Hydraulic Engineering

Lecture Notes for CIVE 401

Part III

Open Channels

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In this third part, we will focus on calculating the flow in open channels like canals and rivers. We will learn to solve differential equations with finite difference numerical schemes. We will also explore important concepts regarding optimization, sustainability and engineering ethics.

1. Open Channels (*week 1*)

1.1. Open channel geometry

Open channels convey surface waters in natural waterways like streams and rivers, or man-made channels like irrigation canals. The cross-section of ach channel is measured in the direction perpendicular to the main flow direction. The geometric elements of an open channel are defined as follows:

Surface Width: B

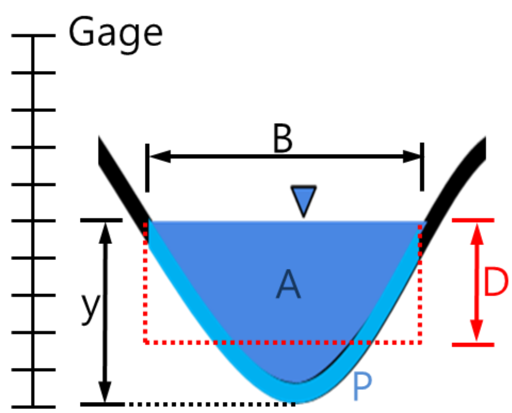
Wetted Perimeter: P

Flow Depth: y

Cross-Section Area: A

Averaged Depth: D = A/B

Hydraulic Radius: R = A/P

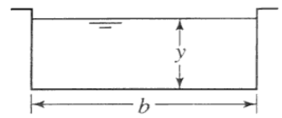
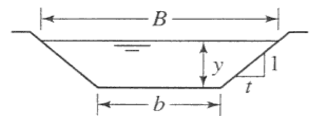
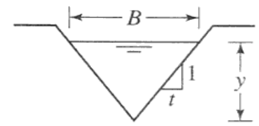
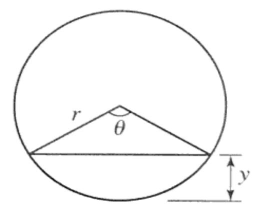
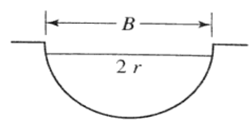
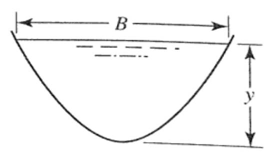


※ Cross-section normal to the velocity vector

Figure. Definition sketch for cross-section geometric elements

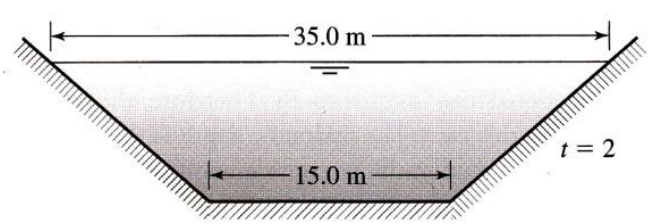
Man-made channels can have specific geometric shapes. For instance, trapezoidal sections have been used extensively in unlined earth banks. Rectangular sections often have unstable earthen banks, but they are reasonable approximations for wide rivers. Triangular sections are not common but can be found in small ditches, gutters, and laboratory experimental channels. Sewers and culverts are often circular. The main geometric characteristics of various open channel cross-sections are detailed in Table 3.1.

Table 3.1 Geometric Properties of Typical Channel Cross Sections

Section	Area, A	Wetted Perimeter, P	Hydraulic Radius, R	Top Width, B	Hydraulic Depth, D	Cross Section
Rectangular	by	$b + 2y$	$by/(b + 2y)$	b	y	
Trapezoidal	$(b + ty)y$	$b + 2yw$, $w = (1 + t^2)^{0.5}$	A/P	$b + 2ty$	A/B	
Triangular	ty^2	$2yw$	$ty/(2w)$	$2ty$	A/B	
Circular	$(\theta - \sin\theta)\frac{d^2}{8}$	$r\theta$	$\left(1 - \frac{\sin\theta}{\theta}\right)d/4$	$2r\sin(\theta/2)$	A/B	 $\theta = 2\cos^{-1}\left(1 - 2\frac{y}{d}\right)$
Semicircular	$\pi r^2/2$	πr	$r/2$	$2r$	$\pi r/4$	
Parabolic Section	$2/3By$	$B + (8/3)y^2/B$*	$2B^2y/(3B^2 + 8y^2)$	$3A/(2y)$	$2/3y$	

*Approximation for the interval $0 < \frac{4y}{B} < 1$.

Example 1: Compute the hydraulic radius and average depth for a trapezoidal channel with a 10 m bottom width, a 5 m flow depth and a side slope t = 2H/1V.



Solution: $B = b + 2ty = 15 + 2 \times 2 \times 5 = 35.0$ m.

$$P = b + 2y\sqrt{1 + t^2} = 15.0 + 2 \times 5\sqrt{5} = 37.36 \text{ m}$$

$$A = \tfrac{1}{2}(15 + 35) \times 5 = 125 \text{ m}^2$$

$$R = \frac{A}{P} = \frac{125}{37.36} = 3.35 \text{ m}$$

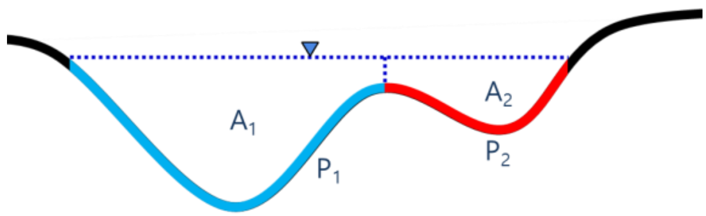
$$D = \frac{A}{B} = \frac{125}{35} = 3.57 \text{ m}$$

1.2. Resistance to flow

Manning's Equation is the primary resistance to flow equation for open channel flows.

$$\mathbf{V} = \frac{1}{n} R_h^{\frac{2}{3}} S_f^{\frac{1}{2}}$$ in SI unit (R_h in m, V in m/s)

$$\mathbf{V} = \frac{1.49}{n} R_h^{\frac{2}{3}} S_f^{\frac{1}{2}}$$ in English unit (R_h in ft, V in ft/s)



$$A = \sum_{i=1}^{N} A_i \qquad P = \sum_{i=1}^{N} P_i \qquad R = \frac{A}{P}$$

Values for n is described in the Table 3.2 and the flow discharge

$$Q = AV = A\frac{1}{n} R_h^{\frac{2}{3}} S_f^{\frac{1}{2}} = kS_f^{\frac{1}{2}}$$ where k is the conveyance parameter

$k = \frac{A R_h^{\frac{2}{3}}}{n}$ (in SI) , or $k = \frac{1.49 A R_h^{\frac{2}{3}}}{n}$ (in Eng.) and the basic discharge relationship becomes

$$\mathbf{Q = kS_f^{\frac{1}{2}}}$$ which can be rewritten as

$$S_f = \left(\frac{Q}{k}\right)^2 = \left[\frac{nQ}{1.49AR^{2/3}}\right]^2 = \left[\frac{nV}{1.49R^{2/3}}\right]^2$$ in English unit

$$S_f = \left(\frac{Q}{k}\right)^2 = \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left[\frac{nV}{R^{2/3}}\right]^2$$ in SI unit

By definition, the **normal depth $y=y_0$** corresponds to steady and uniform flow (i.e. $S_f = S_0$)

$$AR_h^{\frac{2}{3}} = \frac{nQ}{S_0^{1/2}}$$ (at normal depth, $S_f = S_0$ for steady and uniform flow)

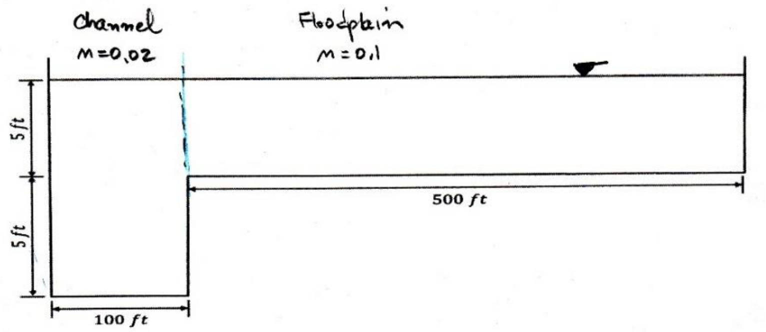
Table 3.2 Darcy-Weisbach *f* and Manning *n* for natural and vegetated channels

