Case Study: Equivalent Widths of the Middle Rio Grande, New Mexico

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Abstract: Successive reaches of the Rio Grande have maintained equivalent channel widths of 50 and 250 m, respectively, over long periods of time. It is hypothesized that alluvial channels adjust bed slope to match the long-term changes in channel width. Analytical relationships show that wider river reaches develop steeper slopes. A modeling approach using daily water and sediment discharges simulates the transient evolution of bed elevation changes. The analytical and numerical models are in very good agreement with the longitudinal profile measurements of the Bosque del Apache reach of the Rio Grande, NM, from 1992 to 1999. The slope of the 50 m wide reach was 50 cm/km and the slope of the 250 m wide reach of the same river increased to 80 cm/km. This unsteady daily transient model compares well with a steady transient solution at a constant discharge close to the mean annual flow. The transient slope adjustments can also be approximated with an exponential model. Accordingly, it takes about 20–25 years for the Rio Grande to achieve about 90% of its slope adjustment.

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CE Database subject headings: Streams; Hydraulics; Geometry; Channels; Numerical models; New Mexico; Width.

Introduction

For several decades, methods have been developed to predict the equilibrium downstream hydraulic geometry of alluvial channels. Formulations typically relate a dominant discharge, sediment discharge, sediment size, flow velocity, bankfull width, flow depth, and bed slope. Usually, three out of the seven variables are specified and the remaining variables are solved by means of three basic equations: a water continuity equation, a flow resistance equation, and a sediment transport equation. The downstream hydraulic geometry is typically defined in terms of width, depth, velocity, and slope under steady water and sediment discharges at a given sediment size. For instance, Griffiths (1989) used this approach to design a stable single thread channel equivalent in water and bedload discharge to a given braided gravel-bed river. Water discharge and sediment size were always assumed to be known in the single-thread channel together with at least two of the following variables: width, depth, bed slope, gravel load, and resistance coefficient. The results indicate that the single-thread channel had a flatter slope than the wider braided reach to convey the same water discharge and gravel load. This trend was supported by the laboratory data from Ashmore (1985, 1988). Griffiths (1989) also developed an approach for unsteady flow using a flow discharge and sediment-rating curve method.

Other researchers have introduced a fourth condition (bank stability, extremal hypotheses, etc.) in an attempt to explain the self-adjustment mechanism of alluvial channels. The purpose is generally to define unique equilibrium channel geometry out of the family of solutions (Kirkby 1977; Chang 1979, 1980; Carson and Griffiths 1987; Yang 1988; Stevens and Yang 1989; Huang and Nanson 2000; Yu and Smart 2003; Singh et al. 2003). For instance, Huang and Nanson (2000) reduced the number of dependent variables to three (width to depth ratio, slope, and velocity) and used three basic equations (water continuity, Lacey's resistance equation, and Duboys' sediment transport equation) to propose an analytical solution that relates the sediment discharge to the width-depth ratio, slope, water discharge, sediment size, and critical shear stress. Julien and Wargadalam (1995) also successfully replaced the sediment transport equations with the Shields parameter. Accordingly, the channel width, flow depth, mean velocity, and Shields parameter are then defined as a power function of bankfull discharge, sediment size and channel bed slope (Julien 2002). A regression analysis by Lee and Julien (2006) also used a large data set and obtained similar exponents for discharge, slope, and median grain diameter to define the downstream hydraulic geometry of alluvial channels in terms of channel width, mean flow depth, mean velocity, and Shields parameter.

Rivers adjust vertically and horizontally to balance water and sediment discharges (Surian and Rinaldi 2003; Rinaldi 2003). It is a common practice in river engineering to constrain the width of rivers to protect adjacent lands from flooding. In order to sustain an imposed channel width for a long period of time (e.g., decades), the banks of the channels must be able to withstand the forces generated under the natural variability of flow discharges. Current methods are generally limited to steady-state input variables to define equilibrium conditions. Existing methods are also deficient to represent the transient behavior of rivers under variable water discharge and sediment load. There is a need to develop analytical and numerical methods to replicate the transient

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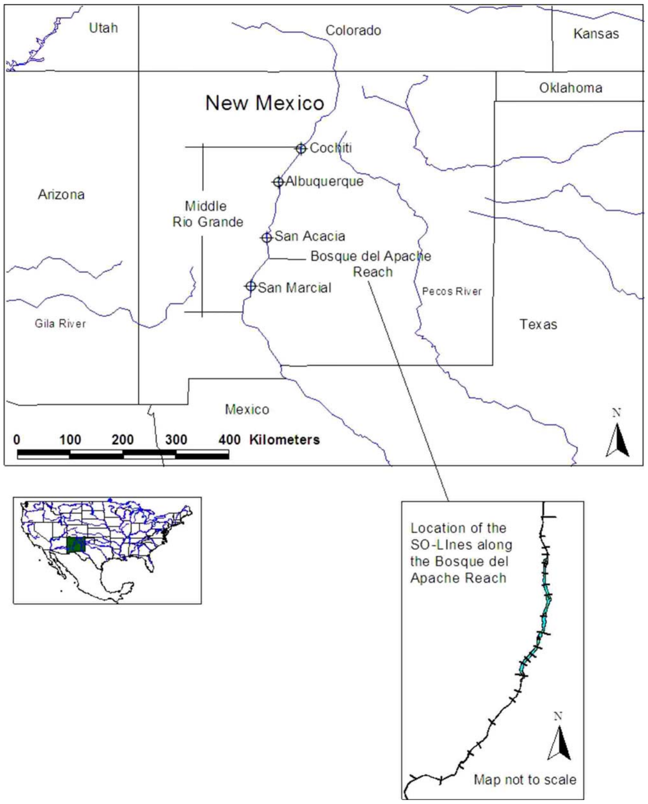

Fig. 1. Location map of Middle Rio Grande, N.M. Bosque del Apache reach and SO lines are indicated.

changes in reach averaged slopes due for channels with different channel widths under variable and constant water discharge and sediment load. It is also important to validate model results with quantitative field data of hydraulic geometry of alluvial channels over long periods of time.

The hypothesis to be developed further in this paper is that a single channel with different channel widths maintained for a long period of time will adjust vertically to reach different equilibrium bed slopes. Accordingly, the term equivalent width refers to channels expected to adjust their bed slope to accommodate the water and sediment input through successive river reaches with different widths.

This paper develops analytical and numerical approaches to represent the transient behavior of rivers under variable and constant water discharge and sediment load. Quantitative field data of hydraulic geometry of alluvial channels over long periods of time are used to validate the model results.

