1D and 2D Numerical Models for River Routing

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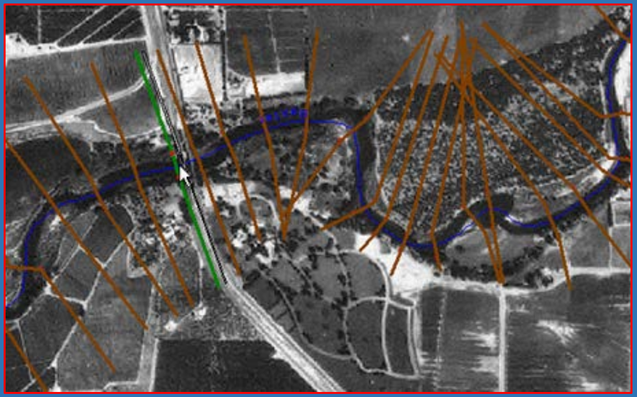
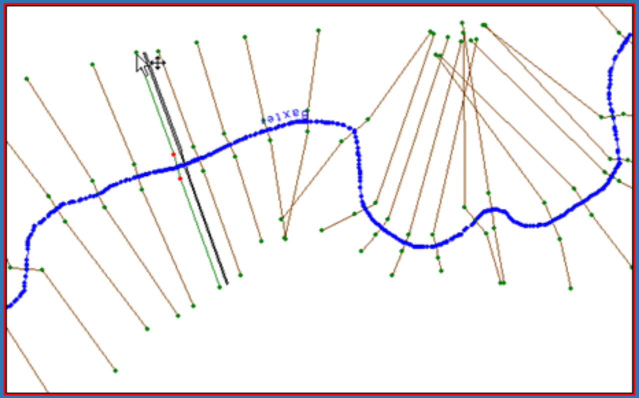
CIVE 717 – River Mechanics – Spring 2022





Motivation for Use of Numerical Models

Design along river corridors includes analysis of complex problems such as *floodwave propagation, floodplain inundation, backwater effects,* and others.

Numerical models are built to analyze space & time solutions for the differential equations governing hydrodynamic processes.

Models can address one or more of the following principles with relatively limited user input:

- Conservation of Mass
- Conservation of Momentum
- Resistance to Flow
- Sediment Transport

Mathematical Foundations

Numerical river models solve approximations to the fundamental **conservation of mass and momentum equations** for incompressible flows.

Conservation of mass:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

For depth-integrated 2-D rivers, the above equation becomes:

$$\frac{\partial h}{\partial t} + \frac{\partial hV_x}{\partial x} + \frac{\partial hV_y}{\partial y} + i_b - i = 0$$

Mathematical Foundations

Conservation of Momentum:

The depth-integrated 2D momentum equation in the x-direction is given by:

$$\frac{\partial V_x}{\partial t} + \frac{V_x \partial V_x}{\partial x} + \frac{V_y \partial V_x}{\partial y} = g\overline{S_{0_x}} + gS_{0_x} - \frac{g\partial h}{\partial x} - \frac{\tau_{0_x}}{\rho h}$$

Here, the sum of forces acting upon the flow are shown on the RHS of the equation. These forces are gravity, bed elevation, pressure, and bed shear; respectively.

Mathematical Foundations

Resistance to Flow:

Due to the highly complex and heterogeneous nature of actual riverine systems, empirical equations for estimating the bed shear force in the momentum equation are often incorporated in to numerical models. One such equation is the Manning-Strickler relationship:

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_f^{\frac{1}{2}}$$

Sediment transport: In addition to computing hydrodynamic properties of the flow 1D and 2D numerical river models are also able to couple sediment mass transport equations, such as the well-known MPM equation:

$$q_{bv} = 8(\tau_* - \tau_{*c})^{1.5}[(G-1)gd_s^3]^{0.5}$$

Implicit Assumptions

Common assumptions made for 1D and 2D numerical river models:

- flow is steady and incompressible.
- channel geometry is wide and rectangular.
- Acceleration in the z-direction is negligible.
- Pressure distribution in the vertical is hydrostatic.
- Boussinesq approximation ($\beta = 1$)

Simplified equations for 1-D numerical models:

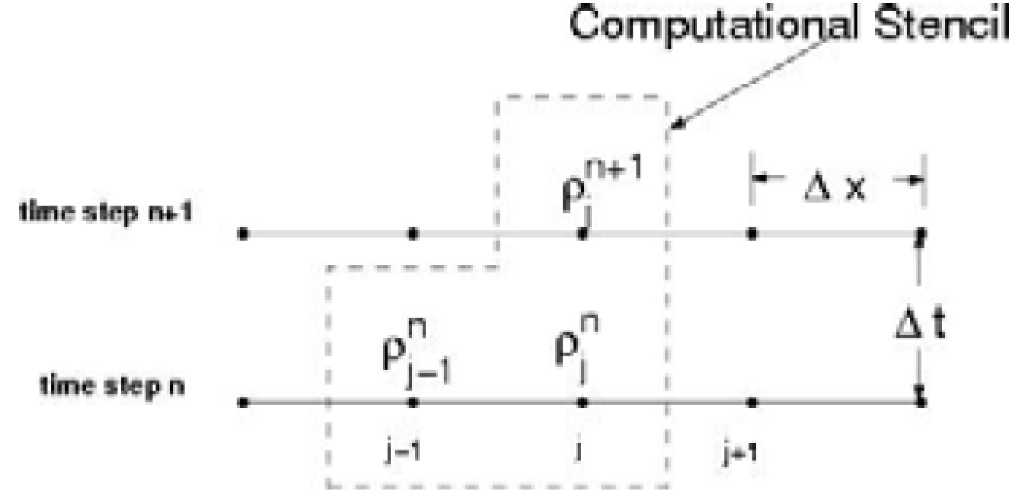
Continuity: $\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$ (neglects rainfall, lateral flow, and flow through channel boundary.)

Momentum (Saint-Venant Eq.): $\frac{\partial v_x}{\partial t} + \frac{v_x \partial v_x}{\partial x} \approx g S_0 - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z}$ (applicable to wide and rectangular channels, neglects lateral velocity).

Numerical Methods

The basic numerical methods for approximating the aforementioned differential equations are finite differencing (for raster-based data sets) and finite-element analysis (for vector-based data).

Both of these methods require **discretization** of the model domain in space and time. These considerations must be made carefully, in the interest of preserving the **numerical stability** of the model.



Example of computational domain for finite-differencing technique.

Model Selection Based on Application

Three general types of numerical models

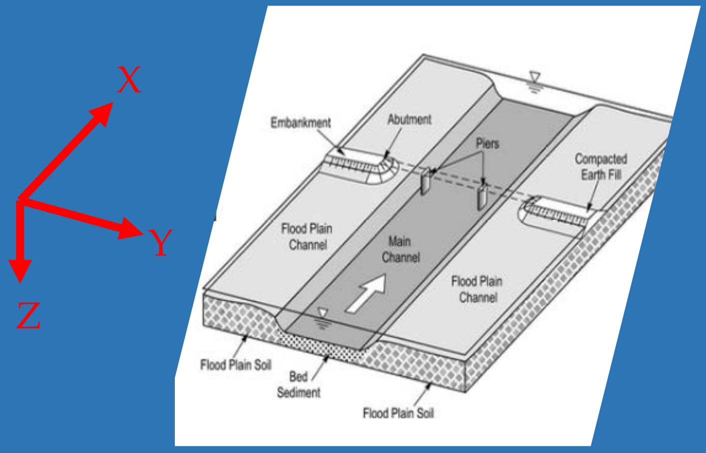
- 1 Dimensional (1D) Model
- 2 Dimensional (2D) Model
- 3 Dimensional (3D) Model

1D & 2D Models are the most common and computationally viable. They are the focus of this presentation.

Flow Direction Analysis by Model Type:

- 1D – X direction flow
  - Assumes constant y & z
- 2D – X & Y direction flow
  - Assumes constant z
- 3D – X, Y, & Z direction flow

Standard Riverine Coordinate Convention



Model Selection Based on Application

2D models have an additional degree of freedom of movement, which adds greater complexity over 1D

- 2D models add additional boundary conditions which are the means of addressing the increased complexity

Modelers choose 1D vs. 2D based on

- Available Information
- Minimum complexity required to adequately answer the problem

Common Model Applications:

1D Model

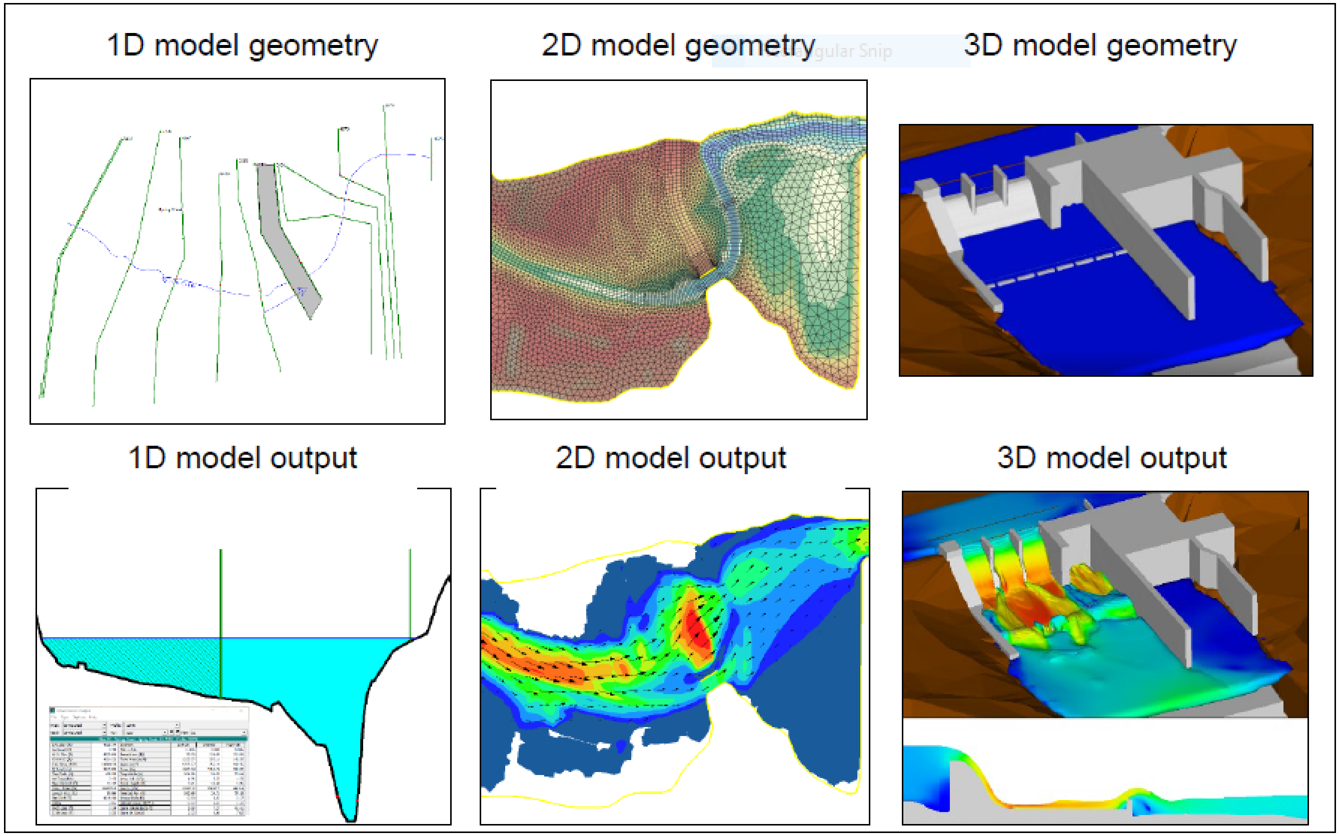
- Steady Flow Water Surface Profile
- Unsteady Flow Simulations
- Movable-Boundary Sediment Transport Computations
- Water Quality Analysis

2D Model

- Meandering or Braided Systems
- Channel Migration
- Flow at Abrupt Bends
- Bridge Scour
- Floodplain Mapping

Model Selection Based on Application

The image below exhibits the difference in Geometry Input & Model Output for 1D, 2D, and 3D Models: (Note the increasing complexity)


1D model geometry

2D model geometry

3D model geometry

1D model output

2D model output

3D model output

Figure 1.2. Example 1D, 2D, and 3D model geometry and output.