Roughness type	Name	Number	Roughness coefficients: Darcy-Weisbach *f* Min.	Mean	Max.	Manning *n* Min.	Mean	Max.
Natural channels (1,865 data)	Sand	172	0.011	**0.115**	2.188	0.014	**0.036**	0.151
	Gravel	989	0.010	**0.251**	6.121	0.011	**0.045**	0.250
	Cobble	651	0.015	**0.465**	21.462	0.015	**0.051**	0.327
	Boulder	53	0.034	**0.794**	14.592	0.023	**0.080**	0.444
Vegetated channels (739 data)	Grass	281	0.016	**0.271**	6.121	0.015	**0.045**	0.250
	Shrub	150	0.015	**0.580**	12.910	0.016	**0.057**	0.250
	Tree	308	0.030	**0.434**	21.462	0.018	**0.047**	0.310

Other typical Manning *n* roughness values	
plexiglass	0.010-0.014
timber and concrete	0.011-0.020
riveted metal	0.015-0.020
corrugated metal	0.022-0.030
straight earth channel	0.013-0.025
sand dunes and ripples	0.018-0.040
antidunes	0.013-0.020
rocky irregular channel	0.026-0.060
overbank short grass	0.025-0.035
tall grasses and reeds	0.030-0.050
floodplain crops	0.020-0.050
sparse vegetation	0.035-0.050
medium to dense brush	0.070-0.016

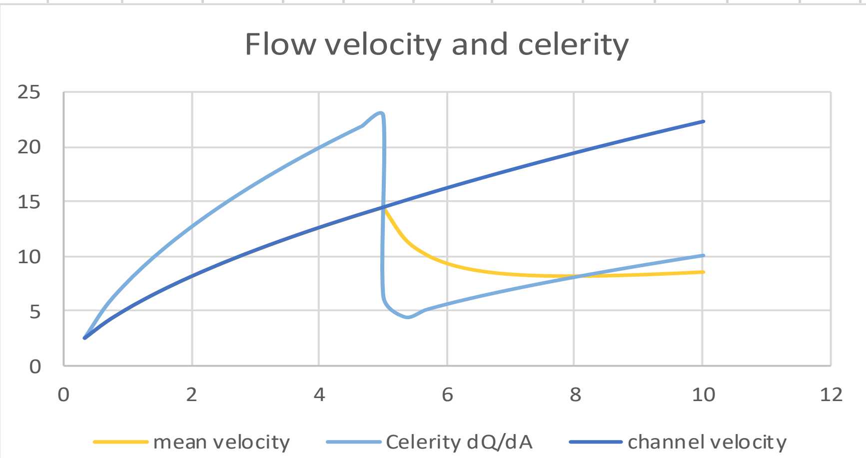
Channel Type	Condition	Manning *n*
Timber lined	Planed, carefully laid	0.010–0.012
Timber lined	Unplanned	0.012–0.014
Concrete lined	Unfinished	0.014–0.020
Concrete lined	Finished	0.011–0.016
Metal lined	Steel, painted	0.012–0.017
Metal lined	Steel, riveted	0.015–0.020
Metal lined	Corrugated	0.022–0.026
Natural earth	Straight, good condition	0.018–0.020
Natural earth	Regular, weathered	0.022–0.025
Natural earth	Regular, stones and weeds	0.028–0.040
Natural earth	Winding, irregular	0.035–0.050
Natural earth	Sluggish, deep pools	0.050–0.080
Natural earth	Weedy, obstructed with debris	0.050–0.15
Overbanks	Short grass	0.025–0.035
Overbanks	High grass	0.030–0.050
Overbanks	Mature crops	0.035–0.050
Overbanks	Light brush and trees	0.040–0.080
Overbanks	Medium to dense brush	0.070–0.16

Example 2: Consider the channel geometry given the slope S = 0.005, with Manning n=0.02 for the main channel and n = 0.1 for the floodplain. Separate the flow into two separate areas for the main channel and floodplain. Calculate every 4" until a maximum stage elevation of 10 ft. Calculate the hydraulic radius, mean flow depth, the flow velocity and discharge for each, and plot the flow velocity and the celerity dQ/dA. Can you repeat the calculations with n = 0.02 on the floodplain and compare the results?



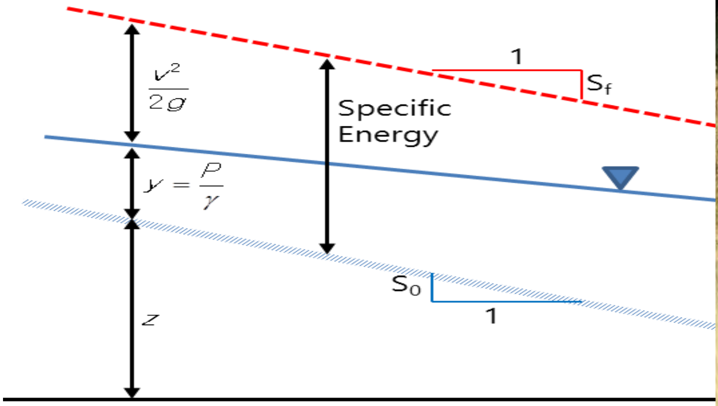
Spread sheet for Q based on R_h

slope 0.005				n channel		0.02						n floodpla		0.1				
			channel						floodplain									
y in	y ft	B ft	A ft2	P ft	Rh ft	n	V ft/s	Q cfs	A ft2	P ft	Rh ft	n	V ft/s	Q cfs	Q tota	A tota	V ft/s	dQ/dA
4	0.3	100	33.33	100.7	0.3	0.02	2.52	84.05							84.05	33.3	2.52	2.52
8	0.7	100	66.67	101.3	0.7	0.02	3.98	265.7							265.7	66.7	3.98	5.45
12	1	100	100	102	1	0.02	5.2	519.9							519.9	100	5.2	7.63
16	1.3	100	133.3	102.7	1.3	0.02	6.27	836.1							836.1	133	6.27	9.49
20	1.7	100	166.7	103.3	1.6	0.02	7.25	1208							1208	167	7.25	11.1
24	2	100	200	104	1.9	0.02	8.15	1629							1629	200	8.15	12.7
28	2.3	100	233.3	104.7	2.2	0.02	8.99	2098							2098	233	8.99	14
32	2.7	100	266.7	105.3	2.5	0.02	9.79	2609							2609	267	9.79	15.4
36	3	100	300	106	2.8	0.02	10.5	3162							3162	300	10.5	16.6
40	3.3	100	333.3	106.7	3.1	0.02	11.3	3753							3753	333	11.3	17.7
44	3.7	100	366.7	107.3	3.4	0.02	11.9	4381							4381	367	11.9	18.8
48	4	100	400	108	3.7	0.02	12.6	5044							5044	400	12.6	19.9
52	4.3	100	433.3	108.7	4	0.02	13.2	5740							5740	433	13.2	20.9
56	4.7	100	466.7	109.3	4.3	0.02	13.9	6469							6469	467	13.9	21.8
60	5	100	500	110	4.5	0.02	14.5	7227							7227	500	14.5	22.8
60	5	100	500	110	4.5	0.02	14.5	7228	0.05	500	0	0.1	0	1E-04	7228	500	14.5	6.35
64	5.3	100	533.3	110.3	4.8	0.02	15.1	8032	167	500	0.33	0.1	0.51	84.42	8117	700	11.6	4.44
68	5.7	100	566.7	110.7	5.1	0.02	15.7	8869	333	501	0.67	0.1	0.8	267.8	9136	900	10.2	5.1
72													1.05	526.2	10262	1100	9.33	5.63
76													1.27	849.5	11482	1300	8.83	6.1
80													1.48	1232	12790	1500	8.53	6.54
84													1.67	1668	14180	1700	8.34	6.95
88													1.85	2156	15650	1900	8.24	7.35
92													2.02	2692	17195	2100	8.19	7.73
96													2.18	3274	18813	2300	8.18	8.09
100													2.34	3901	20501	2500	8.2	8.44
104													2.49	4571	22258	2700	8.24	8.78
108													2.64	5282	24080	2900	8.3	9.11
112													2.78	6033	25967	3100	8.38	9.43
116													2.92	6823	27917	3300	8.46	9.75
120													3.06	7651	29928	3500	8.55	10.1