The Middle Rio Grande (MRG), New Mexico, shown in Fig. 1, has been managed to protect irrigated lands and urban areas from flooding. Managing the vegetation has controlled the width of the Bosque del Apache Reach, located in the southern portion of the MRG. The channel has developed into a sequence of narrow and wide reaches where the channel widths have been maintained almost constant for a recent 20 year period. The presence of successive wide and narrow reaches provides an excellent opportunity to test our hypothesis and examine the adjustment of reach averaged bed slope (vertical changes) due to spatial changes in channel width (horizontal changes) under equivalent water and sediment discharges.

The purpose of this paper is to examine the bed adjustments in successive channel reaches with different widths under steady and unsteady water and sediment discharges. The specific objectives are to: (1) determine the long-term equilibrium slope as a function of equivalent channel width, or width-depth ratio, for sand bed alluvial channels; (2) test the analytical solution, apply a numerical model to the Bosque del Apache reach of the Rio Grande, NM, and compare the steady-state results with field measurements; (3) use a one-dimensional numerical model to simulate and compare transient changes in reach averaged slope under unsteady flows from 1992 to 1999; and (4) estimate the time scale of channel slope adjustments to equivalent channel widths from a simple transient model.

Analytical Development

Long-Term Equilibrium Slope

This engineering study is framed in the time span of 10–20 years, in which rivers adjust their shape, form, and gradient to accommodate the water and sediment discharge imposed upon them. The water and sediment loads reflect changes in climate, geology, vegetation, soil, and anthropogenic conditions and as such they define the two most important independent variables controlling river morphology. For the purpose of this paper, a channel is considered in equilibrium when it develops dynamic characteristics (gradient, shape, pattern, and dimensions) in which the rates of transport and sediment supply are equal.

Although natural rivers are characterized by unsteady nonuniform flows, steady and uniform state mathematical models have been previously used to describe the state of equilibrium in rivers. A steady-state model is examined for a first approximation of the hydraulic characteristics of a river reach in equilibrium. Uniform hydraulic conditions are assumed to develop in a long river reach in equilibrium when a constant channel width has been maintained for long periods of time and channel slope has adjusted to transport the incoming water and sediment supply. Over a long period of time, a constant water and sediment discharge is expected to produce fixed channel dimensions and bed slope.

The problem to be specifically solved consists of estimating the long-term equilibrium channel slopes of two adjacent channel reaches with different widths, both capable of transporting the same sediment load and water discharge. As sketched in Fig. 2, the upstream width W_1 of a rectangular channel of slope S is suddenly increased to W_2. If the channel widths are held constant over long periods of time, the slopes of both subreaches are expected to gradually adjust to accommodate the water and sediment discharge until equilibrium is reached ($Q_{s1}=Q_{s2}$).

The following assumptions are made to solve this problem: (1) both reaches are straight and have rectangular cross sections; (2) there is uniform sand size d_s with the same friction factor (Manning n) for both reaches; and (3) water discharge Q and sediment discharge Q_s are constant for steady-state equilibrium.

The following three equations are used to solve the steady-flow condition: (1) $Q=VA$, where Q=water discharge; V=mean flow velocity; and A=cross-sectional area; (2) Manning flow resistance equation $V=\varphi R_h^{2/3}S^{1/2}/n$, where $\varphi=1.49$ for English units and 1 for metric units; n=Manning friction factor; R_h=hydraulic radius; and S=friction slope; and (3) Julien's (2002) simplified sediment transport equation $Q_s=18W\sqrt{g}d_s^{3/2}\tau_*^2$, where Q_s =sediment discharge by volume; W=channel width; g =gravitational acceleration; d_s=particle size; and τ_*=Shields parameter defined as $\tau_*=R_hS/[(G-1)d_s]$ where G=specific gravity

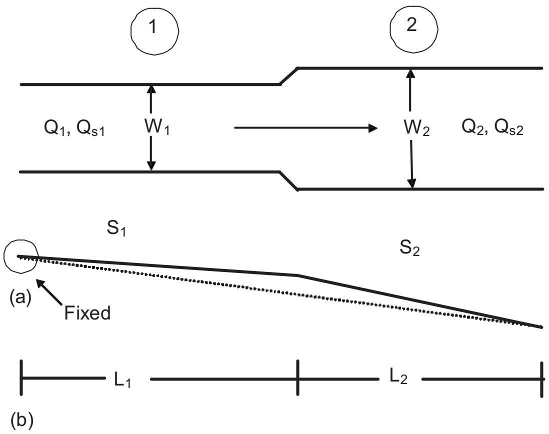

Fig. 2. (a) Plan view of channel; (b) longitudinal profile. Dotted line represents reach-average bed slope. Solid line represents slope of each reach: W=width; S=slope; L=reach length; Q=water discharge; and Q_s=sediment discharge.

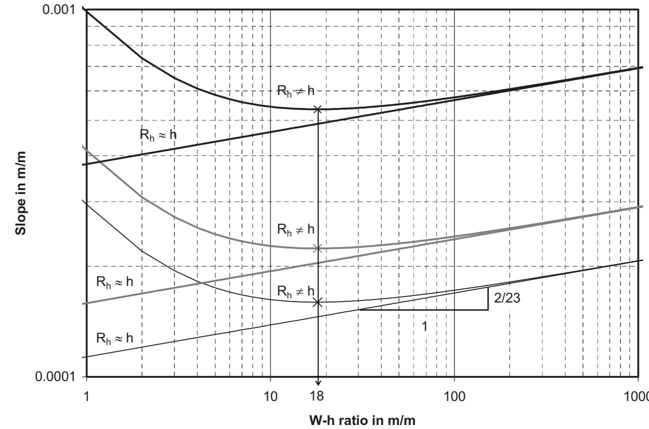

Fig. 3. Family of curves representing slope versus width-depth ratio relationships for three different input data sets. Minimum width-depth ratio is always 18.

of sediment. Julien's simplified sediment transport equation was developed for $0.1 < \tau_* < 1.0$. Note that other sediment transport methods can be used as well, but are not prone to simple analytical solutions.

A simplified analytical solution can be found if the hydraulic radius is approximated to the flow depth ($R_h \approx h$). Under sediment transport equilibrium the following conditions are met: $Q_1 = Q_2$ and $Q_{s1} = Q_{s2}$. Replacing the resistance equation with the discharge equation and approximating the R_h to the flow depth h gives

$$Q = \varphi h^{2/3} S^{1/2} h W/n = \varphi h^{5/3} S^{1/2} W/n \tag{1}$$

For constant Manning n, the ratio of Q_1 and Q_2 is a function of the width, depth, and slope ratios denoted with subscript r

$$Q_2/Q_1 = 1 = W_2 h_2^{5/3} S_2^{1/2} / W_1 h_1^{5/3} S_1^{1/2} = W_r h_r^{5/3} S_r^{1/2} \tag{2}$$

Solving for S yields the first important relationship

$$S_r = 1/(W_r^2 h_r^{10/3}) \tag{3}$$

where W_r=ratio of the widths; h_r=ratio of the flow depths; and S_r=ratio of the slopes.

