# 1 GVF Flow Profiles

*Is the equation for gradually varied flow the result of conservation of momentum or conservation of energy? Can you explain the difference?*

The equation for steady gradually varied flow is given by:

$$\frac{\partial h}{\partial x} = \frac{S_0 - S_f}{1 - Fr^2} \tag{1}$$

The equation for gradually varied flow is the result of **conservation of momentum**. Beginning from the Saint-Venant equation for conservation of linear momentum of steady 1D flow in wide-rectangular open channels:

$$S_f = S_0 - \frac{\partial h}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} \tag{2}$$

Using the identity:

$$\frac{1}{2g}\frac{\partial V^2}{\partial x} = \frac{2V}{2g}\frac{\partial V}{\partial x} = \frac{V}{g}\frac{\partial V}{\partial x} \tag{3}$$

Rearranging Eq. 2:

$$S_0 - S_f = \frac{\partial}{\partial x}\left(h + \frac{V^2}{2g}\right) = \frac{\partial E}{\partial x} = \frac{\partial E}{\partial h}\frac{\partial h}{\partial x} \tag{4}$$

Knowing $E = h + \frac{V^2}{2g}$:

$$E = h + \frac{Q^2}{2gA^2} \tag{5}$$

$$\frac{\partial E}{\partial h} = 1 - \frac{2Q^2}{2gA^3}\frac{\partial A}{\partial h} \tag{6}$$

Knowing $\frac{\partial A}{\partial h} \approx W$

$$\frac{\partial E}{\partial h} = 1 - \frac{V^2}{g}\frac{W}{A} = 1 - \frac{V^2}{gh} = 1 - Fr^2 \tag{7}$$

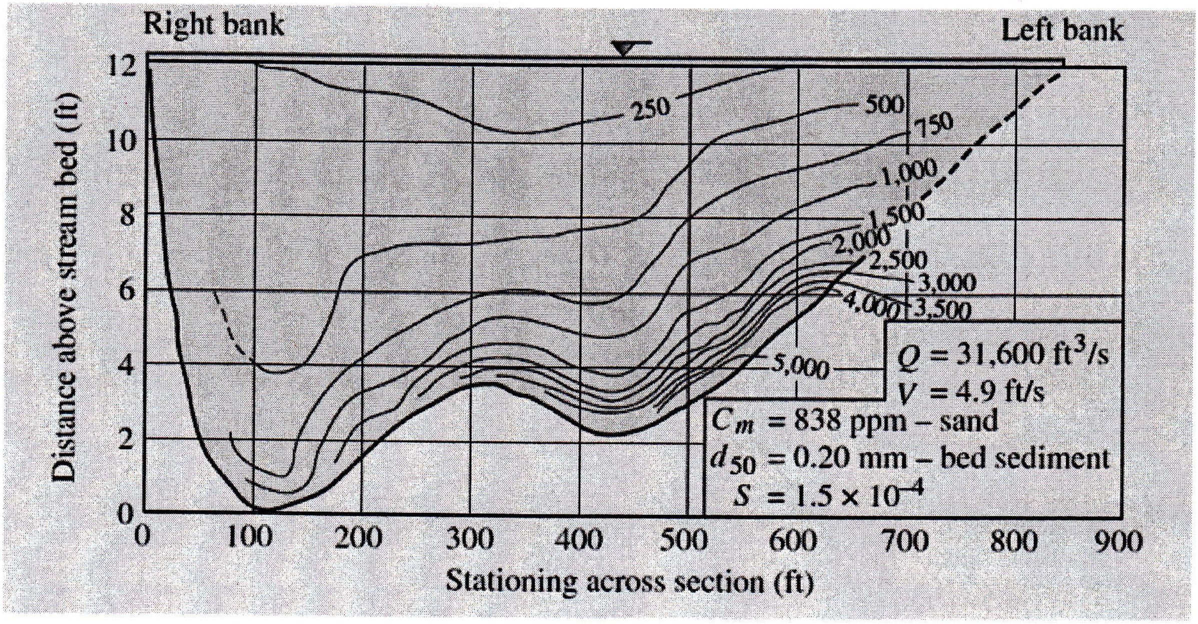
Knowing $\partial E/\partial h = 1 - Fr^2$:

$$\therefore \frac{\partial h}{\partial x} = \frac{S_0 - S_f}{1 - Fr^2}$$

## Problem 2: At-a-Station Hydraulic Geometry

**Given:**

Consider the cross-section of the Missouri River below and




**Find:**

1. Estimate Manning n
2. Develop a spreadsheet for the hydraulic geometry parameters as a function of flow depth every 1 ft until 12 ft.
3. Estimate the velocity and discharge assuming constant n.
4. Plot at-a-station hydraulic geometry relationships on a log-log scale.
5. Compare flow velocity and flood wave celerity c = dQ/dA.
6. Discuss the results