2. Rapidly-varied Flow   (*week 2*)

2.1. Specific energy in open channels



The total energy level is obtained from the Bernoulli sum

$E_T = z + y + \frac{V^2}{2g}$ and this sum defines the **Energy Grade Line EGL**

Specific energy is the energy level above the channel floor

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} \qquad (Q=AV,\ V=Q/A)$$

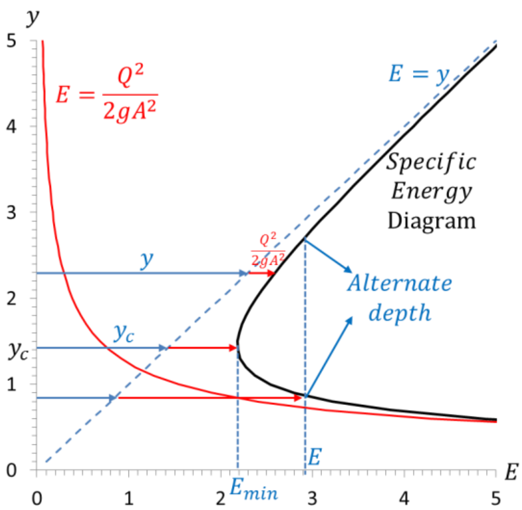
Specific Energy Diagram [Q, g are constant and A=f(h)]

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

Alternate depths: two depths with same energy level

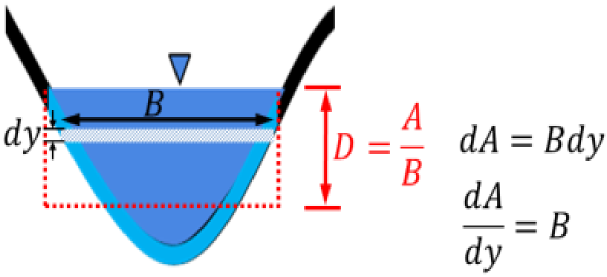
Critical depth: flow depth at the minimum energy level

When the energy level is below the minimum energy level, there is insufficient energy to pass then desired flow discharge. We refer to this condition as **choking** the flow.



This means that the flow will accumulate upstream until the minimum energy level is reached.

Example 3: Let us define the critical flow depth condition for any cross-section geometry with a constant discharge



$E = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2gA^2(y)}$ and

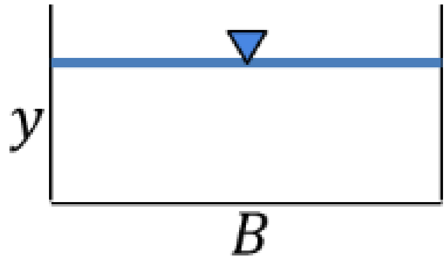
$\frac{dE}{dy} = 0$ when y=y_c (critical depth)

$\frac{dE}{dy} = 1 + \frac{Q^2}{2g}\left(\frac{-2}{A^3(y)}\frac{dA}{dy}\right) = 0$, if y = y_c

$\frac{dE}{dy} = 1 + (-1)\frac{Q^2B}{gA^3} = 0$ => $\boldsymbol{\frac{Q^2B}{gA^3} = 1} = \frac{V^2B}{gA} = \frac{V^2}{gD} = \boldsymbol{Fr^2 = 1}$

※ Also, we find $\frac{V^2}{gD} = 1$ => $\frac{V^2}{g} = D$ => $\frac{V^2}{2g} = \frac{D}{2}$ (when Fr=1)

Example 4: How is the critical depth defined for a rectangular channel? For a rectangular channel *A=By*



$Q = AV = ByV$, the unit discharge $q = \frac{Q}{B} = Vy$

$$\boldsymbol{E = y + \frac{Q^2}{2gA^2}} = y + \frac{Q^2}{2gB^2y^2} = y + \frac{q^2}{2gy^2}$$

Let's find the critical depth when the unit discharge q is constant

$\frac{dE}{dy} = 0$ when y=y_c (critical depth) $\mathrm{E = y + \frac{q^2}{2gy^2}}$

$\frac{dE}{dy} = 1 + \frac{q^2}{2g}\left(\frac{-2}{y^3}\right) = 0$, when y = y_c => $\boldsymbol{1 = \frac{q^2}{gy_c^3}}$ and the critical depth is

$$\therefore \; \boldsymbol{y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}}$$ (for rectangular channels only)

2.2. Specific Energy Applications

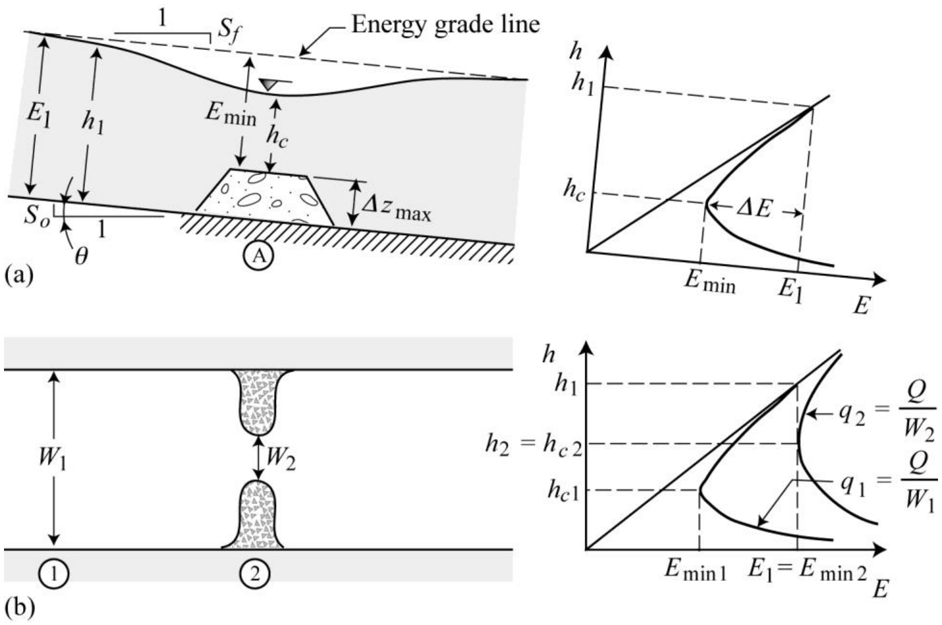
Consider a rectangular channel where the flow depth $y = h$ and width $B = W$

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2gB^2y^2}$$

1) What is the maximum step height Δz_{max} possible in a channel without raising the upstream water level, i.e. without choking the flow?

The maximum step height $\Delta z_{max} = E - E_{min}$.

If $\Delta z > \Delta z_{max}$, it will raise the upstream water level until $E = E_{min} + \Delta z$



2) What is the minimum channel width that will not raise the upstream water level?

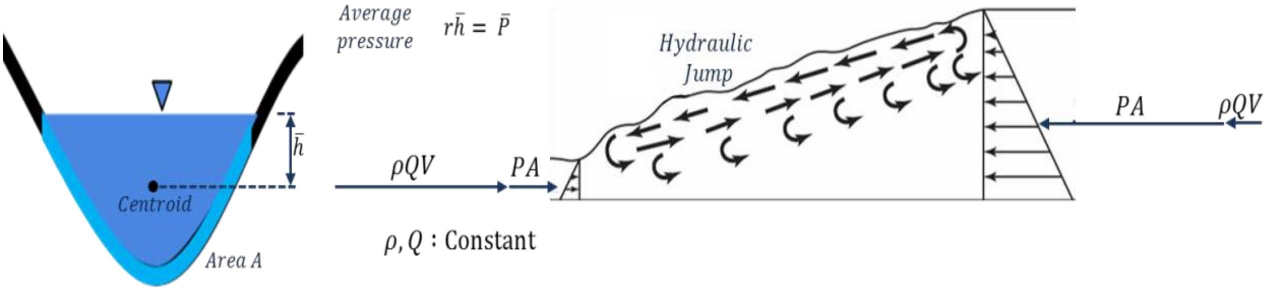
The specific energy at section 1 equals the minimum energy at section 2,

$E_1 = E_{\min 2} = 1.5 y_2$, and at section 2, $1 = \frac{q_2^2}{g y_{c2}^3} = \left(\frac{Q}{B_2}\right)^2 \frac{1}{g y_{2c}^3} = \left(\frac{Q}{B_2}\right)^2 \frac{1}{g\left(\frac{2E_1}{3}\right)^3}$,

and therefore, $B_{2\min} = \frac{Q}{\sqrt{g(0.667E_1)^3}}$.