Repeating the procedure for sediment load with $R_h \approx h$, a fixed grain size d_s and specific gravity of sediment G gives the ratio of Q_{s2} and Q_{s1} as

$$Q_{s2}/Q_{s1} = 1 = (W_2 h_2^2 S_2^2)/(W_1 h_1^2 S_1^2) = W_r h_r^2 S_r^2 \tag{4}$$

Solving S_r gives the second important relationship

$$S_r = 1/(W_r^{1/2} h_r) \tag{5}$$

Combining Eqs. (3) and (5) to eliminate one more variable (h_r or W_r) results in the following approximate relationships for slope ratio as a function of width and depth ratios for a constant supply of water and sediment:

$$S_r = W_r^{1/7} \tag{6}$$

$$S_r = h_r^{-2/9} \tag{7}$$

Alternatively, Eqs. (3) and (5) can be solved as a function of the width-depth ratio $\xi_r = W_r/h_r$, where one obtains

$$S_r = (W_r/h_r)^{2/23} = \xi_r^{2/23} \tag{8}$$

Consequently, Eqs. (6) and (8) indicate that an increase in channel width, or width-depth ratio, requires an increase in channel slope to satisfy the continuity of sediment transport. Fig. 3 shows a typical example using Eq. (8).

A more complex formulation is obtained when the hydraulic radius is not approximated to the flow depth ($R_h \neq h$). It can be demonstrated (Leon 2003) that

$$S = [C_v^{16/23}(G-1)^{32/33} d_s^{8/23} \varphi^{18/23} \times (\xi+2)^{20/23}]/(Q^{2/23} 18^{16/23} g^{8/23} n^{18/23} \xi^{18/23}) \tag{9}$$

where C_v=volumetric sediment concentration.

For large width-depth ratios ($\xi \to \infty$), the hydraulic radius R_h approximates the flow depth h and Eq. (9) reduces to

$$\lim_{\xi\to\infty} S = [C_v^{16/23}(G-1)^{32/33} d_s^{8/23} \varphi^{18/23} \times (\xi)^{2/23}]/(Q^{2/23} 18^{16/23} g^{8/23} n^{18/23}) \tag{10}$$

This equation simply reduces to Eq. (8) for constant C_v, G, d_s, φ, g, and n, and the slope is a function of the width-depth ratio to the power of 2/23. Fig. 3 compares Eqs. (9) and (10) for three different sets of flow and sediment discharges, and shows that the minima [in Eq. (9)] are caused by the difference between the flow depth and the hydraulic radius. For all input data sets, both equations yield similar solutions for width-depth ratios greater than ~70.

The minimum of the curve for the case when $R_h \neq h$ can be calculated analytically by taking the derivative of Eq. (9) with respect to ξ and equating it to zero. It can be demonstrated that the minimum slope occurs at a width-depth ratio $\xi = 18$ as shown in Fig. 3. This minimum slope is obtained without any extremal hypothesis. Accordingly, the equilibrium condition will always occur at a width-depth ratio of 18 when using Manning's resistance equation and Julien's (2002) simplified sediment transport equation. Similar results were also obtained from different sediment transport relationships (Leon 2003).

Transient Model

A one-dimensional transient model was used to simulate the evolution of channel bed elevation for two sequential channel reaches with different widths. The long-term changes in channel slope are investigated for constant flow and sediment discharges, and also for variable daily discharges.

The one-dimensional numerical model computes bed aggradation and degradation processes driven by steady or unsteady water discharge Q on a daily basis is similar to HEC-6 and is given only a very brief description here. The model consists of fully uncoupled hydraulic and sediment components solved by an explicit finite-difference scheme, forward in time and backward in space (FTBS) (Hoffmann and Chiang 2000). The backwater profile for steady one-dimensional gradually varied flow is calculated with the following equation of motion (Chow 1959; Henderson 1966):

$$dh/dx = (S_0 - S_f)/(1 - \mathsf{F}^2) \tag{11}$$

where S_0=bed slope; S_f=friction slope; F=Froude number; h=flow depth; and x=downstream distance along the channel. Changes in channel bed elevation z_b were computed with the equation of conservation of sediment mass without sediment source (Vanoni 1977; Julien 1998) given by

$$(\partial z_b/\partial t) + \partial Q_s/[W(1 - p_o)\partial x] = 0 \tag{12}$$

where z_b=bed surface elevation; W=width of the channel; p_o=porosity of the sediment; and Q_s=volumetric sediment discharge.

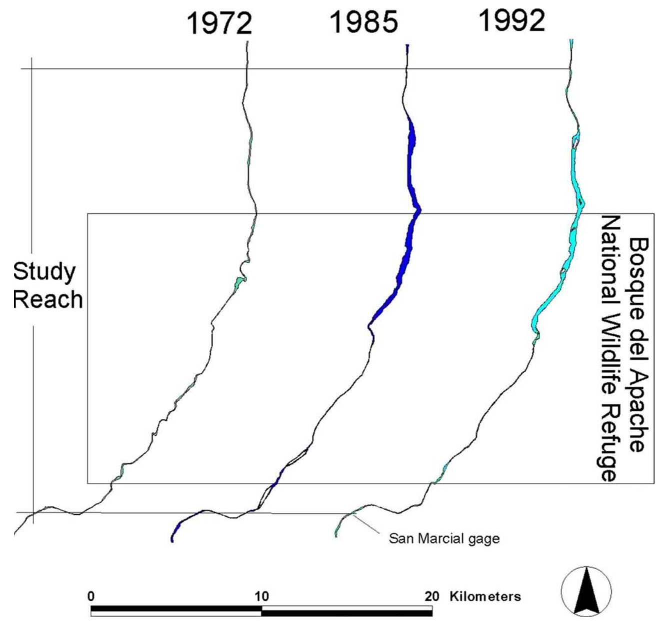

Fig. 4. Digitized planform of nonvegetated channel from 1972 to 1992

Field Application to Middle Rio Grande, New Mexico

Field applications are presented for the Bosque del Apache reach of the Middle Rio Grande, N.M. Field measurements consist of geometry, hydraulic, and sediment data collected from 1992 to 1999.