**Solution:**

First, point coordinates were estimated to create a replica of the given cross section and are plotted below in Figure 1. Each coordinate is composed of the depth of the channel on the y-axis and stationing as the x-axis. These coordinates will be used to estimate the manning's n and to compute the hydraulic geometry parameters as a function of flow depth.

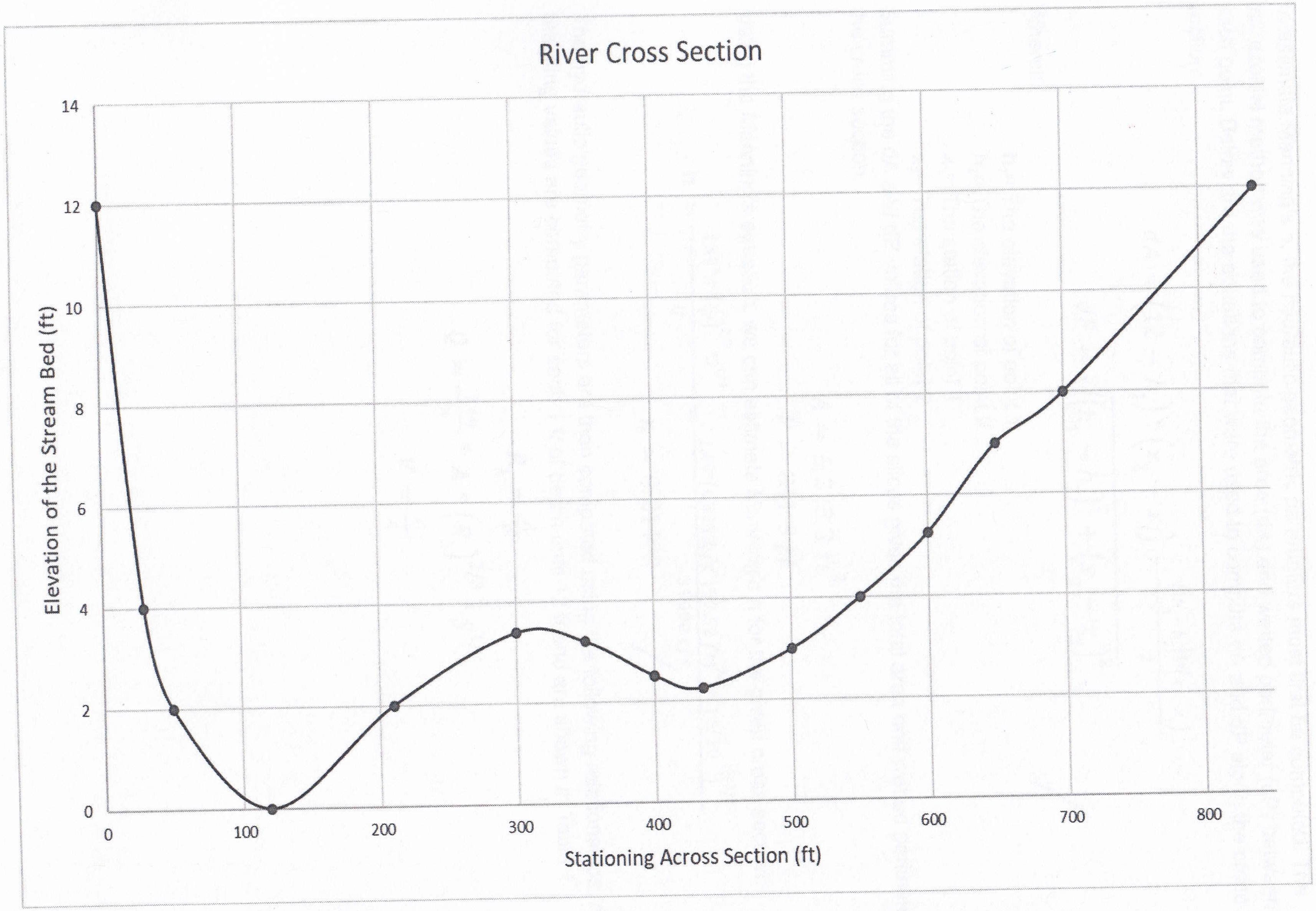



**Figure 1.** Plot of given river cross section

To estimate Manning's n, the hydraulic geometric parameters must first be computed. The trapezoidal method was used to compute the area (dA) and wetted perimeter (dP) between each point. Below are the equations that were used to compute dA and dP along the cross section:

$$dA = \left(12 - h_1\right) * \left(x_2 - x_1\right) + \frac{\left(\left|h_1 - h_2\right|\right) * \left(x_2 - x_1\right)}{2}$$

$$dP = \sqrt{\left(h_1 - h_2\right)^2 + \left(x_2 - x_1\right)^2}$$

Wherein,

$h_1$= The elevation of point 1
$h_2$= The elevation of point 2
$x_1$= The station of point 1
$x_2$= The station of point 2

Summing the dA and dP values for all of the slices gives the total area and wetted perimeter for the cross section.

$$A = 6,328.3\ ft^2$$
$$P = 841.3\ ft$$

Using the Manning's equation, we can estimate Manning's n for the given cross section:

$$n \simeq \frac{1.49 * A * \left(\frac{A}{P}\right)^{2/3} * S^{1/2}}{Q} = \frac{1.49 * \left(6,328.3\ ft^2\right) * (7.52\ ft)^{2/3} * \left(1.5 * 10^{-5}\right)^{1/2}}{31,600\ cfs}$$

$$n \simeq 0.01403$$

The hydraulic geometry parameters are then computed using the following relationships. All resulting values are computed for each 1 ft of depth until 12 ft and are shown in Table 1.

$$R_h = \frac{A}{P}$$

$$Q = \frac{1.49}{n} * A * \left(R_h\right)^{2/3} * S^{1/2}$$

$$V = \frac{Q}{A}$$

**Table 1.** Hydraulic geometry parameters

| Hydraulic Parameters as a Function of Flow Depth | | | | | |
|---|---|---|---|---|---|
| H (ft) | A ($ft^2$) | P (ft) | $R_h$ (ft) | V (ft/s) | Q ($ft^3/s$) |
| 1 | 40 | 80 | 0.5 | 0.82 | 33 |
| 2 | 160 | 160 | 1 | 1.3 | 208 |
| 3 | 412 | 370 | 1.1 | 1.4 | 576 |
| 4 | 881 | 520 | 1.7 | 1.85 | 1627 |
| 5 | 1423 | 564 | 2.5 | 2.41 | 3429 |
| 6 | 2005 | 599 | 3.3 | 2.91 | 5833 |
| 7 | 2620 | 632 | 4.1 | 3.36 | 8797 |
| 8 | 3278 | 686 | 4.8 | 3.69 | 12101 |
| 9 | 3983 | 725 | 5.5 | 4.05 | 16133 |
| 10 | 4726 | 764 | 6.2 | 4.38 | 20722 |
| 11 | 5508 | 802 | 6.9 | 4.7 | 25874 |
| 12 | 6328 | 841 | 7.5 | 4.99 | 31600 |

Using the values in the table below, the hydraulic geometry relationships can be plotted on a log-log scale as shown below in Figures 2 through 5:

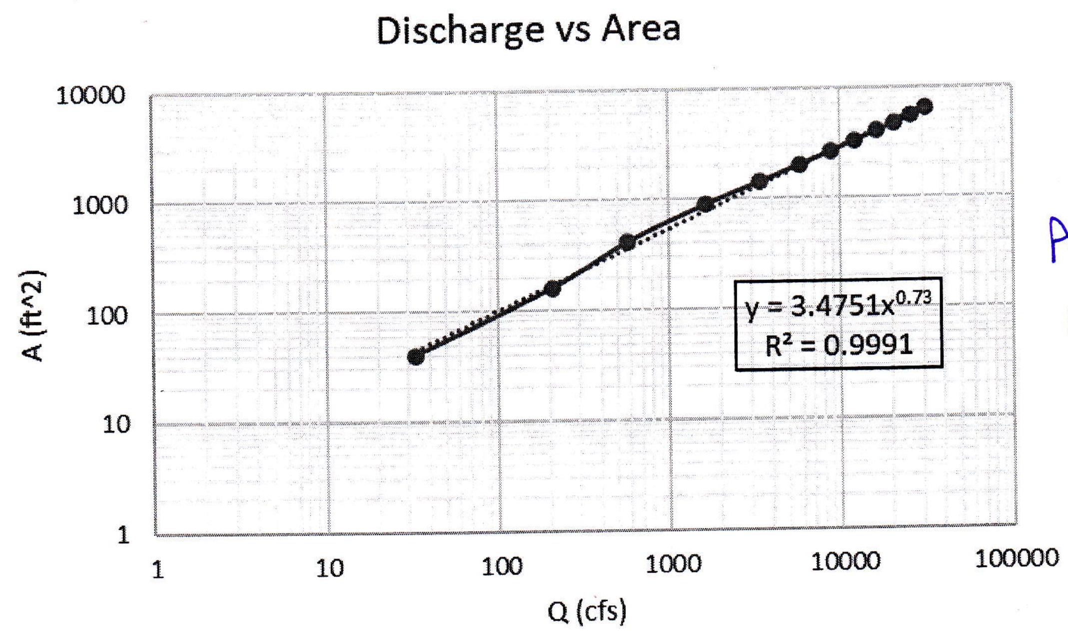



**Figure 2.** Plot of relationship between discharge and flow area.

$A = 3.47\,Q^{0.73}$

$Q = \left(\frac{A}{3.47}\right)^{\frac{1}{0.73}}$

$c = \frac{\partial Q}{\partial A}$.

