. The relatively large number of storms in even a few years of record make estimation of these parameters fairly accurate.

2. It can also represent conveniently the distributions of the two critical time periods, the duration t_r of precipitation during which infiltration and/or surface runoff occurs and the interval between storms t_b during which evapotranspiration occurs. The distribution of storm inter-arrival time t_a of storm depth h and hence that of its average intensity i is also important.

The atmospheric temperature T_a is assumed to have a small coefficient of variation and hence is replaced by its mean (over the rainy season).

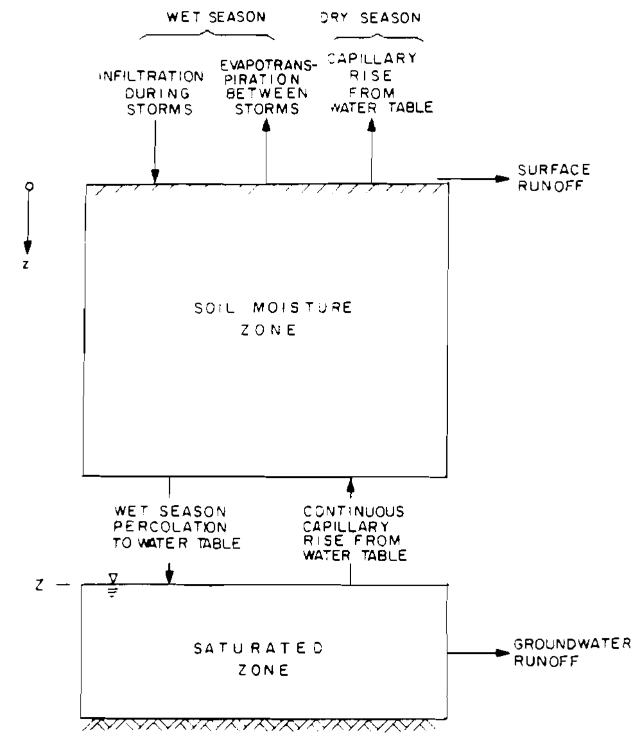

Fig. 6. Schematic representation of soil column.

The potential rate of evaporation e_p provides an upper bound to the actual rate of bare soil evaporation e_T. It also is assumed to have a small coefficient of variation during the rainy season and is thus replaced by its seasonal average value $\bar{e}_p$. This rate, and hence the average annual potential evapotranspiration $E[E_{pA}]$, is calculated from climate and land surface parameters by using the modified Penman equation [*Van Bavel*, 1966].

The soil is assumed to be homogeneous, unsaturated vertical flow of soil moisture being based for the bare soil fraction upon the *Philip* [1969] equation to which a distributed sink has been added to account for moisture extraction by plant roots. The vegetated fraction is modeled after *Van den Honert* [1948], with the added assumption that a vegetal canopy in equilibrium with its environment must operate in the unstressed state on the average.

Actual evapotranspiration is given by the net upward flow to the composite surface as bounded by the potential value. Groundwater is generated by the difference between wet season gravitational percolation to and continuous capillary rise from a fixed water table at elevation Z. These subprocesses are illustrated schematically in Figure 6.

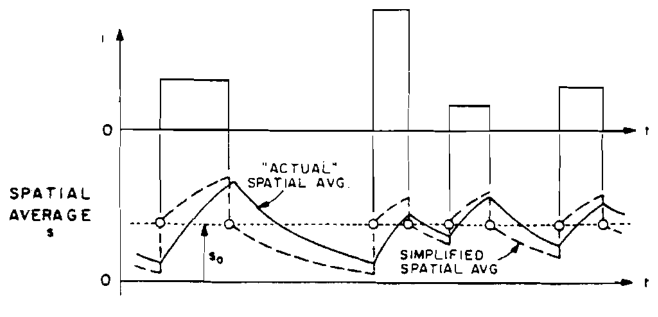

Fig. 7. Soil moisture representation.

When analyzed separately the respective rates of soil moisture movement are superimposed as though the system were linear.

Soil properties. The permeability $k(s)$ and matrix potential $\Psi(s)$ of the homogeneous soil are represented by functions of the following forms:

$$k(s) = k(1)s^c \tag{6}$$

and

$$\Psi(s) = \Psi(1)s^{-1/m} \tag{7}$$

where

- s effective soil moisture concentration (i.e., effective volume of soil moisture divided by effective volume of voids);
- $k(s)$ effective intrinsic permeability, square centimeters;
- $\Psi(s)$ matrix potential, centimeters (suction);
- c pore disconnectedness index;
- m pore size distribution index.