Description of Reach

The study reach starts about 47 km downstream of the San Acacia diversion dam (Fig. 1). This study reach comprises the Bosque del Apache National Wildlife Refuge and consists of three reaches with one wide section located between two narrow, straight reaches. This reach was selected because its width has remained almost constant between 1985 and 2000. In addition, the river has not migrated across the floodplain during that time period. Digitized aerial photos of the active nonvegetated channel show that this reach narrowed from 1918 to 1972. Channel modifications took place in this reach between 1949 and 1972 (U.S. Bureau of Reclamation 2000). Accordingly, much of the narrowing, during this period, is due to human intervention. The wide reach observed in the planform plots after 1972 is the result of river bank vegetation clearing. The width of the channel corresponds to the channel clearing width and has remained the same between 1985 and 2000 (Fig. 4).

Geometry Data

The geometry data input to the model consists of: (1) the thalweg elevations and channel widths at 23 cross sections; (2) historical thalweg elevations of the first upstream cross section from 1992 to 1999; and (3) maximum erodible bed material thickness. Cross-section data from the Socorro lines (SO lines) have been collected in the field from 1987 to 1999. The data input to the model correspond to field measurements collected in 1992 because most cross sections were surveyed during that year. The active channel width corresponds to the nonvegetated channel delineated by the geographic information system (GIS) and Remote Sensing Group of the United States Bureau of Reclamation (USBR) in Denver from aerial photos and topographic maps. Fig. 4 shows the active channel width of the SO lines in 1985 and 1992. The nonvegetated channel width has not changed significantly during this period, except for two short stretches in the upstream and downstream narrow sections of the river. Figs. 5 and 6 show the digitized nonvegetated channel geometry in 1992 along with the 2000 aerial photo of the upstream reaches (narrow) and the middle reach (wide), respectively.

Bed elevation profiles at the first cross section define the upstream boundary condition of the model. A total of 38 elevation measurements at the upstream cross section were available from 1992 to January 1999. A linear interpolation of the field measurements was performed in order to run the model continuously.

To define reach average values, the geometry data consist of the reach averaged widths, width-depth ratios, and slopes of the wide and narrow reaches. Nonvegetated channel widths for each cross section were weighted to compute the reach-averaged widths. Channel widths at each station were multiplied by a weighting factor equal to half the sum of the distances between the station in consideration and the adjacent upstream and downstream stations. For the first upstream and last downstream stations, only the distances to the downstream and upstream adjacent stations, respectively, were used in the weighting factor. Then the sum of all the products of width and weighting factors was divided by the sum of the weights to come up with a reach averaged width. As a result, the reach averaged width of the wide section is 229 m, and the reach averaged width of the narrow downstream section is 46 m.

Hydraulic Data

The mean daily flow discharge record at the San Acacia gauge from 1992 to 1999 was routed through the channel. The water regime during this time period is fairly homogeneous as evidenced by the flow discharge mass curve at the San Acacia gauge

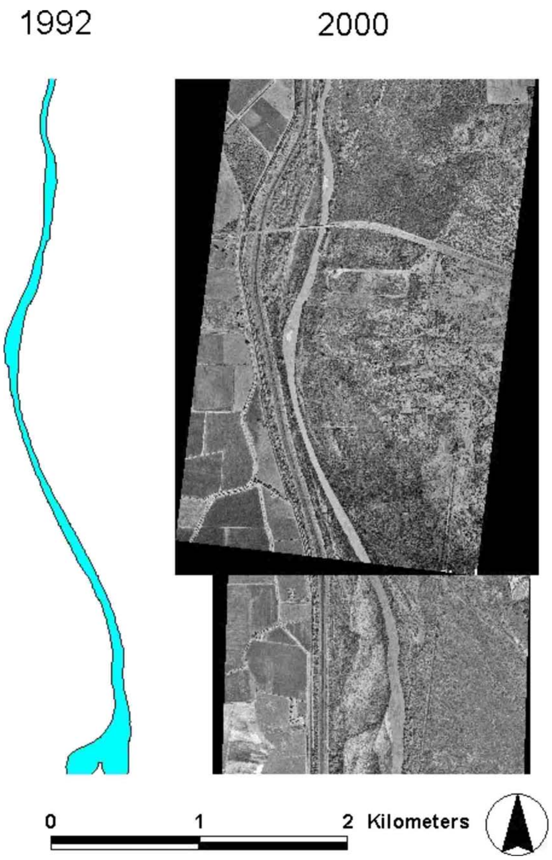

Fig. 5. Digitized nonvegetated planform in 1992 and aerial photo in 2000 of upstream (narrow) reach

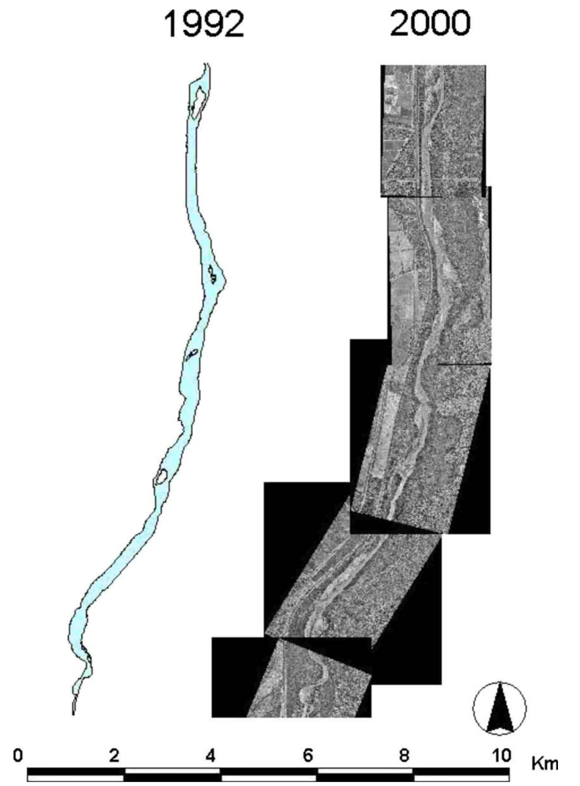

Fig. 6. Digitized nonvegetated channel in 1992 and aerial photo in 2000 of middle (wide) reach

(Fig. 7). A flow duration curve for the 1978–2000 period at the San Acacia gauge is also shown in Fig. 8 to characterize the flow regime of this reach.

For the downstream boundary condition, flow depth was estimated from depth-discharge relationships developed at the San Marcial gauge. Based on flow discharge, channel width, and cross-sectional area measurements at the gauge from 1992 to 1999, the mean flow depth was estimated and plotted against the discharge. Depth-discharge relationships were recalculated each year because the bed elevation changed with flow regime and caused stage-discharge relationships to shift over time. There were few data values for 1996 and 1997, and the 1995 equation was used for these years. The equations are of the power form: $h=aQ^b$, where h=the flow depth (m); Q=flow discharge (m^3/s), and a and b=coefficients determined from a nonlinear regression fitted through the data. Table 1 summarizes the regression coefficients.