The maximum lateral contraction $\Delta B_{\max} = B_1 - B_2 = B_1 - \frac{Q}{\sqrt{g(0.667E_1)^3}}$.

2.3. Momentum in open channels



The concept of momentum balances the pressure and momentum forces

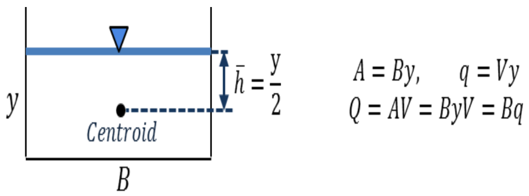
$$F = pA + \rho QV = \gamma \bar{h}A + \rho QV = \gamma \bar{h}A + \frac{\rho Q^2}{A} = \gamma \bar{h}A + \frac{\gamma}{g}\frac{Q^2}{A} \qquad (\gamma = \rho g)$$

Specific momentum is the momentum force divided by the specific weight of the fluid

$$\boldsymbol{M = \frac{F}{\gamma} = \bar{h}A + \frac{Q^2}{gA}}$$

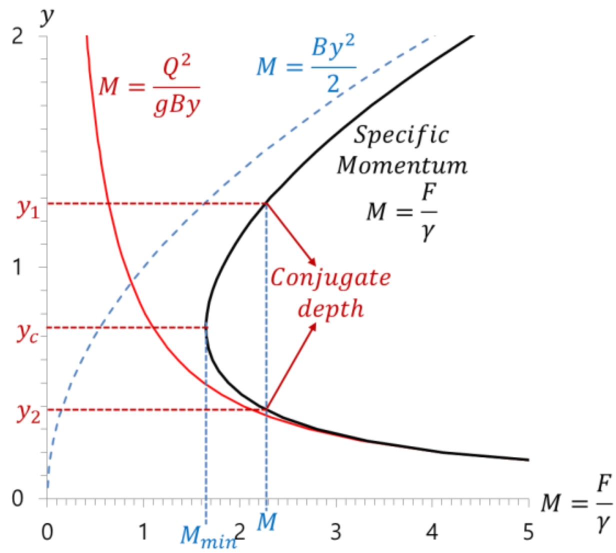
Conjugate (sequent) depths are two flow depths with the same specific momentum

In the case of a rectangular section



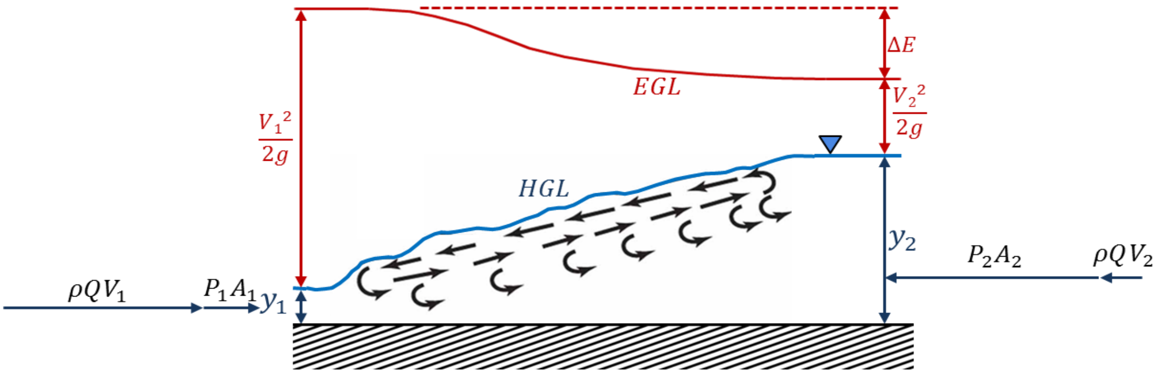
$$M = By\frac{y}{2} + \frac{Q^2}{gBy} = \frac{By^2}{2} + \frac{Q^2}{gBy}$$

where B, Q, g are constants



2.4. Hydraulic jumps

Consider rapidly-varied flow in a rectangular channel.



Here y_1 and y_2 are conjugate depths (same momentum)

$$M = \frac{F}{\gamma} = A_1\overline{h_1} + \frac{Q^2}{gA_1} = A_2\overline{h_2} + \frac{Q^2}{gA_2}$$

We know that $M = \frac{By^2}{2} + \frac{Q^2}{gBy}$ in a rectangular channel, $A = By, \bar{h} = \frac{y}{2}, \frac{Q}{B} = q$

$\frac{M}{B} = \frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_2}$, or $\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{q^2}{gy_2} - \frac{q^2}{gy_1} = \frac{q^2}{g}[\frac{1}{y_2} - \frac{1}{y_1}]$ or

$\frac{1}{2}(y_1 + y_2)(y_1 - y_2) = \frac{q^2}{g}\left[\frac{(y_1-y_2)}{y_1y_2}\right]$ or $y_1y_2(y_1 + y_2) = \frac{2q^2}{g}$

Which is a quadratic equation to be solved for y_1 or y_2 as

$$\boldsymbol{y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right], \text{ or } \quad y_1 = \frac{y_2}{2}\left[-1 + \sqrt{1 + 8Fr_2^2}\right]}$$

The energy loss in the hydraulic jump is

$\Delta E = y_1 + \frac{q^2}{2gy_1^2} - \left[y_2 + \frac{q^2}{2gy_2^2}\right] = (y_1 - y_2) + \frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right]$, which is combined with $y_1y_2(y_1 + y_2) = \frac{2q^2}{g}$ to eliminate q and g such that

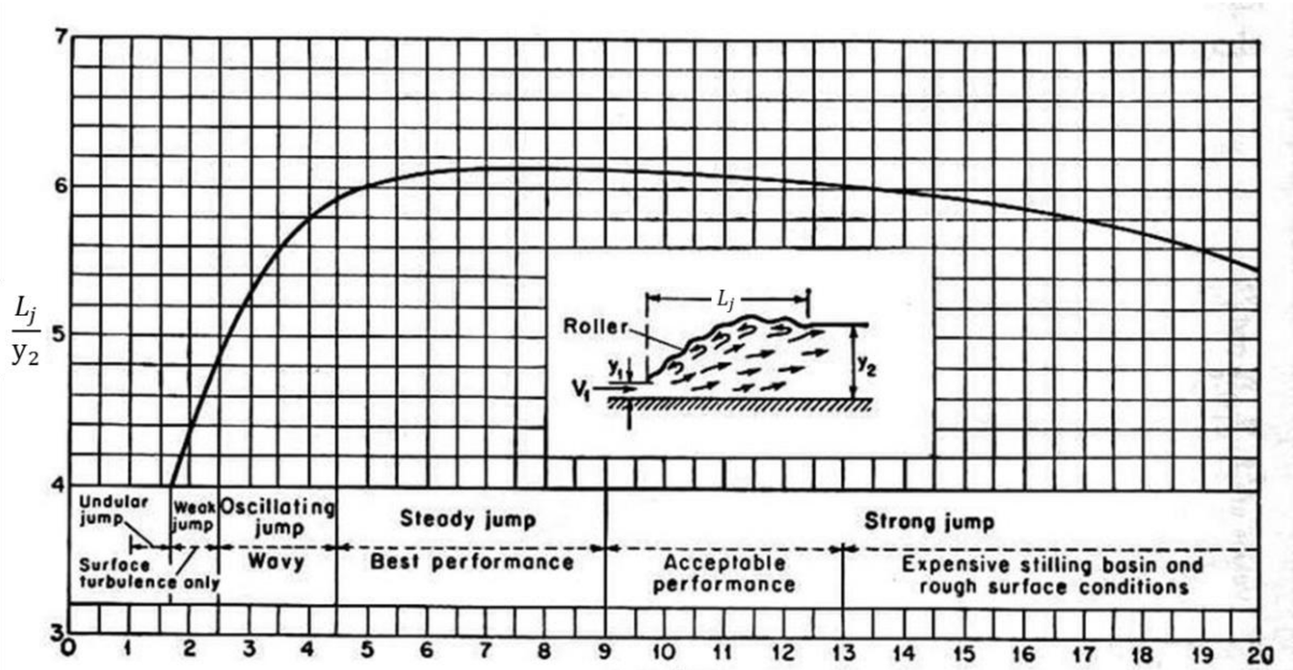
$$\Delta E = (y_1 - y_2) + \frac{y_1y_2(y_1 + y_2)}{4}\left[\frac{y_2^2 - y_1^2}{y_1^2y_2^2}\right] = (y_2 - y_1)\left[-1 + \frac{(y_1 + y_2)^2}{4y_1y_2}\right]$$

$$\Delta E = (y_2 - y_1)\left[\frac{(y_2 - y_1)^2}{4y_1y_2}\right]$$

Head loss, ΔE in a hydraulic jump is therefore

$$\boldsymbol{\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}}$$

Hydraulic jumps change supercritical flow to subcritical flow. The two depths are conjugate depths. The length of the hydraulic jump depends on the upstream Froude number and is scaled to the downstream flow depth.