Also $s = 1$ signifies effective saturation. The behavior of soil moisture is described [*Eagleson*, 1978*b*] in terms of three independent soil parameters: n, $k(1)$, and c, where n is the effective porosity (i.e., the effective volume of voids divided by the total volume).

Soil moisture representation. A principal climatic effect upon the water budget results from the random intermittency of the processes by which the soil moisture storage is discharged and recharged through the land surface. The forces causing this are (1) the unidirectional gravitational force and (2) a reversing capillary force. The latter is the result of a space and time variable matrix potential gradient which is itself a function of the soil moisture concentration.

In response to saturation of the surface soil a wave of soil moisture moves downward through the dry soil column. Similarly, following surface drying of the wet soil column, a negative wave of soil moisture propagates downward. This complex wavelike process demands computer-based numerical representation for faithful reproduction of internal soil moisture variations. Such detailed analysis requires, for consistency, equally detailed knowledge of the heterogeneity of the soil properties, information which is seldom available outside the laboratory. Here, where we are attempting to establish aggregate system behavior by a one-dimensional model, it hardly seems appropriate or necessary to include such detail.

If we represent the time and space variable soil moisture by a time-varying spatial average, we might expect the time variations shown in Figure 7 in response to rectangular pulses of precipitation at the surface. This simplified representation highlights the dominant role which soil moisture plays in controlling the water budget. The soil moisture level at the beginning of precipitation sets the 'receptor' matrix potential governing the amount of water which will be infiltrated during the subsequent storm. Similarly, the soil moisture level at the cessation of precipitation sets the rejector matrix potential governing the amount of water which will be exfiltrated and transpired during the subsequent interstorm period. To account for this time variation while a spatial average is used requires bookkeeping of the additions to and subtractions from a storage volume whose lower boundary must be defined. Prescription of this fictitious limit may be avoided by assuming that the initial (spatial average) soil moisture s_0 at the beginning of all precipitation and interstorm periods is a constant value determined by the climate, soil, and vegetation properties. The resulting time variation of s is illustrated qualitatively by the dashed line in Figure 5. The magnitude of s_0 will be set at the time average of the spatial average and will be found by equating the expected annual additions to and subtractions from soil moisture (given by (5)).

It should be noted that this key assumption is really a neglect of the carryover effect of one storm or interstorm episode upon the following episode. This will result in a consistent overestimate of average soil moistures during periods of infiltration and a consistent underestimate of average soil moistures during periods of desorption. The net effect of this approximation will be overestimation of total yield primarily through exaggeration of the surface runoff component.

This assumption establishes the time and spatial average soil moisture s_0 as a primary climate-soil-vegetation variable in terms of which all soil moisture processes will be defined.

OUTLINE OF SOLUTION

The following is an outline of the solution.

1. In terms of the soil and soil moisture models just described the physics of the soil moisture movement processes are used to derive the following equations

Infiltration depth during the jth storm [*Eagleson*, 1978*d*]

$$I_j = g_1[i, t_r; s_0, n, k(1), c, Z] \tag{8}$$

Evapotranspiration depth during the jth interstorm period [*Eagleson*, 1978*c*]

$$E_{T_j} = g_2]t_b, h, e_p; s_0, n, k(1), c, M, k_v, h_0, Z] \tag{9}$$

(Here h is storm depth, and h_0 is surface retention capacity, M is vegetal canopy density, and k_v is plant coefficient potential transpiration efficiency.)

Evaporation from surface retention during the jth interstorm period [*Eagleson*, 1978*c*]

$$E_{r_j} = g_3[h, t_b, M, h_0] \tag{10}$$

Uniform rate of flow to water table [*Eagleson*, 1978*b*]

$$r_g = K(1)s_0^c - w[n, k(1), Z] \tag{11}$$

(Here $K(1)$ is the saturated effective hydraulic conductivity. The first term represents steady gravitational percolation during the wet season only and w represents the steady year-round rate of capillary rise from the water table to a dry surface.)

2. Short-period observations of storm series are used to define the climatic probability density functions for storm intensity i, storm duration t_r, storm depth h, time between storms t_b, and storm interarrival time t_a [*Eagleson*, 1978*a*]. Analytically tractable marginal distributions are fitted to these observations: the gamma distribution for h and exponentials for i, t_r, t_b, and t_a. Independence of t_r and i and of h and t_b is assumed as an analytical expedient.