Sediment Diameter and Manning n

The banks and bed of the Bosque del Apache reach consist mostly of sand-sized material. Bed material particle size distributions have been collected at the SO lines from 1990 to 1999. Several field samples were collected in 1992 to generate the arithmetic average curve shown in Fig. 9 (Nordin and Culbertson 1961).

Measurements at the San Acacia gauge from 1988 to 1999 were used to estimate the overall Manning roughness coefficient. This period of record was selected, because the sediment and water regimes have not changed during this time period at the San Acacia Gage (Massong et al. 2000). The data collected at the gauge consist of flow discharge, channel width, cross-sectional area, and mean flow velocity. In addition, reported channel slope estimates indicate that the slope in the 5.6 km reach below the San Acacia Diversion dam varies from 60 to 80 cm/km (Massong et al. 2000), with a mean value of 70 cm/km. The Manning roughness coefficient at the gauge was estimated based on these data. Fig. 10 shows Manning n as a function of discharge. The overall roughness coefficient n=0.026 obtained from Fig. 10 was input into the model. This roughness coefficient is in good agreement with the value n=0.024 obtained from the calibration of sediment transport models in the upper reach of Elephant Butte Reservoir.

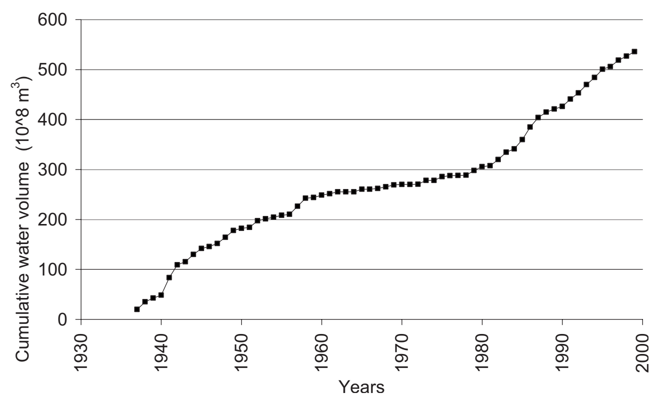

Fig. 7. Flow discharge mass curve at Rio Grande at San Acacia gauge

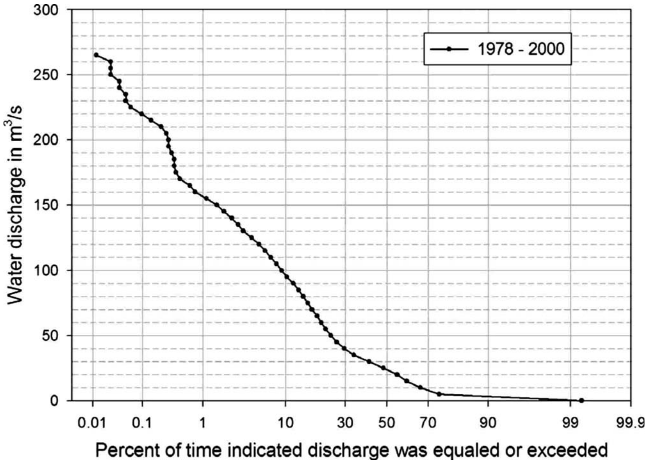

Fig. 8. Flow duration curve at San Acacia gauge from 1978 to 2000

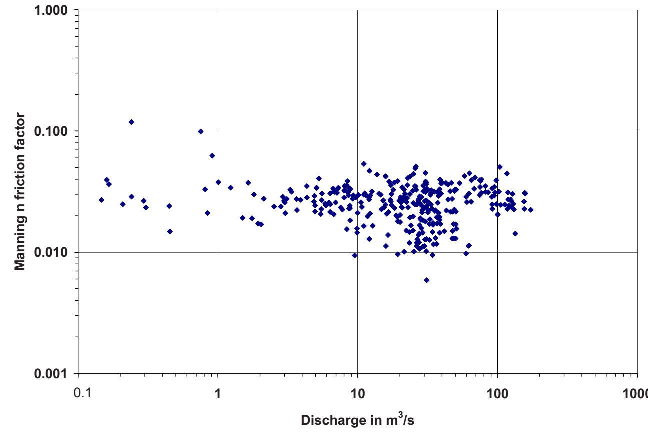

Fig. 10. Manning n as function of flow discharge at San Acacia gauge (1988–1999)

Sediment Transport

The sediment loads at the San Acacia and San Marcial Gages were constant but not equal between 1992 and 1999. The San Marcial gage transported more sediment indicating that sediment was being mined from the bed. The bed profile in the Bosque Del Apache reach is essentially the same between 1992 and 1999 therefore, the sediment mining was upstream of the modeled reach, and this reach is suitable for model testing. The bed material load at San Acacia and San Marcial are shown in Fig. 11. At a given discharge, the sediment load can vary by a couple orders of magnitude at discharges between 10 and 100 m^3/s, but the high sediment loads are fairly consistent as discharges larger than 100 m^3/s. Such scatter in the sediment rating curve is hard to fit with any sediment transport equation. It is expected that the high values of sediment discharge will have the greatest impact on the changes in channel bed elevation and bed slope. The equations of Yang (1973) and Julien (2002) are shown in comparison with the field measurements. Both equations overpredict the low rates of sediment transport, but sediment transport rates at low flows are so low that they will not significantly affect the river bed.

The mean annual flow (35 m^3/s) corresponds to a sediment load of approximately 4,000 t/day. The mean annual sediment load is approximately 3.5 million t/year (Leon 2003). The equivalent mean daily sediment load of 10,000 t/day corresponds to a flow discharge close to 60 m^3/s. Fig. 11 shows that both sediment transport equations compare very well with the field measurements of the sediment load at discharges larger than the mean annual flow (35 m^3/s).

Julien's (2002) equation has been used in the analytical development for the determination of the equilibrium slope. Yang's equation has been used in the numerical models. This equation has been successfully used by the United States Bureau of Reclamation for the modeling upstream of Elephant Butte Reservoir and its results are in good agreement with field measured sediment transport rates.