Image: (Robinson, Zundel et. al, 2019)

Example 1D Model: HEC-RAS 1D

Example Inputs:

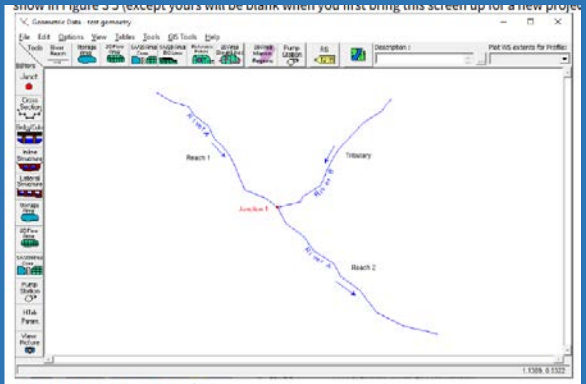


Figure 3 5 Geometric Data Window

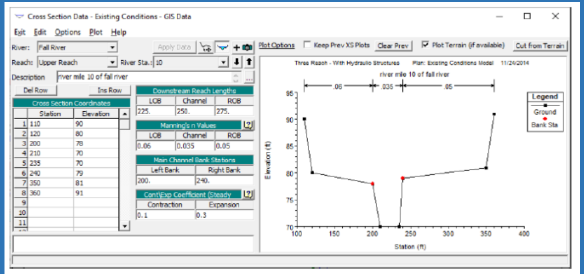


Figure 36. Cross Section Data Editor

A 1D Numerical Flow Model developed by the US Army Corps of Engineers.

- Version 1.0 was released in July 1995
- Version 6.2 was released May 2022
- Capabilities:
  - 1D Steady Flow Calculation
  - 1D Unsteady Flow Calculation
  - Sediment Transport / Mobile Bed Computation
  - Water Temperature / Water Quality Modeling

Example Outputs:

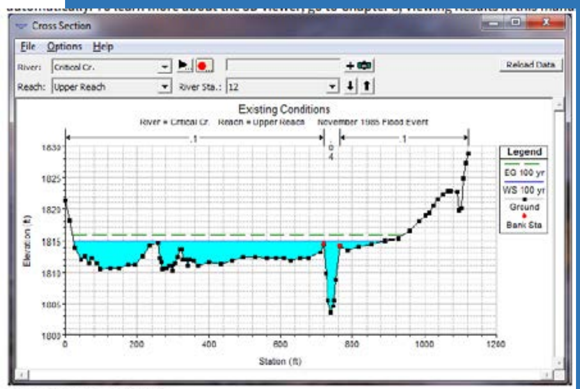


Figure 39. Cross Section Plot

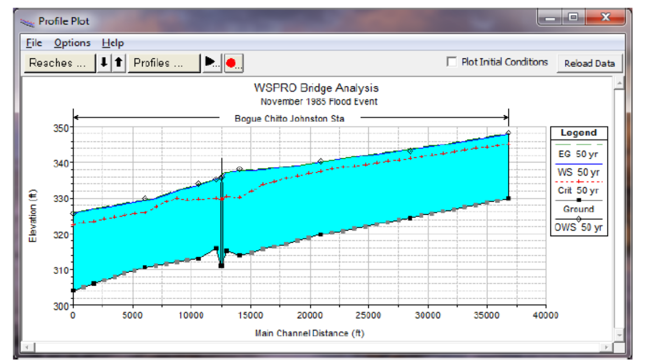
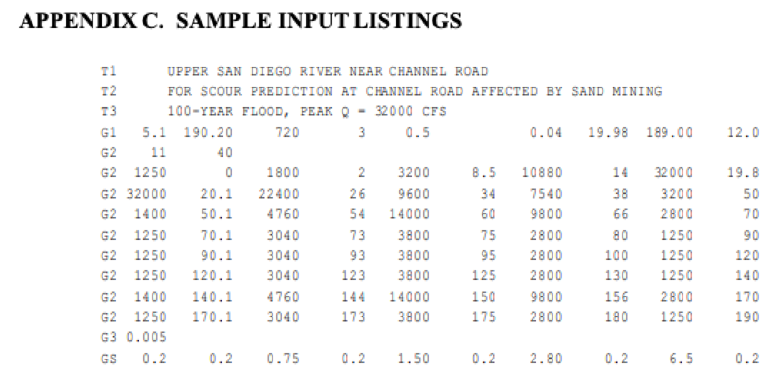


Figure 310. Profile Plot

Images:
(HEC-RAS User's Manual, 2022)