3. Expected values of I_j, E_{T_j}, and E_{r_j} are derived by using these density functions and (8), (9), and (10).

4. Annual average values of I, E_T, and E_r are found by summing over all events in the year in the following manner:

$$E[E_{r_A}] = E\left[\sum_{j=1}^{\omega\tau} E_{r_j}\right] = E[\omega\tau]E[E_{r_j}] \tag{12}$$

where $\omega^{-1} = E[t_a]$ is the average storm interarrival time and τ is the length of the rainy season.

For infiltration this gives [*Eagleson*, 1978*d*]

$$E[I_A]/E[P_A] = 1 - e^{-G-2\sigma}\Gamma(\sigma + 1)\sigma^{-\sigma} \tag{13}$$

where G is the gravitational infiltration parameter and σ is the capillary infiltration parmeter. Both are functions of s_0, soil and climate parameters.

For soil moisture evaporation we get [*Eagleson*, 1978*c*, *e*]

$$E[E_{T_A}^*]/E[E_{p_A}^*] = J(E, M, k_v) \tag{14}$$

where

- $J(E, M, k_v)$ evapotranspiration function;
- E capillary exfiltration parameter, a function of s_0, soil and climate parameters;
- $E_{T_A}^*$ annual soil moisture evapotranspiration, equal to $E_{T_A} - E_{r_A}$;
- $E_{p_A}^*$ annual potential evapotranspiration from soil moisture, equal to $m_v E[t_b]\bar{e}_p^* - E_{r_A}$;
- $\bar{e}_p^*$ weighted average rate of potential evapotranspiration, equal to $[1 - M + Mk_v]\bar{e}_p$.

The average number of storms per year m_v is given by

$$m_v = E[\omega\tau] = E[P_A]/E[h]$$

For groundwater runoff we average across the variable season length to obtain [*Eagleson*, 1978*e*]

$$E[R_{g_A}] = m_\tau K(1)s_0^c - Tw \tag{15}$$

where m_τ is the mean season length in seconds and T equals 3.15×10^7 s/yr.

5. A natural selection hypothesis is proposed which suggests that in the short term, natural vegetal systems of given k_v reach a 'growth equilibrium' density $M = M_0$ at which the soil moisture is maximum because in this state, stress is minimized. Application of this hypothesis determines M.

An extension of this hypothesis to evolutionary time scales suggests that the species mix (e.g., k_v) will evolve from one growth equilibrium to another, provided nutrition or light is not limiting, unitl an 'evolutionary equilibrium' is reached at which the rate of production of biomass is maximized. This implies maximization of the rate of soil moisture utilization and hence maximization of the product $M_0 k_v$. For water-limited cases this hypothesis determines $k_v = k_{v_0}$.

6. Equation (5) can now be expanded to give the mean annual water balance equation [*Eagleson*, 1978*e*],

$$E[P_A]\{1 - e^{-G-2\sigma}\Gamma(\sigma + 1)\sigma^{-\sigma}\} = E[E_{p_A}^*]J(E, M, k_v) + m_\tau K(1)s_0^c - Tw \tag{16}$$

(The term to the left of the equal sign is infiltration, the first term to the right is soil moisture evapotranspiration, and the last two terms give the groundwater runoff.)

Equation (16) serves to define the dimensionless dependent variable s_0 in terms of a set of 10 independent dimensionless variables which arise from the (simplified) physics of the climate-soil-vegetation coupling and hence form the basis for the dynamic similarity of the average annual water balance [*Eagleson*, 1978*e*].

An asymptotic analysis of this equation [*Eagleson*, 1978*e*] yields expressions for the $E[P_A] \to 0$ and $E[P_A] \to \infty$ asymptotes of $E[E_{T_A}^*]$ and $E[I_A]$, the intersections of which define, in comparison with $E[P_A]$, rational classifications of climate-soil-vegetation systems into arid, semiarid, subhumid, and humid categories.