Table 1. Regression Coefficients of Depth-Discharge Relationships at San Marcial Gauge

Year	*a*	*b*
1992	0.156	0.363
1993	0.105	0.475
1994	0.178	0.348
1995	0.109	0.490
1998	0.230	0.254
1999	0.215	0.291

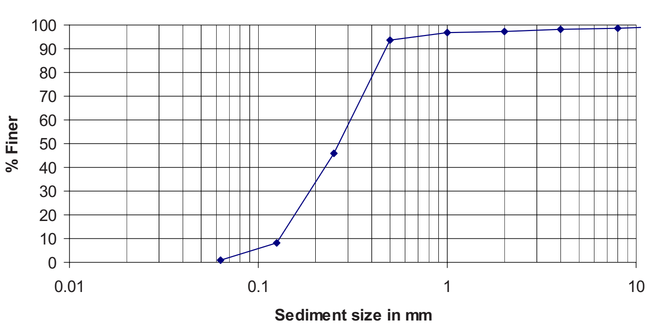

Fig. 9. Averaged bed particle size distribution at cross section SO 1470.5 in 1992

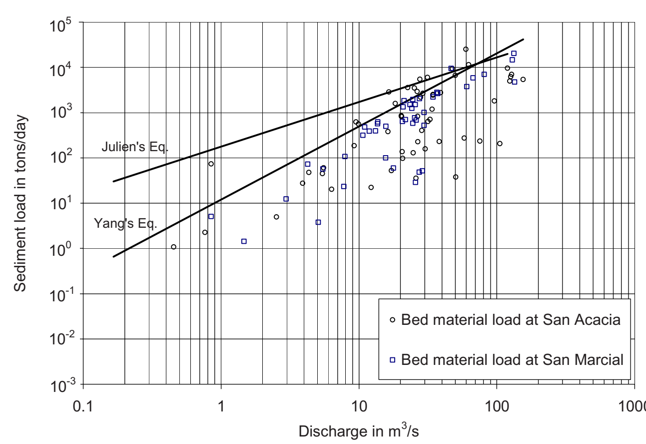

Fig. 11. Comparison of bed material load measured at San Acacia and San Marcial gauges from 1990 to 1997 with potential transport capacities computed with Julien's equation (Julien 2002) and Yang's equation (Yang 1973)

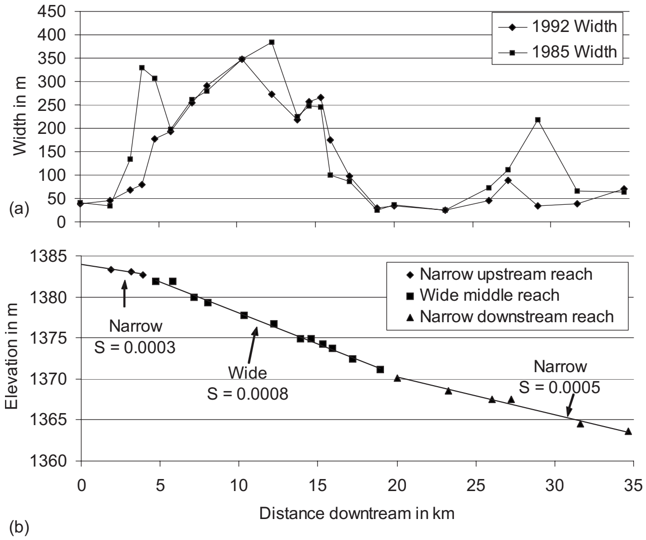

Fig. 12. (a) Nonvegetated channel widths for 1985 and 1992; (b) Thalweg profile in 1999 reach-averaged slopes S in m/m for each subreach are indicated

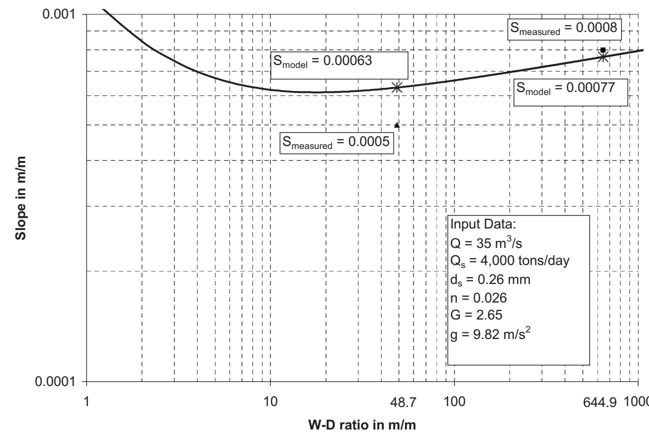

Fig. 14. Comparison of analytical model between slope and width-depth ratio with 1999 field data; Q=water discharge; d_s=sediment size; n=Manning roughness coefficient; G=specific gravity of sediment; g=gravitational acceleration; and S=channel slope

Model Results

Analytical Model Results

The data input to the steady-state analytical solutions consists of field measurements collected in 1999. The reach averaged width of the wide section is 229 m, and the reach averaged width of the narrow downstream section is 46 m. The corresponding reach averaged slopes are shown in Fig. 12. The width-depth ratios were estimated from the measured reach averaged widths and the computed normal depths at each reach corresponding to the reach averaged slopes, the median grain size of 0.26 mm, the mean annual discharge of $Q=35\ \mathrm{m}^3/\mathrm{s}$, and Manning $n=0.026$. The analytical model results compare field measurements with model results of slope versus width in Fig. 13 and slope versus width-depth ratio in Fig. 14. The field measurements confirm the analytical model results that the slope of wider reaches is larger than the slope of narrow reaches. The quantitative agreement between theory and field measurements is also quite satisfactory.

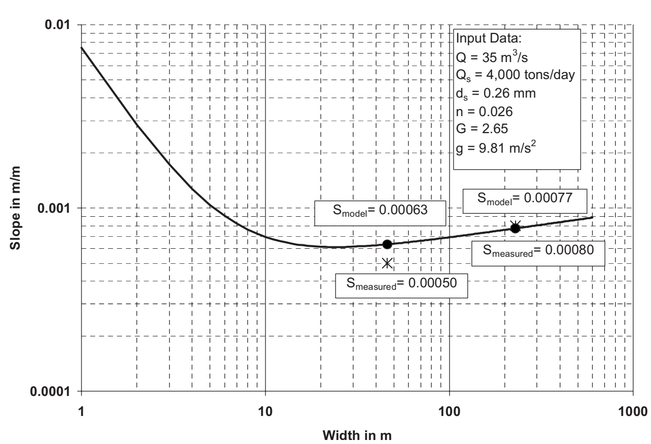

Fig. 13. Comparison of steady-state solution with 1999 field data: Q=water discharge; Q_s=sediment discharge; d_s=sediment size; n=Manning roughness coefficient; G=specific gravity of sediment; g=gravitational acceleration; and S=channel slope

Daily Transient Model Results

A daily transient model has been developed to provide comparisons with field data. The daily transient model considered unsteady flow with measured daily discharges from February 1992 to January 1999. The variable channel width used in this model simulation corresponds to the field measurements of 1992 shown in Fig. 12(a). Also, the measured thalweg elevations at the first cross section were used as upstream boundary conditions because this upstream elevation only changed very slightly during the simulation. Fig. 15 shows a comparison of the model results with the field measurements of 1999. The results show that the wide reach is steeper than the two narrow reaches. In addition, the reach averaged slopes from the simulations are comparable to the reach averaged slopes obtained from the field data.