Compare with V

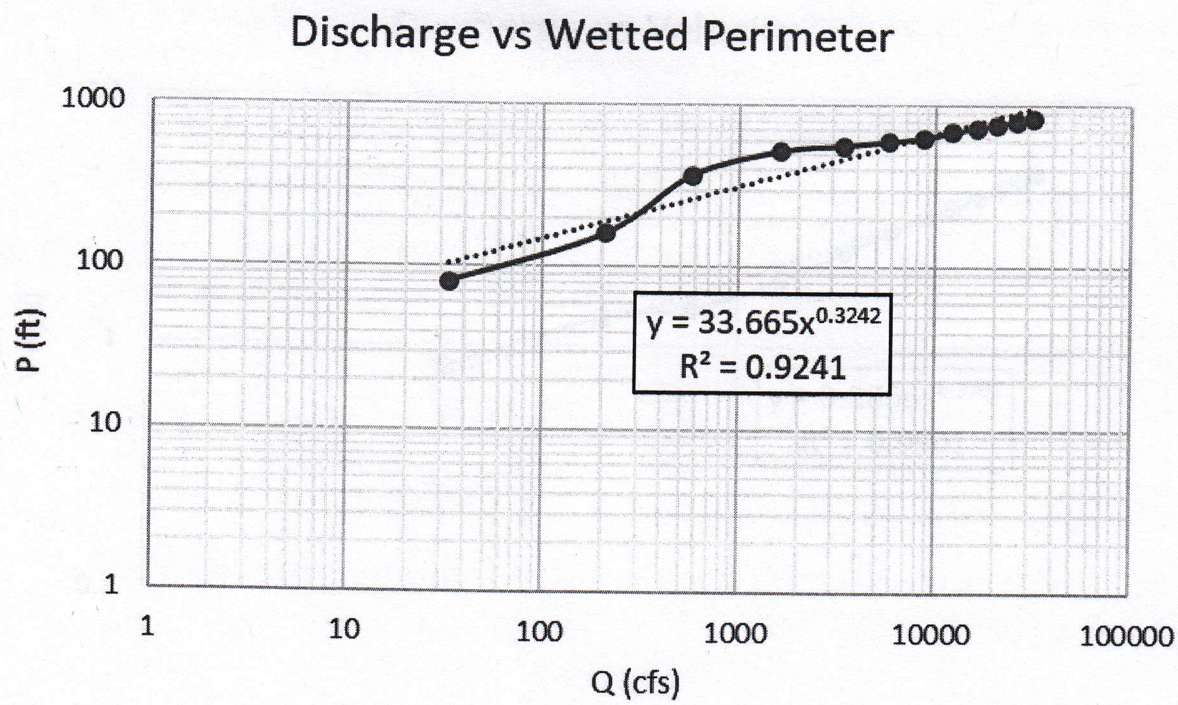



**Figure 3.** Plot of relationship between discharge and wetted perimeter.

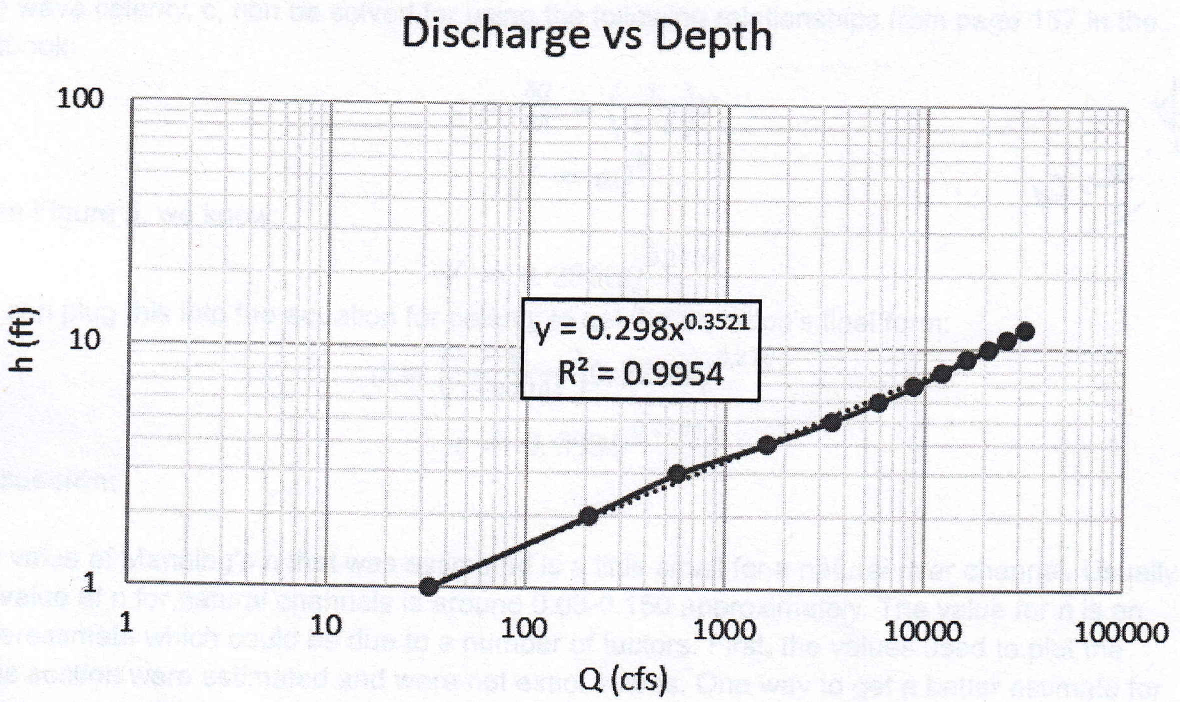



**Figure 4.** Plot of relationship between discharge and flow depth.

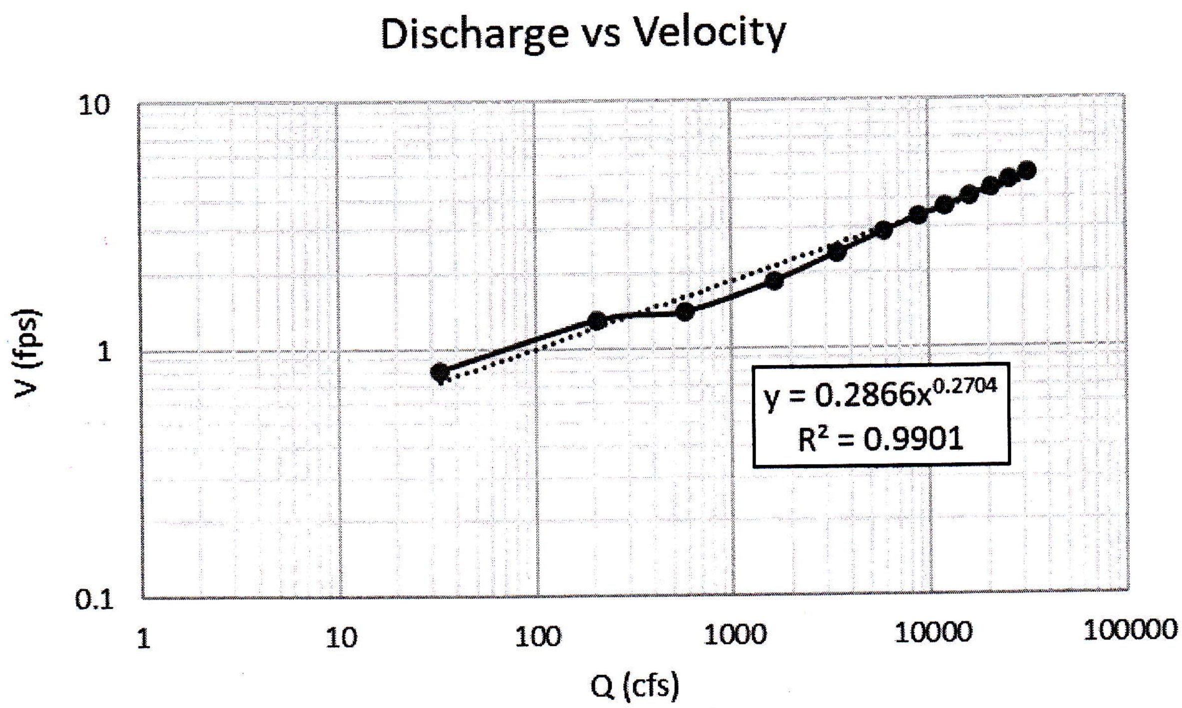



**Figure 5.** Plot of relationship between discharge and flow velocity.

The wave celerity, c, can be solved for using the following relationships from page 157 in the textbook:

$$c = \frac{\delta Q}{\delta A} = \left(\frac{1}{1-b}\right)V$$

$$V = aQ^{b}$$

Nice. 5

From Figure 5, we know:

$$V = 0.2866Q^{0.2704}$$

We can plug this into the equation for celerity to get the equation's final form:

$$c = \left(\frac{1}{1-0.2704}\right)0.2866Q^{0.2704}$$

$$c = 0.393Q^{0.2704}$$

**Discussion:**

The value of Manning's n that was estimated is a little small for a natural river channel. Usually the value of n for natural channels is around 0.03-0.150 approximately. The value for n is an underestimate which could be due to a number of factors. First, the values used to plot the cross section were estimated and were not exact values. One way to get a better estimate for the computed hydraulic geometry parameter values would be to use cross section averaged flow depths. Fixing the approximations of the flow area and perimeter would also make the estimate for Manning's n to be more accurate.