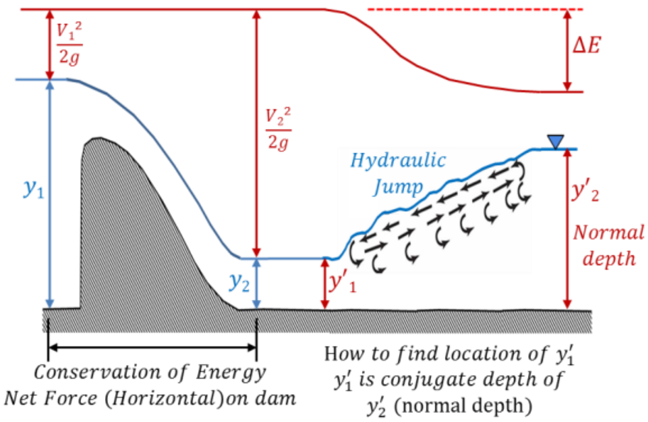


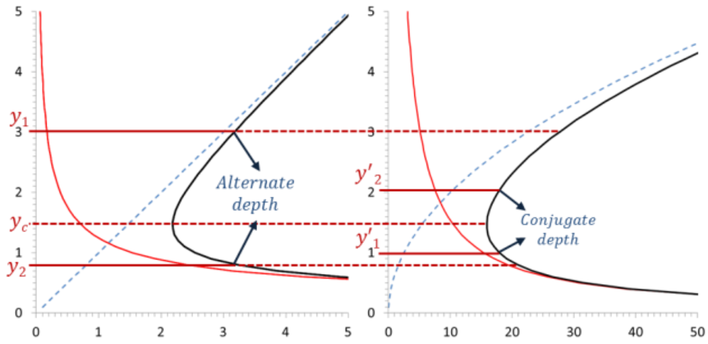
2.5. Momentum and/or energy

The question often arises in rapidly varied flows as to when do we use momentum and when do we use energy concepts. The answer is usually that potential energy can be converted into kinetic energy in accelerating flows. However, the conversion of kinetic energy into potential energy usually dissipates energy while momentum is conserved.

$$E = y + \frac{q^2}{2gy^2} \qquad Energy$$

$$\frac{M}{B} = \frac{y^2}{2} + \frac{q^2}{gy} \qquad Momentum$$





Example 5: Construct the specific energy and specific momentum diagrams for a trapezoidal channel section with b=2 m, t=1, Q=6.0 m^3/s, and y_n=2.0. Find the critical depth. Solve it using a spreadsheet and also plot the specific energy diagram on the same graph. [Hint: use E/Emin and M/Mmin on the horizontal axis.]

$$\frac{Q^2 B}{gA^3} = 1 \quad (when \quad y = y_c), \qquad \frac{Q^2}{g} = \frac{A^3}{B} = \frac{36}{9.8} = \frac{[(2 + 1 \times y_c)y_c]^3}{2 + 2 \times 1 \times y_c}$$

(Goal seek by Excel or calculator) $y_c = 0.839m$

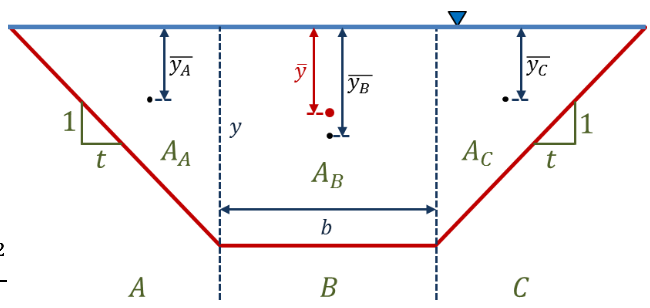
$$M = \frac{F}{\gamma} = A\bar{y} + \frac{Q^2}{gA}$$

(Geometrical moment of area) for $\bar{y}$

$$\bar{y}A = \overline{y_A}A_A + \overline{y_B}A_B + \overline{y_C}A_C$$

$$\bar{y}(b + ty)y = \frac{y}{3}\frac{ty^2}{2} + \frac{y}{2}by + \frac{y}{3}\frac{ty^2}{2}$$

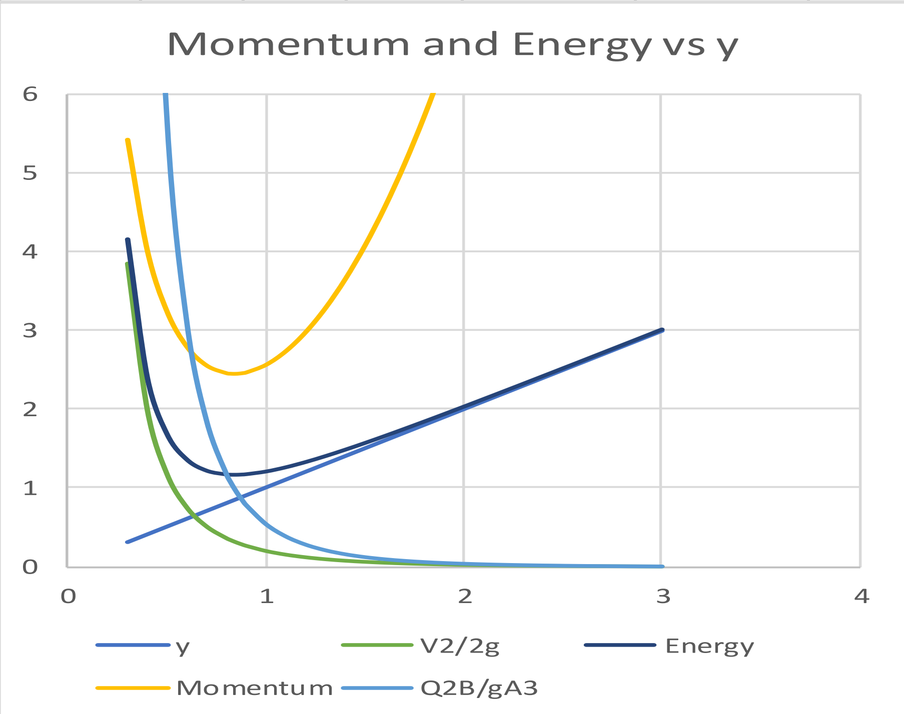
$$\bar{y}(by + y^2) = \frac{2ty^3}{6} + \frac{by^2}{2} = \frac{2ty^3 + 3by^2}{6}$$



$$\bar{y}=\frac{2ty^3+3by^2}{6(by+y^2)}=\frac{2ty^2+3by}{6(b+y)}=\frac{y}{6}\left(\frac{2\times 1\times y+3b}{b+y}\right)=\frac{y}{6}\left(\frac{2y+6}{2+y}\right)=\frac{y}{3}\left(\frac{3+y}{2+y}\right)$$

$$M=\frac{F}{\gamma}=A\bar{y}+\frac{Q^2}{gA}=(2+y)y\frac{y}{3}\left(\frac{3+y}{2+y}\right)+\frac{6^2}{9.81\times(2+y)y}$$