Equation (16) can be solved (numerically) to find

$$s_0 = s\{E[P_A], E[E_{p_A}^*], m_\tau K(1)\} \tag{17}$$

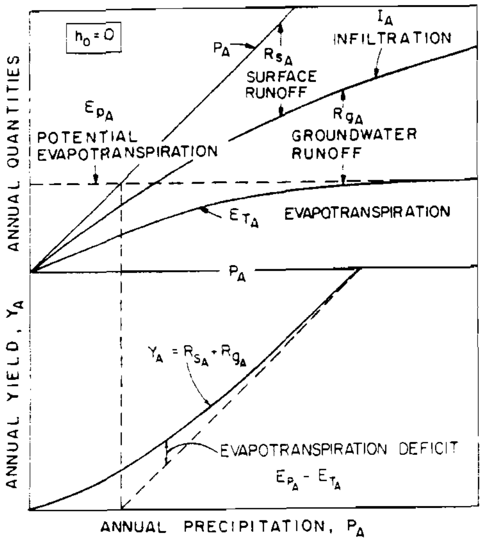

Fig. 8. Climatic influence on the annual water balance (negligible surface retention).

7. The soil moisture relation, (17), is used to eliminate s_0 from the separate terms of the conservation relation, (2), and these long-term averages are used in a first-order analysis [*Benjamin and Cornell*, 1970, p. 180] to define the relation among annual volumes for a particular year. Note that this involves the implicit assumption of no change in water storage over the given water year. This is, of course, only an approximation of reality which is most nearly correct in a humid climate where the coefficients of variation of the climatic random variables are usually quite small. In many climates the coefficient of variation of P_A will far exceed that of E_{P_A} and of τ. In such cases, we leave the latter variables expressed in terms of their means to write the first-order approximation to (2) for a given climate-soil-vegetation system:

$$\begin{aligned} Y_A &\simeq P_A - E_{T_A}(P_A) \\ Y_A &\simeq R_{s_A}(P_A) + R_{g_A}(P_A) \end{aligned} \qquad (18)$$

This allows graphical presentation of the principal climatic effects on the annual water balance as is indicated schematically in Figure 8 for the case of negligible surface retention.

8. The total annual precipitation P_A is represented as the sum of Poisson storm arrivals over a rainy season. The cumulative distribution function (cdf) $F_{P_A}(p)$ is then derived in terms of the climatic parameters of the Poisson storm arrival process and the parameters of the probability density function of storm depth h to give, for constant season length [*Eagleson*, 1978*a*],

$$F_{P_A}(x) = \text{Prob}\left[\frac{P_A}{m_{P_A}} < x\right] = e^{-\omega m_\tau} \cdot \left\{1 + \sum_{\nu=1}^{\infty} \frac{(\omega m_\tau)^\nu}{\nu!} P[\nu\kappa, \omega m_\tau \kappa x]\right\} \qquad (19)$$

where κ is the 'shape' parameter of gamma distribution of storm depths, m_{P_A} is the mean annual (seasonal) precipitation, equal to $E[P_A]$, and $P[a, x]$ is Pearson's incomplete gamma function

$$P[a, x] = \gamma[a, x]/\Gamma(a) \qquad (20)$$

9. Because (18) gives a monotonic relationship between P_A and Y_A, we finally obtain the cumulative distribution function for annual yield $F_{Y_A}(z)$ by direct transformation [*Benjamin and Cornell*, 1970, p. 133]. To do this, we solve (18) for P_A, obtaining (numerically)

$$P_A = g^{-1}(Y_A) \qquad (21)$$

Since

$$F_{Y_A}(z) = F_{P_A}[g^{-1}(z)] \qquad (22)$$

equation (19) becomes [*Eagleson*, 1978*f*]

$$F_{Y_A}(z) = \text{Prob}\left[\frac{Y_A}{m_{P_A}} < z\right] = e^{-\omega m_\tau} \cdot \left\{1 + \sum_{\nu=1}^{\infty} \frac{(\omega m_\tau)^\nu}{\nu!} P[\nu\kappa, \omega m_\tau \kappa g^{-1}(z)]\right\} \qquad (23)$$

The cdf of the separate elements of the annual water balance can be found in a similar fashion.

SUMMARY

The average annual one-dimensional water balance is expressed for natural surfaces in terms of physically significant dimensionless parameters thereby providing the basis for dynamic similarity of the process and for an improved understanding of climate-soil-vegetation coupling.

An asymptotic analysis of the average annual water balance equation provides rational criteria for classification of climate-soil-vegetation systems.

A hypothesis of natural selection suggests equilibrium criteria for the canopy density and plant coefficient of natural vegetal systems.

A first-order analysis of the average annual water balance gives an equation for the annual water balance, which is used to determine the cumulative distribution functions for the components of the annual water balance in terms of observable parameters of the physical system and thus provides the basis for assessing the risk of physical changes and for estimating the recurrence interval of such water balance components as basin yield.

The principal assumptions and simplifications are the following.