# Problem 3

Subject: Numerical Diffusion

## Purpose

The purpose of this exercise is to determine how contaminant concentrations move through a 30 km reach of a river during specific pulses. The summary of pulses and channel characteristics are described below. The purpose is to describe the contaminant concentration vs time at 10 km and 20 km from the source. The key to this exercise is using a method that does not have any numerical diffusion to skew the results.

*Pulse Scenarios*

| First Pulse | Second Pulse |
|---|---|
| 15 min at 100,000 mg/l at t = 0 | 3 min at 200,000 mg/l at t = 1 hr |
| 3 min at 500,000 mg/l at t = 6 mins | 3 min at 200,000 mg/l at t = 1 hr |

*Channel Characteristics*

| | |
|---|---|
| Slope, S | $4\text{x}10^{-3}$ |
| Flow depth, h | 3 m |
| Mean velocity, V | 2 m/s |
| Dispersion Coefficient, Kd | 257 m2/s |

## Method

1. For simplicity of boundary conditions, set Cu = 1 and the change in time = 180 secs.
   a. Calculate the change is x using the following equation. Velocity is given above.

   $$C_u = \frac{V\Delta t}{\Delta x}$$ , **change is x = 360 m.**

   b. Calculate Ck using the equation below.

   $$C_k = \frac{K_d\Delta t}{\Delta x^2}$$ , **Ck = 0.357.**

   c. Calculate $P_\Delta$ using the equation below.