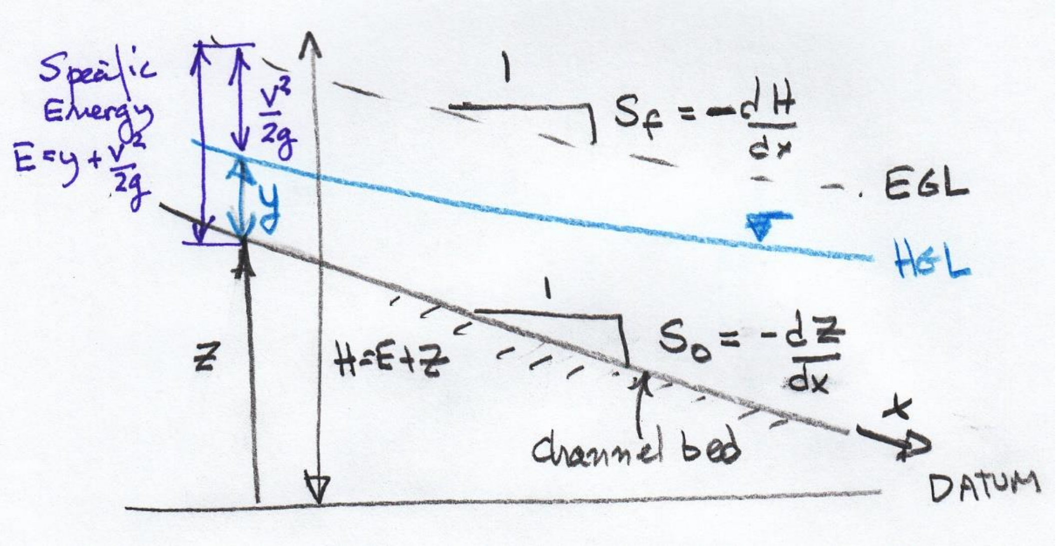
trapezoidal		t = 1			b = 2	Q = 6	
y	B	A	ybar	M	Q2B/gA3	v2/2g	E
m	m	m2	m				
0.1	2.2	0.21	0.05	17.49	871.7627	41.60686	41.70686
0.2	2.4	0.44	0.1	8.383	103.3919	9.477595	9.677595
0.3	2.6	0.69	0.14	5.417	29.04421	3.853943	4.153943
0.4	2.8	0.96	0.19	4.004	11.61389	1.990953	2.390953
0.5	3	1.25	0.23	3.227	5.636697	1.174312	1.674312
0.6	3.2	1.56	0.28	2.784	3.093212	0.75397	1.35397
0.7	3.4	1.89	0.32	2.546	1.848107	0.513665	1.213665
0.8	3.6	2.24	0.36	2.449	1.175417	0.365685	1.165685
0.82	3.64	2.31	0.37	2.443	1.080304	0.343145	1.163145
0.83	3.66	2.35	0.37	2.442	1.036385	0.332563	1.162563
0.839	3.677	2.38	0.38	2.441	1.000809	0.323906	1.162406
0.85	3.7	2.42	0.38	2.442	0.955089	0.312663	1.162663
0.86	3.72	2.46	0.39	2.444	0.917451	0.303301	1.163301
0.88	3.76	2.53	0.4	2.45	0.84761	0.285662	1.165662
0.9	3.8	2.61	0.4	2.459	0.784324	0.269353	1.169353
1							3874
1.1							7796
1.2							4435
1.3							9699
1.4							0982
1.5							5571
1.6							5304
1.7							6377
1.8							9219
1.9							3417
2							2867
2.1							4751
2.2							1491
2.3							3759
2.4							5454
2.5							4498
2.6							2827
2.7							1394
2.8							0158
2.9	7.8	14.2	1.16	16.8	0.009976	0.009087	2.909087
3	8	15	1.2	18.24	0.008699	0.008155	3.008155



3. Gradually-varied Flow (*week 3*)

In gradually-varied flows, the hydraulic conditions (depth and velocity) change slowly in the downstream direction.

3.1. Gradually-varied flow equation



H= *Z* + *E* such that $\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dE}{dx}$, and we remember $Fr^2 = \frac{Q^2 B}{gA^3}$, with $dA = Bdy$

From the figure, $S_f = -\frac{dH}{dx}$, and $S_0 = -\frac{dZ}{dx}$,

$$\frac{dH}{dx} = -S_f = -S_0 + \frac{d}{dx}\left[y + \frac{V^2}{2g}\right] = -S_0 + \frac{d}{dx}\left[y + \frac{Q^2}{2gA^2}\right]$$

$$S_0 - S_f = \frac{d}{dy}\left[y + \frac{Q^2}{2gA^2}\right]\frac{dy}{dx} \text{ with } Q \text{ and } g \text{ constant}$$

$$S_0 - S_f = \left[\frac{dy}{dy} + \frac{Q^2}{2g}\left(\frac{-2}{A^3}\right)\frac{dA}{dy}\right]\frac{dy}{dx} = \left[1 - \frac{Q^2 B}{gA^3}\right]\frac{dy}{dx} = (1 - Fr^2)\frac{dy}{dx}$$

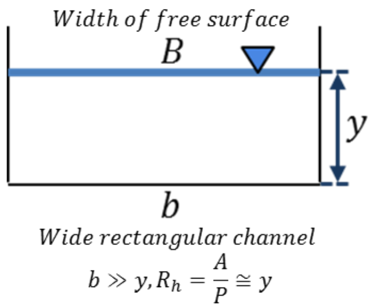
$$\boldsymbol{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

This is the general form of the backwater equation for open channels. The backwater equation is used to calculate changes in flow depth y as a function of x.

3.2. Rectangular channels

Let's examine the case of backwater in a rectangular channel.

$A = by;$
$P = b + 2y;$
$q = Vy$
$Q = bq$



1) From the definition of **critical depth**,

$$Fr^2 = \frac{V^2}{gy} = \frac{q^2}{gy^3} = \left(\frac{y_c}{y}\right)^3$$

$$\rightarrow \ \mathbf{1 - Fr^2 = 1 - \left(\frac{y_c}{y}\right)^3}$$

Note that $\boldsymbol{y_c = \sqrt[3]{\frac{q^2}{g}}}$ (critical depth when $y = y_c,\ Fr_c = 1$)

2) Resistance to flow (Manning equation) defines the **normal flow depth**

$$q = yV = y\frac{1}{n}y^{\frac{2}{3}}S_f^{\frac{1}{2}} = y\frac{1.49}{n}y^{\frac{2}{3}}S_f^{\frac{1}{2}}$$

(SI) (English)

In SI, $q = yV = y\frac{1}{n}y^{\frac{2}{3}}S_f^{\frac{1}{2}} = \frac{S_f^{\frac{1}{2}}}{n}y^{\frac{5}{3}} \ \rightarrow \ y = \left(\frac{nq}{S_f^{\frac{1}{2}}}\right)^{\frac{3}{5}}$, or $S_f = \left(\frac{nq}{y^{\frac{5}{3}}}\right)^2$

3) In steady-uniform flow, **normal depth** $\boldsymbol{y = y_n}$ **when** $\boldsymbol{S_f = S_0}$,

$y_n = \left(\frac{nq}{S_0^{\frac{1}{2}}}\right)^{\frac{3}{5}}$, and $S_0 = \left(\frac{nq}{y_n^{\frac{5}{3}}}\right)^2$, and $\boldsymbol{\frac{S_f}{S_0} = \left(\frac{nq}{y^{\frac{5}{3}}}\right)^2 \left(\frac{y_n^{\frac{5}{3}}}{nq}\right)^2 = \left(\frac{y_n}{y}\right)^{\frac{10}{3}}}$

And the gradually-varied flow equation for rectangular channels becomes

$$\boldsymbol{\frac{dy}{dx}} = \frac{S_0\left[1 - \left(\frac{S_f}{S_0}\right)\right]}{\left[1 - F_r^2\right]} = \boldsymbol{\frac{S_0\left[1 - \left(\frac{y_n}{y}\right)^{\frac{10}{3}}\right]}{\left[1 - \left(\frac{y_c}{y}\right)^3\right]}}$$