Long-Term Unsteady Transient Model Results

Long-term unsteady simulations were carried out to determine the time required to reach the new equilibrium slope. To simulate transient conditions over a long period of time, a sequence of two reaches with different widths similar to Fig. 2 was modeled. For this case, reach averaged width and slope were used because of the lack of field data for the entire simulation period (1979–1999). In this example, the total channel length was 32 km and the upstream bed elevation was kept constant. The upstream reach averaged width was $W_1=100$ m and the downstream reach averaged width was $W_2=200$ m. The initial channel slope was $S=80$ cm/km, the Manning friction factor was $n=0.023$, and the sediment size was $d_s=0.3$ mm. These channel geometry, sediment, and roughness characteristics are typical values for the middle Rio Grande downstream from the San Acacia gauge (Fig. 1).

The results of the long-term daily transient model in Fig. 16 show that most of the bed elevation changes occur during high flows and changes are negligible at low flows. The sequence of high flow peaks between Year 12 and 19 did not change the channel slope as much as the sequence of peaks from Years 1 to 9. This indicates that the channel slope is asymptotically approach-

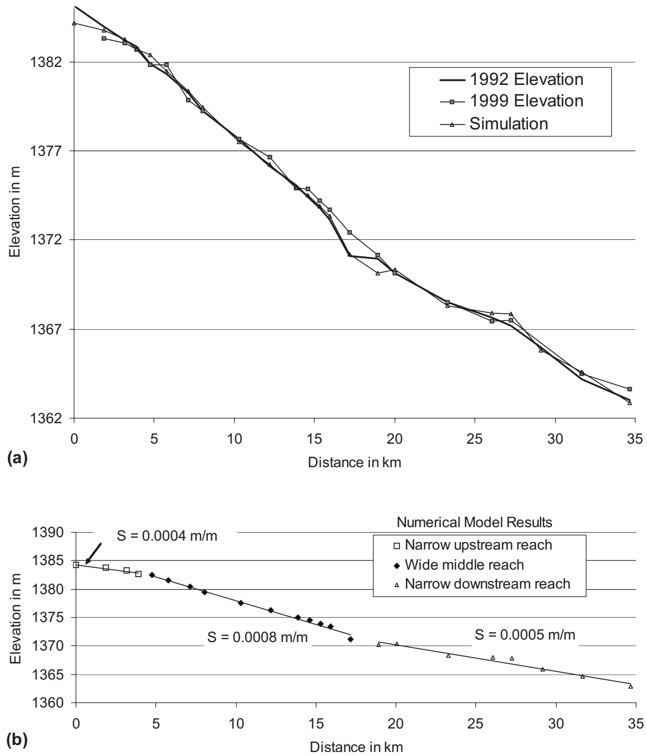

Fig. 15. (a) Comparison of resulting channel profile from daily transient model with field measurements of 1992 and 1999; (b) reach-averaged slopes of each subreach from simulation

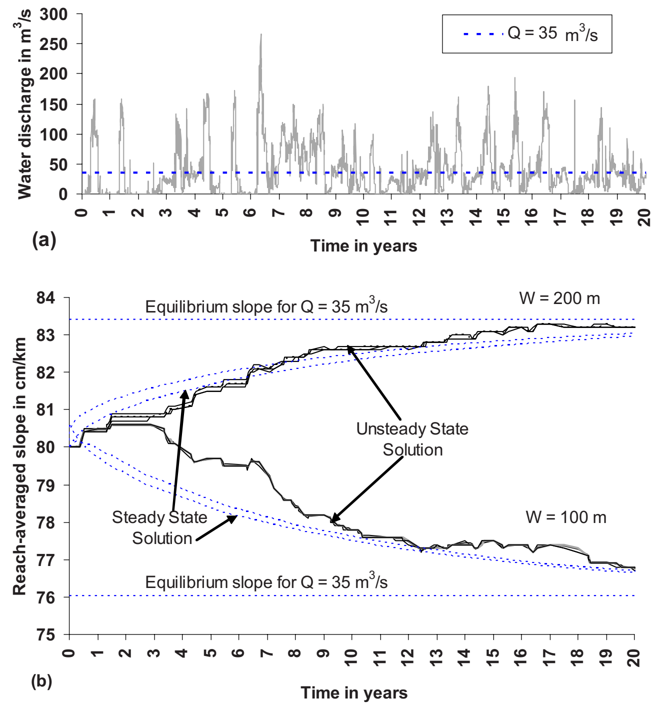

Fig. 16. Daily transient model results with variable and constant discharges: (a) input hydrograph from San Acacia gauge from 1979 to 1999; mean annual flow of $Q=35\ m^3/s$; (b) family of curves representing change in reach-averaged slope with time from daily transient model with variable discharge (unsteady) and from numerical model with constant discharge (steady) equal to mean annual flow of $Q=35\ m^3/s$

ing an equilibrium slope. This reach of the Middle Rio Grande needs more than 20 years to reach a new equilibrium slope.

Long-Term Steady Transient Model Results

For comparison, a long-term steady transient simulation was carried out for long-term simulations at a constant discharge $Q_m=35\ m^3/s$, which corresponds to the mean annual flow from 1979 to 1999. The values of the reach averaged slopes depend on which nodes or stations along the channel are selected for the calculation. Then, rather than a unique curve, a family of curves better represents the time variability in channel slope of each reach. Fig. 16 shows the steady flow simulation results in comparison with the steady transient simulations. The steady flow simulations show a gradual slope increase for the wide reach and an overall slope decrease for the narrow reach. This simulation clearly shows that the new equilibrium slope is reached asymptotically.

Exponential Transient Model

A simple exponential transient model is formulated based on the hypothesis that the slope S changes more rapidly, the further it is from the equilibrium slope S_e. Thus

$$dS/dt = -k(S - S_e) \tag{13}$$

where dS=change in slope over the time interval dt. Rearranging Eq. (13) and solving for S gives the following exponential function:

$$S = S_e + (S_0 - S_e)e^{-kt} \tag{14}$$

where k=constant; S_0=initial channel slope at time t_o; S_e=final slope describing the equilibrium condition; and S=channel slope at time t in days. This model has a form similar to the exponential model proposed by Richard (2001) and Richard et al. (2005) to describe the narrowing trend of the channel width of the Middle Rio Grande.