   $$P_\Delta = \frac{C_u}{C_k}$$ , **$P_\Delta$ = 2.80.**

   d. Check for stability using Figure 7.4 (Julien, River Mechanics). When Cu = 1, Ck = .367, and $P_\Delta$ = 2.80, it is stable. Since $P_\Delta$ is close to 2, the results will be relatively smooth, but more leaky and might show a loss of concentration over time.
2. Use the general form of the Leonard scheme shown below:

$$\phi_j^{k+1} = A\phi_{j-2}^k + B\phi_{j-1}^k + C\phi_j^k + D\phi_{j+1}^k$$

Where:

$$A = C_kC_u + \frac{C_u}{6}(C_u^2 - 1)$$ , **A = 0.357**

$$B = C_k(1 - 3C_u) - \frac{C_u}{2}(C_u^2 - C_u - 2)$$, **B = 0.286**

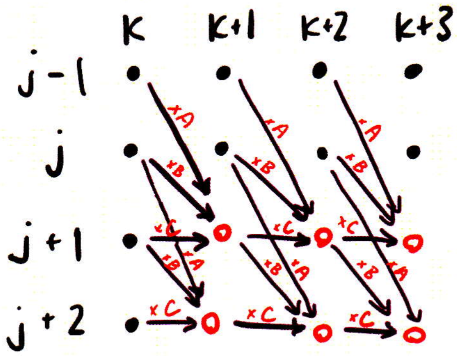
☺

$$C = 1 - \left[C_k(2 - 3C_u) - \frac{C_u}{2}(C_u^2 - 2C_u - 1)\right]$$, **C = 0.357**

$$D = C_k(1 - C_u) - \frac{C_u}{6}(C_u^2 - 3C_u + 2)$$, **D = 0.0**

a. Check to make sure A+B+C+D = 1, 0.357 + 0.286 + 0.357 + 0.0 = 1, so the model should be stable.

b. The scheme works as shown below, two upstream boundaries are needed and one boundary for all j's at t=0 is needed. Because D = 0, a downstream boundary is not needed.




Nice

3. For the boundary conditions, see the summary below for each pulse scenario. Because Cu = 1, we know that for each change in x, the concentration is transposed by 1 change in time, so that is how the second upstream boundary is set.

Great

a. Pulse Scenario 1:

| | Time (mins) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **x, km** | **0** | **3** | **6** | **9** | **12** | **15** | **18** | **54** | **57** | **60** | **63** |
| -0.36 | 100000 | 100000 | 100000 | 100000 | 100000 | 0 | 0 | 0 | 200000 | 0 | 0 |
| 0.00 | 100000 | 100000 | 100000 | 100000 | 100000 | 100000 | 0 | 0 | 0 | 200000 | 0 |
| 0.36 | 0 | 64305.52 | 87259.04 | 95452.18 | 98376.68 | 99420.56 | 64098.69 | 0.274197 | 0.097873 | 71389 | 82704 |
| 0.72 | 0 | 35694.48 | 66833.92 | 84516.22 | 93171.97 | 97098.32 | 98798.48 | 3.060033 | 1.170713 | 0.445883 | 91814.26 |
| 1.08 | 0 | 0 | 33166.08 | 62107 | 80420.9 | 90478.33 | 95564.26 | 19.39141 | 7.895043 | 3.187983 | 25483.2 |
| 1.44 | 0 | 0 | 12740.96 | 37893 | 61462.79 | 78205.33 | 88460.52 | 87.38069 | 37.83043 | 16.18011 | 6.846677 |

b. Pulse Scenario 2:

| | Time (mins) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **x, km** | **0** | **3** | **6** | **9** | **12** | **15** | **18** | **54** | **57** | **60** | **63** |
| -0.36 | 0 | 500000 | 0 | 0 | 0 | 0 | 0 | 0 | 200000 | 0 | 0 |
| 0.00 | 0 | 0 | 500000 | 0 | 0 | 0 | 0 | 0 | 0 | 200000 | 0 |
| 0.36 | 0 | 0 | 178472.4 | 206760 | 73801.9 | 26343.21 | 9403.072 | 0.040224 | 0.014358 | 71388.97 | 82703.99 |
| 0.72 | 0 | 0 | 0 | 229535.2 | 141087.6 | 71475.97 | 33050.04 | 0.528277 | 0.200074 | 0.075523 | 91814.12 |
| 1.08 | 0 | 0 | 0 | 63704.81 | 162213.4 | 124611.1 | 74332.36 | 3.866191 | 1.54552 | 0.614034 | 25482.16 |
| 1.44 | 0 | 0 | 0 | 0 | 100158 | 132522.3 | 108468.6 | 20.02722 | 8.443335 | 3.52741 | 1.46173 |