Example 1D Model: FLUVIAL 12

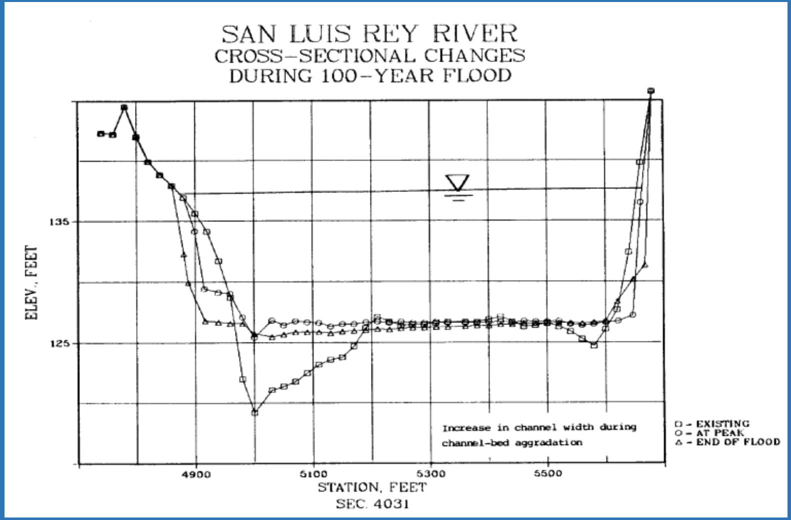
Example Inputs:



APPENDIX C. SAMPLE INPUT LISTINGS

```
T1      UPPER SAN DIEGO RIVER NEAR CHANNEL ROAD
T2      FOR SCOUR PREDICTION AT CHANNEL ROAD AFFECTED BY SAND MINING
T3      100-YEAR FLOOD, PEAK Q = 32000 CFS
G1   5.1  190.20     720       3     0.5            0.04   19.98  189.00    12.0
G2    11      40
G2  1250       0    1800       2    3200     8.5   10880      14   32000    19.8
G2 32000    20.1   22400      26    9600      34    7540      38    3200      50
G2  1400    50.1    4760      54   14000      60    9800      66    2800      70
G2  1250    70.1    3040      73    3800      75    2800      80    1250      90
G2  1250    90.1    3040      93    3800      95    2800     100    1250     120
G2  1250   120.1    3040     123    3800     125    2800     130    1250     140
G2  1400   140.1    4760     144   14000     150    9800     156    2800     170
G2  1250   170.1    3040     173    3800     175    2800     180    1250     190
G3 0.005
GS   0.2     0.2    0.75     0.2    1.50     0.2    2.80     0.2     6.5     0.2
```

Example Outputs:



SAN LUIS REY RIVER
CROSS-SECTIONAL CHANGES
DURING 100-YEAR FLOOD

Increase in channel width during channel-bed aggradation

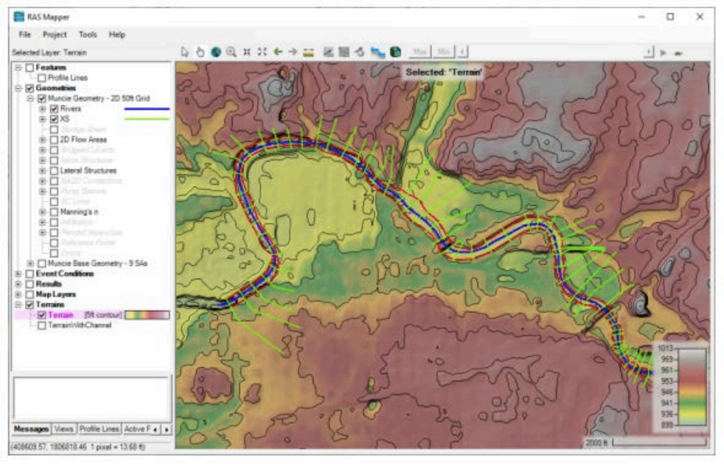
1D Erodible Boundary Numerical Flow Model, which couples flow and sediment transport information to model process-response behaviors in alluvial channels

- Developed by Howard Chang in June 1998
- Capabilities:
  - Erodible Boundary allows changes in channel width, while mobile bed models do not
  - Channel bed scour & fill
  - Channel width variation
  - Impact of channel curvature
  - Channel response to large changes (including gravel mining, channelization, hydraulic structures & others)
  - General scour at bridge crossings