The exponential model was fit to the numerical simulation results with a constant discharge equal to the mean annual flow of $Q_m=35\ m^3/s$ (Fig. 16). The equilibrium channel slope S_e was obtained from the numerical model, and the constant $k=0.00028\ \text{day}^{-1}$ was obtained by minimizing the sum of square errors (SSE). Fig. 17 shows the exponential model fit to the data as well as the data from the simulation. This plot shows the time scale in years, however, the regression was performed with the time in days. The exponential model is in good agreement with the results from the simulation. This simplified exponential model lends itself to easy calculations of the time required T_m to reach the m% of slope adjustment as

$$T_m = \frac{-1}{k}\ln(1 - 0.01m) \tag{15}$$

where m=level of slope adjustment in percentage, and T_m=time required to reach this slope adjustment level. In other words, $m=100\ (S_0-S_m)/(S_0-S_e)$, such that $m=0$ initially when $S_m=S_0$ and $m=100\%$ when the equilibrium slope is reached and $S_e=S_0$. For example, 90% of the slope adjustment will be obtained when $m=90\%$. This will occur at a time $T_{90}=(-1/\kappa)\ln(1-0.9)=2.6/k$. Similarly, half the slope adjustment will be obtained at a time $T_{50}=0.7/k$.

As a practical application of this exponential transient model, one can estimate the time it would take the Rio Grande to reach half the adjustment in channel slope. For instance, the detailed

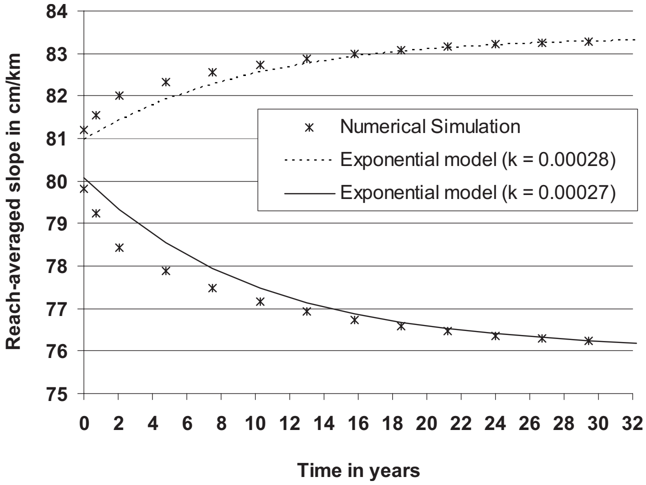

Fig. 17. Change in reach-averaged slope with time in narrow (upstream) and wide (downstream) reaches of channel as predicted with exponential model

simulation results based on daily flow measurements in Fig. 16(b) indicate that the wider reach would reach a slope of 81.7 cm/km (halfway between the initial slope S_0=80 cm/km and equilibrium slope S_e=83.4 cm/km) after about 6 years. Similarly, the narrow reach on the same figure would reach a slope of 78 cm/km (halfway between 80 and 76 cm/cm) in about 9 years. In comparison, the exponential transient model with k=0.00028 day^{-1} from Fig. 17 shows that half the slope adjustment from Eq. (14) would be obtained after T_{50}=2,520 days=7 years. Similarly, 90% of the channel slope adjustment would also be obtained where T_{90} =23 years, which compares reasonably well with the results shown in Fig. 16(b). Of course this exponential model does not account for the daily discharge and sediment fluctuations, which obviously can be better predicted by the daily transient model in Fig. 16.

Conclusions

The Bosque del Apache reach of the middle Rio Grande, NM, has maintained successive reaches with equivalent channel widths of 50 and 250 m over long periods of time. This provides an excellent case study to test the hypothesis that long-term changes in channel width induce changes in bed slope. The conclusions of this paper are as follows:

1. A theoretical derivation defines long-term equilibrium slope conditions for river channel reaches with different widths. It is analytically demonstrated that an increase in equivalent channel width would result in an increase in channel slope.
2. Quantitative field measurements from the Bosque del Apache reach of the Middle Rio Grande, NM, confirm that the wide river reach has a steeper slope than the narrow adjacent reaches both upstream and downstream. The daily transient model simulations of a study reach of the Middle Rio Grande for the period 1992–1999 correctly replicate the field measurements of riverbed slope (Fig. 15).
3. The long-term unsteady transient simulations using daily flow and sediment discharges compare well with the steady transient solution with a constant discharge close to the mean annual flow (Fig. 16). Most of the river bed elevation changes occur at high flows when the sediment load is large.
4. The exponential transient model for constant discharge provides first approximations of the time scale for the changes in channel slope. The equilibrium slope is reached asymptotically with half the slope adjustment on the Rio Grande achieved in 6–9 years. The Rio Grande also needs about 20–25 years to reach 90% of its slope adjustment.

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Notation

The following symbols are used in this paper:

A = cross-section area (m^2);
a = coefficient of at-a-station depth-discharge relationship;
b = exponent of at-a-station depth-discharge relationship;
C_v = volumetric sediment concentration;
d_s = bed material size (mm);
dS = change in bed slope;
dt = change in time (days);
F_r = Froude number;
G = specific gravity of sediment;
g = gravitational acceleration (m/s^2);
h = water depth (m);
h_r = depth ratio;
L = length;
m = level of slope adjustment in percentage;
n = Manning roughness coefficient;
p_o = porosity of sediment;
Q = water discharge (m^3/s);
Q_m = mean annual flow (m^3/s);
Q_s = sediment discharge by volume (m^3/s);
R_h = hydraulic radius (m);
S = channel slope (m/m);
S_e = equilibrium channel slope (m/m);
S_f = friction slope (m/m);
S_r = slope ratio;
S_0 = initial channel bed slope (m/m);
T_{50}, T_{90} = time scale to reach 50 and 90% of slope adjustment, respectively;
t = time;
V = mean flow velocity (m/s);
V_r = velocity ratio;
W = channel width (m);
W_r = width ratio;
x = downstream distance along channel (m);
z_b = bed surface elevation (m);
ξ = width-depth ratio (m/m);
ξ_r = width-depth ratios of two sequential channel reaches;

$\tau*$ = Shields parameter;
φ = coefficient of Manning's resistance equation equal to 1 for SI units and 1.49 for English units; and
6 = constant in the exponential model (e.g., $6=0.00028\ \text{day}^{-1}$).

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