1. General
   a. One-dimensional analysis (only vertical processes) is used.
   b. No consideration is given to snow or ice.
   c. All processes are stationary in the long-term average.
2. Precipitation
   a. Storm series is represented by Poisson arrivals of independent and identically distributed rectangular pulses.
   b. Average interstorm period is much greater than average storm duration.
   c. Interstorm period and storm duration are statistically independent.
3. Soil
   a. Soils are homogeneous.
   b. Movement of water vapor is not considered.
   c. Column is effectively semiinfinite as far as surface processes are concerned.
   d. Infiltration, exfiltration, percolation, and capillary rise from water table are formulated separately and their fluxes are linearly superimposed.
   e. Carryover moisture storage (or deficit) from storm to

interstorm period (and vice versa) is neglected with internal moisture at the start of every period being s_0 the space and time average in the surface boundary layer.

4. Vegetation (natural systems only)
   a. Transpiration occurs at the potential rate.
   b. Rate of soil moisture extraction by the root system is a constant throughout the soil volume above the maximum root depth.
   c. Canopy density seeks a short-term equilibrium state at which soil moisture is a maximum.
   d. In water-limited systems, species evolve in the long term toward maximum water use.
5. Infiltration and surface runoff
   a. No surface inflows from outside the region are considered.
   b. Storm intensity and duration are statistically independent.
6. Evapotranspiration
   a. Vegetation transpires at the potential rate.
   b. Potential rate of evaporation averaged over the interstorm period has a negligible coefficient of variation during the rainy season.
7. Percolation to water table
   a. Percolation is steady throughout rainy season at a rate determined by the average soil moisture s_0.
   b. Percolation is zero during dry season.
8. Capillary rise from water table
   a. Potential rate of evaporation is much greater than rate of capillary rise from water table.
   b. Dry surface matrix potential is much greater than saturated matrix potential.
9. Miscellaneous
   a. Water table is constant (no carryover groundwater storage from year-to-year).
   b. Relation among annual water balance components is given to the first order by the relation among the average annual quantities.

Only the outline of this development is presented. Details and verifications are found in a series of subsequent papers.

NOTATION

c pore disconnectedness index.
E exfiltration parameter.
E_{p_A} annual potential evapotranspiration, centimeters.
$E_{p_A}{}^*$ annual potential soil moisture evapotranspiration, centimeters.
E_r storm surface retention, centimeters.
E_{r_A} annual evaporation from surface retention, centimeters.
E_{T_A} annual total evapotranspiration, centimeters.
$E_{T_A}{}^*$ annual soil moisture evapotranspiration, centimeters.
E_{T_i} interstorm evapotranspiration, centimeters.
e_p potential (soil surface) evaporation rate, centimeters per second.
$\bar{e}_p$ time average potential evaporation rate, centimeters per second.
$\bar{e}_p{}^*$ weighted average potential evapotranspiration rate, centimeters per second.
$\bar{e}_{pv}$ time average potential transpiration rate, centimeters per second.
f_i infiltration rate, centimeters per second.
G gravitational infiltration parameter.
h storm depth, centimeters.
h_0 surface retention capacity, centimeters.
I_A annual infiltration, centimeters.
I_j storm infiltration, centimeters.
i precipitation rate, centimeters per second.
k effective intrinsic permeability, square centimeters.
k_v potential transpiration efficiency (i.e., plant coefficient), equal to $\bar{e}_{pv}/\bar{e}_p$.
k_{v_0} evolutionary equilibrium plant coefficient.
K effective hydraulic conductivity, centimeters per second.
m pore size distribution index.
m_H mean storm depth, centimeters.
m_{P_A} average annual precipitation, centimeters.
m_v mean number of storms per year.
m_τ mean length of rainy season, days.
M vegetated surface fraction.
M_0 growth equilibrium vegetated surface fraction.
n medium porosity (i.e., volume of voids divided by total volume.
P_A annual precipitation, centimeters.
R_{g_A} annual groundwater runoff, centimeters.
R_{s_A} annual surface runoff, centimeters.
$R_{s_A}{}^*$ annual rainfall excess, centimeters.
r_g groundwater runoff rate, centimeters per second.
r_s surface runoff rate, centimeters per second.
s degree of medium saturation (i.e., soil moisture concentration), equal to volume of water divided by volume of voids.
s_0 time and spatial average soil moisture concentration in surface boundary layer.
T 1 yr, seconds.
t_a storm interarrival time, days.
t_b time between storms, days.
t_r storm duration, days.
V_{ss} storage on surface, centimeters.
V_{sg} storage in ground, centimeters.
v_{ss} rate of surface storage, centimeters per second.
w upward apparent pore fluid velocity representing capillary rise from the water table, centimeters per second.
x value of dimensionless annual precipitation.
Y_A annual water yield, centimeters.
y yield rate, centimeters per second.
Z depth to water table, centimeters.
z value of dimensionless water balance term.
β reciprocal of average time between storms, days^{-1}.
κ shape parameter of gamma distribution.
σ capillary infiltration parameter.
τ length of rainy season, days.
Ψ soil matrix potential, centimeters (suction).
$\Psi(1)$ saturated soil matrix potential, centimeters (suction).
ω storm arrival rate, days^{-1}.
$E[\ ]$ expected value of [ ].
$F[\ ]$ cumulative distribution function.
$g(\ )$ functional notation.
$J(\ )$ evapotranspiration function.
$P[\ ,\ ]$ Pearson's incomplete gamma function.
$\Gamma(\ )$ gamma function.
$\gamma[a, x]$ incomplete gamma function.