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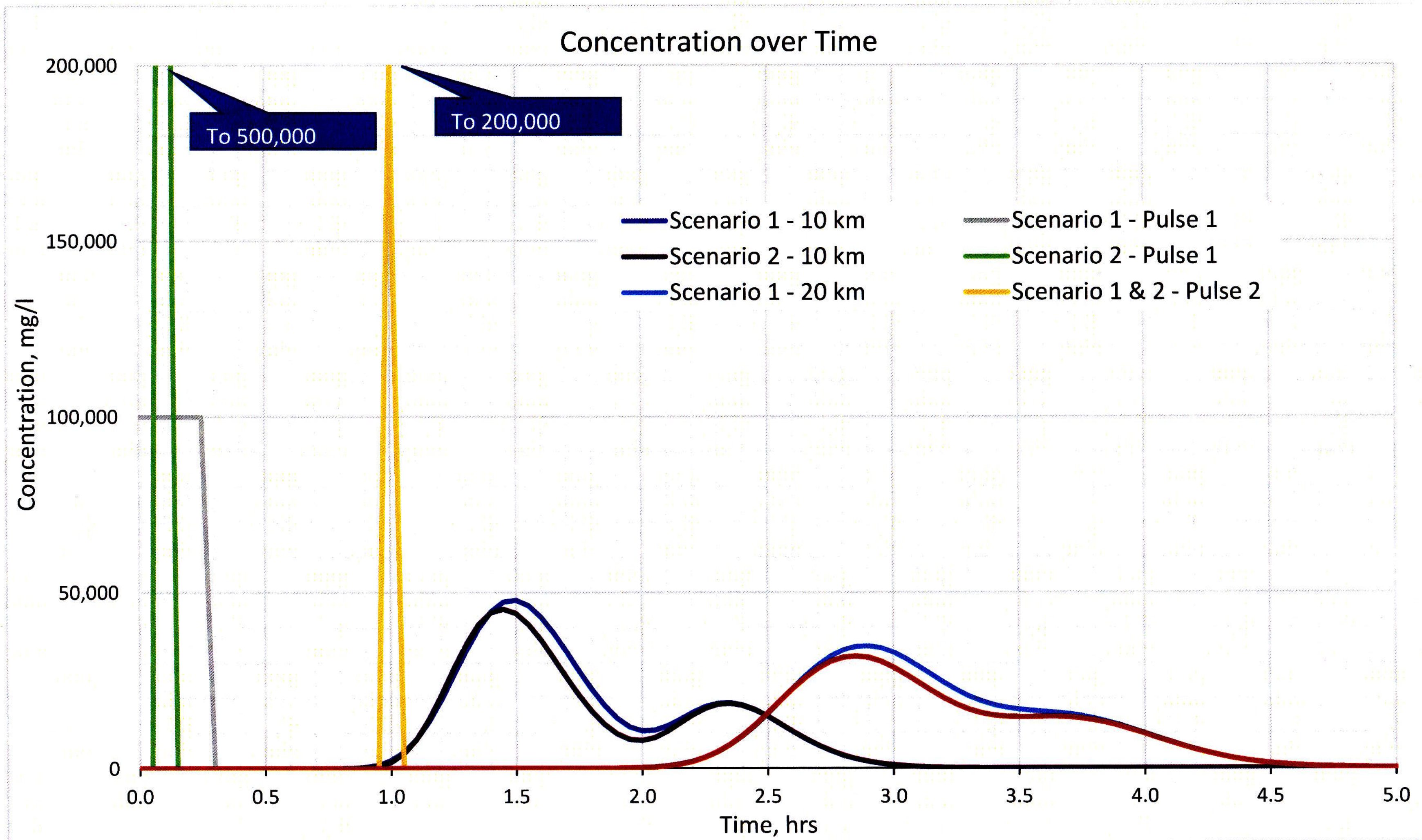
Results and Discussion

The plot below shows the comparison between the two pulse scenarios at 10km and 20km from the point of origin, x = 0 km. The results show very similar results between the two scenarios. The one time, 500,000 mg/L pulse 1 in scenario 2 created slightly lower downstream concentrations than the longer, lower pulse 1 of 100,000 mg/l in scenario 1. The second pulse for both scenarios created the same concentration despite the different first pulses. For both scenarios, the concentrations at 10km and 20 km equaled each other at approximately 2.5 hours.






Great!