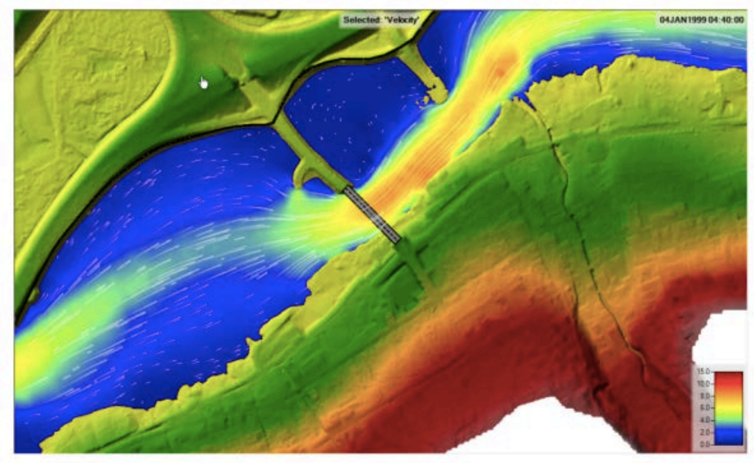
Images: (Chang, 1998)

Example 2D Model: HEC-RAS 2D

Example Inputs:



Example Outputs:



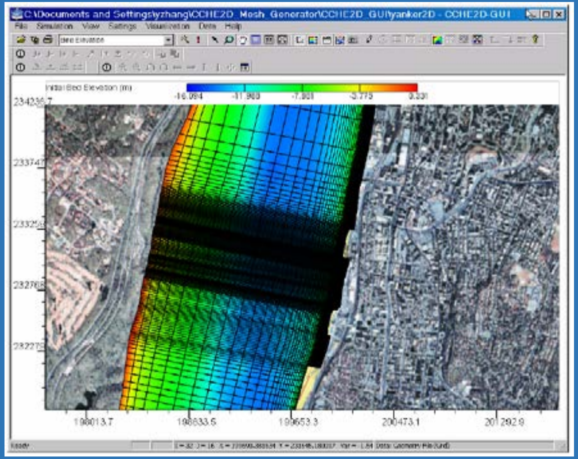
This is an addition to the US Army Corps of Engineers' HEC-RAS program with 2D flow modeling capabilities.

- Version 5.0 was the first with 2D flow capabilities, released in March 2016
- Version 6.2 was released May 2022
- Capabilities:
  - 2D or Combined 1D/2D Steady & Unsteady Flow Modeling
  - Shallow Water Equations or Diffusive Wave Equations in 2D
  - Hydraulic Structures Analysis within 2D Flow Areas
  - Detailed Flood Mapping & Flood Animations
  - Implicit Finite Volume Solution Algorithm (for greater numerical stability & larger time steps.

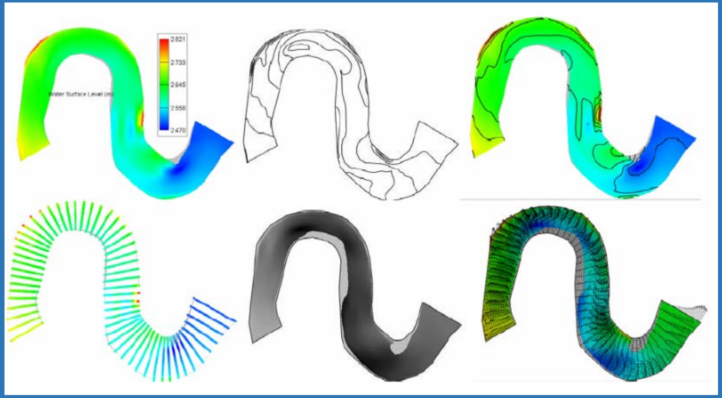
Images: (HEC-RAS 2D User's Manual, 2022)

Example 2D Model: CCHE 2D

Example Inputs:



Example Outputs:



Developed by the National Center for Computational Hydroscience and Engineering at the University of Mississippi.

- Supported by funding from the USDA since 1989.
- Began as simple 1D model, have since added numerous computational for modeling sediment transport, coastal processes, floodplain interactions, and water quality analysis.
- Capabilities:
  - Free-surface flows
  - Sediment transport
  - Morphological processes
  - Pollutant transport and water quality
  - Mesh generation

Images: (CCHE 2D User's Manual, 2002)

Conclusions

- With increasing availability of computational power and rapid model development, numerical river models are increasingly powerful tools for analyzing hydraulic systems.
- However, a lack of knowledge of the underlying assumptions implicit in numerical models can lead to gross misuse in the pursuit of impressive "CFD" (colorful fluid dynamics)!
- Although numerical modeling for river systems has become widespread, trustworthy field data is still required to validate and calibrate our models.

References

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