Acknowledgments. This work was performed in part during the 1975–1976 academic year while the author was a visiting associate in the Environmental Quality Laboratory at the California Institute of

Technology (Caltech) on sabbatical leave from MIT. Robert C. Y. Koh at Caltech and Chiang C. Mei at MIT were particularly helpful with some of the analytical questions, while Pedro Restrepo, research assistant at MIT, performed the necessary machine computation. Publication has been made possible by a grant from the Sloan Basic Research Fund of MIT.

REFERENCES

Benjamin, J. R., and C. A. Cornell, *Probability, Statistics and Decision for Civil Engineers*, McGraw-Hill, New York, 1970.

Budyko, M. I., N. A. Efimova, L. I. Zubenok, and L. A. Strokina, The heat balance of the earth's surface, *Izv. Akad. Nauk SSSR, Ser. Geogr.*, no. 1, 6–16, 1962.

Carlson, R. F., and P. Fox, A northern snowmelt-flood frequency model, *Water Resour. Res.*, *12*(4), 786–794, 1976.

Crawford, N. H., and R. K. Linsley, Digital simulation in hydrology: Stanford watershed model IV, *Tech. Rep. 39*, Dep. of Civil Eng., Stanford Univ., Stanford, Calif., 1966.

Eagleson, P. S., Dynamics of flood frequency, *Water Resour. Res.*, *8*(4), 878–898, 1972.

Eagleson, P. S., Climate, soil, and vegetation, 2, The distribution of annual precipitation derived from observed storm sequences, *Water Resour. Res.*, *14*, this issue, 1978*a*.

Eagleson, P. S., Climate, soil, and vegetation, 3, A simplified model of soil moisture movement in the liquid phase, *Water Resour. Res.*, *14*, this issue, 1978*b*.

Eagleson, P. S., Climate, soil, and vegetation, 4, The expected value of annual evapotranspiration, *Water Resour. Res.*, *14*, this issue, 1978*c*.

Eagleson, P. S., Climate, soil, and vegetation, 5, A derived distribution of storm surface runoff, *Water Resour. Res.*, *14*, this issue, 1978*d*.

Eagleson, P. S., Climate, soil, and vegetation, 6, Dynamics of the annual water balance, *Water Resour. Res.*, *14*, this issue, 1978*e*.

Eagleson, P. S., Climate, soil, and vegetation, 7, A derived distribution of annual water yield, *Water Resour. Res.*, *14*, this issue, 1978*f*.

Houghton, H. G., On the annual heat balance of the northern hemisphere, *J. Meteorol.*, *11*(1), 1–9, 1954.

Philip, J. R., Theory of infiltration, in *Advances in Hydroscience*, vol. 5, edited by V. T. Chow, pp. 215–296, Academic, New York, 1969.

Todorovic, P., A mathematical study of precipitation phenomena, *Rep. CER67-68PT65*, Eng. Res. Center, Colo. State Univ., Fort Collins, 1968.

Van Bavel, C. H. M., Potential evaporation: The combination concept and its experimental verification, *Water Resour. Res.*, *2*(3), 455–467, 1966.

Van den Honert, T. H., Water transport in plants as a catenary process, *Discuss. Faraday Soc.*, *3*, 146–153, 1948.

(Received August 19, 1977;
revised December 23, 1977;
accepted February 16, 1978.)