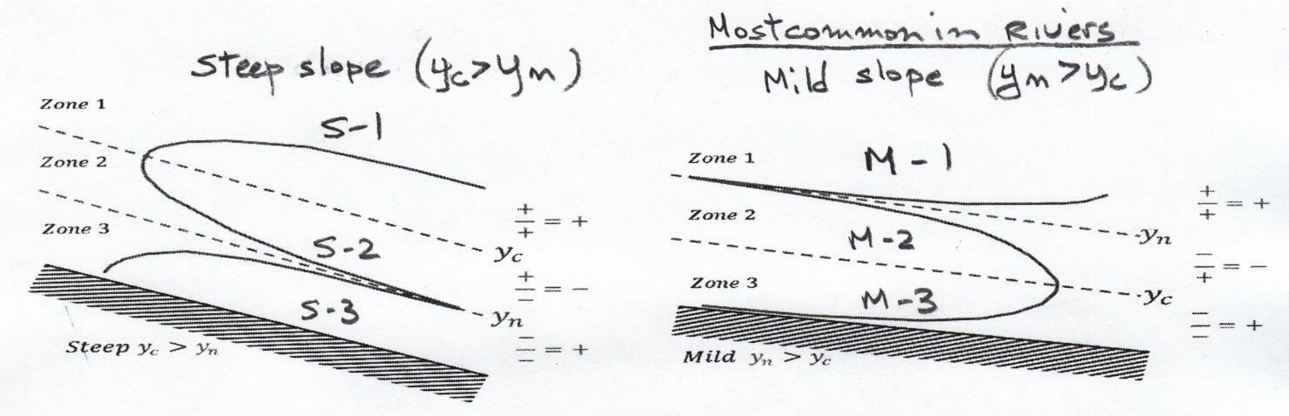
3.3. Backwater profiles

3.1.1. *Shape of backwater profiles*

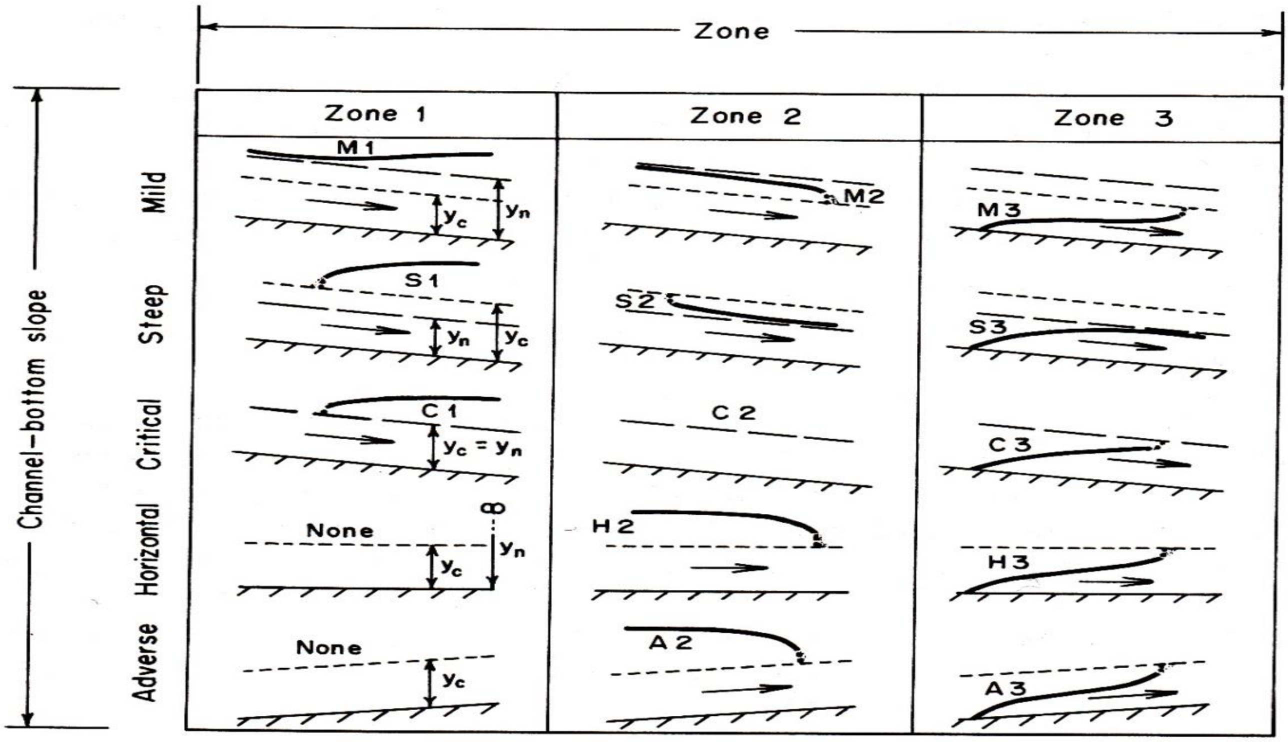
There are three depths involved in backwater calculations: (1) the flow depth y; (2) the normal depth y_n ; and (3) the critical depth y_c. We can then examine whether dy/dx is positive or negative from $\frac{dy}{dx} = \frac{S_0\left[1-\left(\frac{y_n}{y}\right)^{\frac{10}{3}}\right]}{\left[1-\left(\frac{y_c}{y}\right)^3\right]}$

➔ If $y > y_n$, the numerator is $(+)$ ‖ If $y = y_n$, $\frac{dy}{dx} = 0$

➔ If $y > y_c$, the denominator is $(+)$ ‖ If $y = y_c$, $\frac{dy}{dx} = \infty$



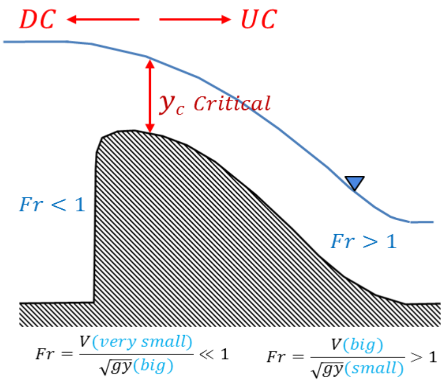
There are five general backwater types: (1) mild when $y_n > y_c$; (2) steep when $y_n < y_c$; (3) critical when $y_n = y_c$; (4) horizontal when $S_0 = 0$; and (5) adverse when $S_0 < 0$.

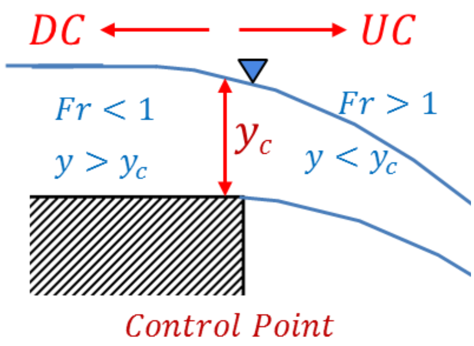


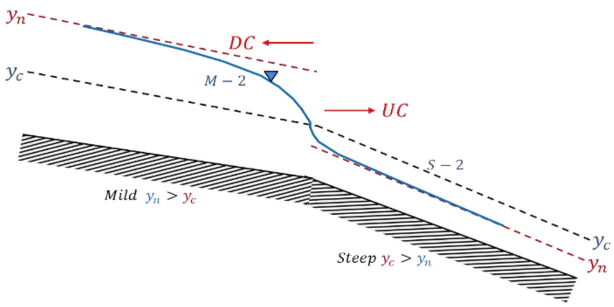
3.1.2. *Control points*

Sub-critical flow Fr < 1, or y > y_c requires downstream control (*DC*) which means that the flow depth depends on a downstream boundary condition (at a dam, a fall, etc.).

Likewise, super-critical flow Fr > 1, y < y_c requires upstream control (*UC*)





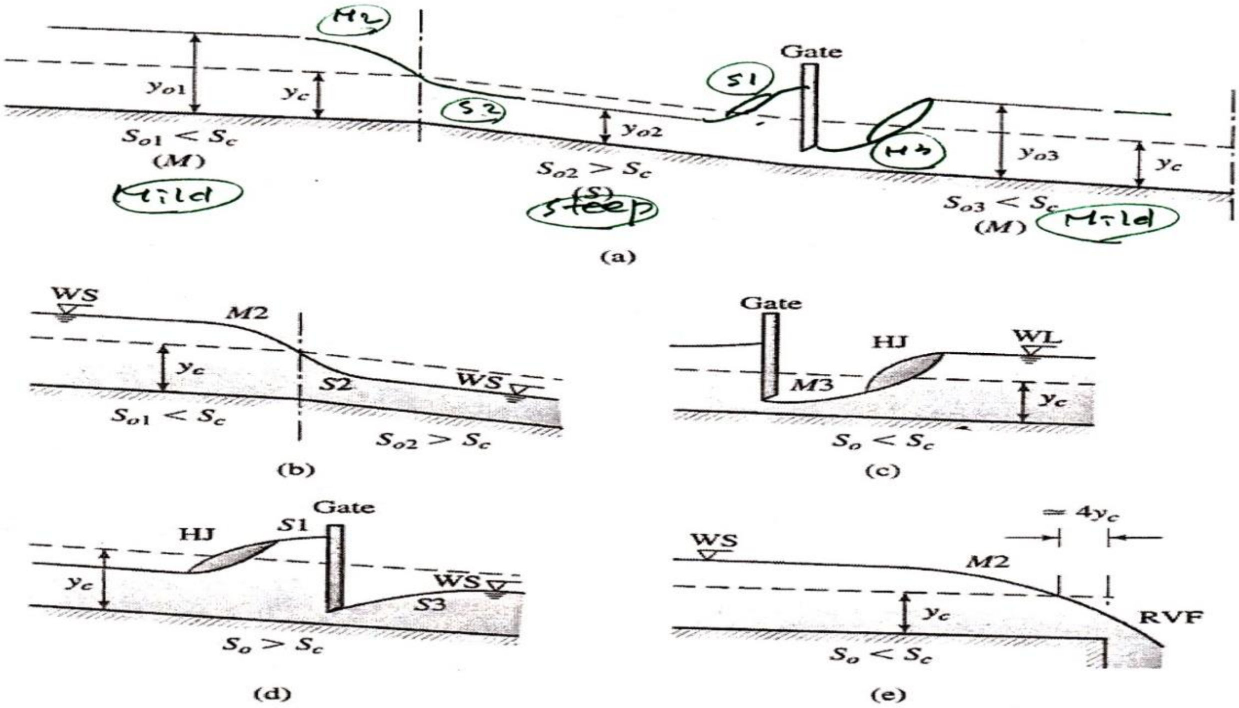


Can you locate the control point on these two photos?

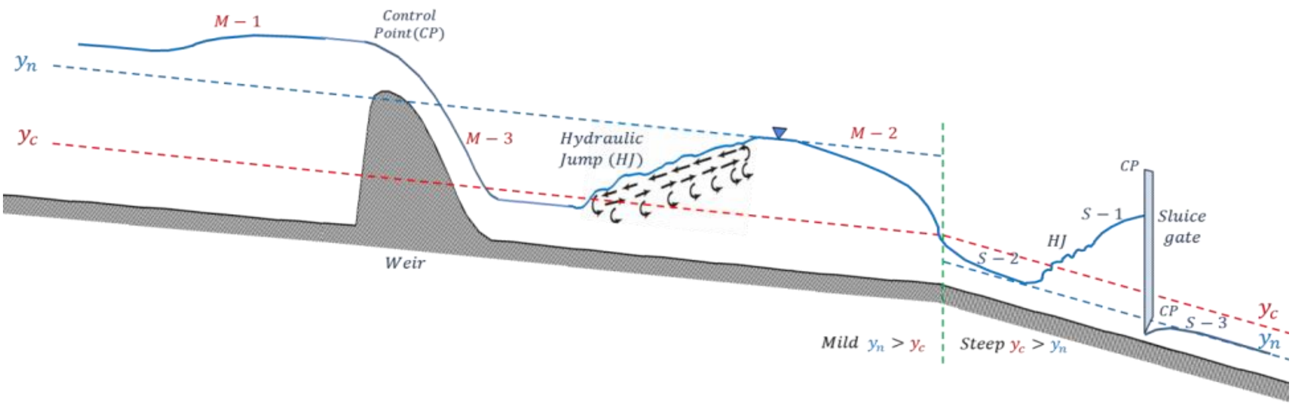
3.1.3. *Sketches of backwater profiles*

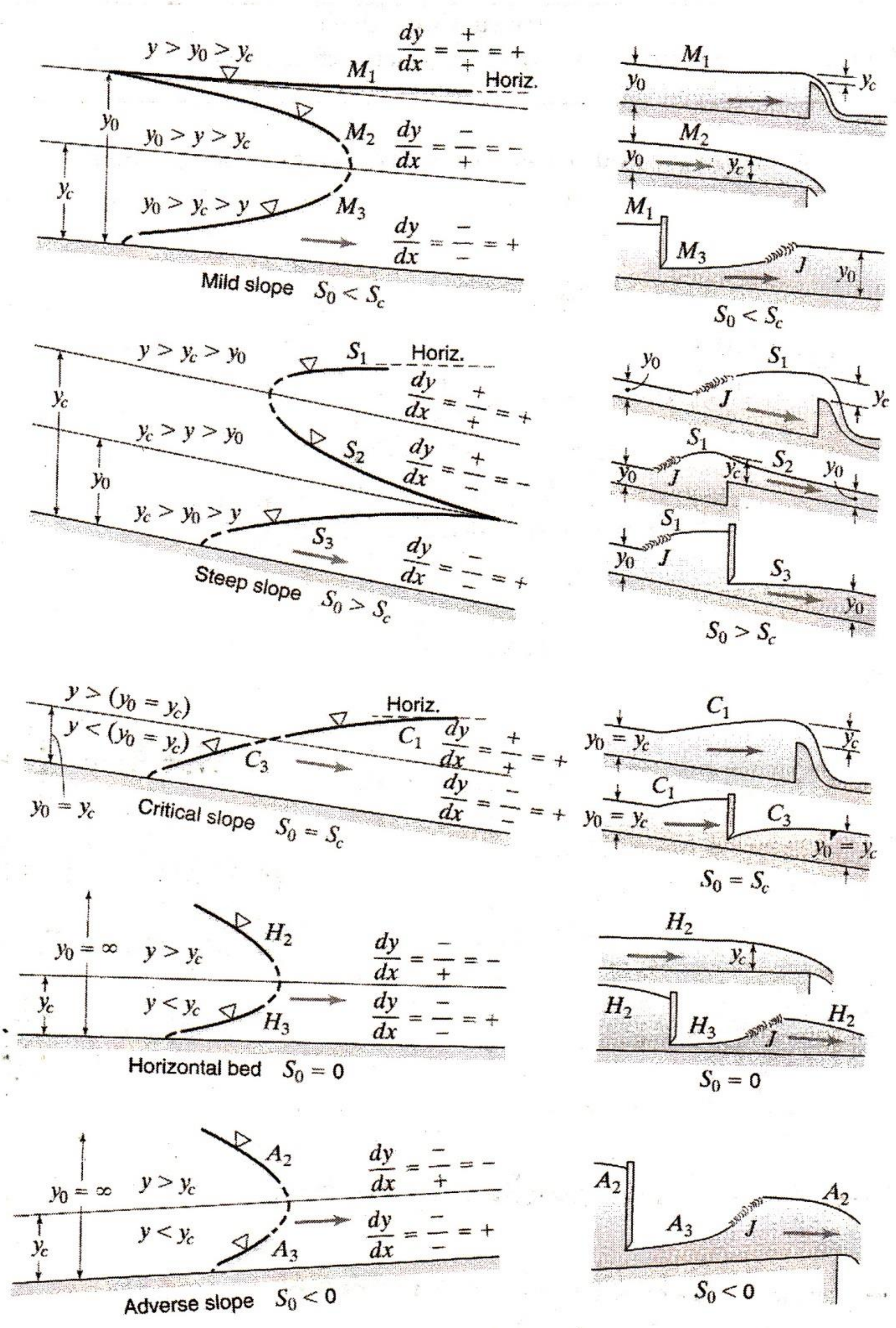
Water surface procedure

1. Determine the bed slope (mild, steep or critical…)
2. Draw the critical depth line (notice that y_c is independent of slope)
3. Draw the normal depth line (y_n depends on slope)
4. Find the control points
5. Proceed from the control points: (1) upstream when $y > y_c$; and (2) downstream when $y > y_c$
6. Sketch the appropriate water surface profile.
7. Identify the backwater profile types (M-1, S-2, etc.), the control points (CP), the hydraulic jumps (HJ).



Example 6: Sketch the water surface profile for the case of a sluice gate in a steep reach downstream of a weir on a mild slope.




y > y0 > yc
dy/dx = +/+ = +
M1
Horiz.
y0
y0 > y > yc
M2
dy/dx = −/+ = −
yc
y0 > yc > y
M3
dy/dx = −/− = +
Mild slope S0 < Sc
S0 < Sc
J
y > yc > y0
S1
Horiz.
dy/dx = +/+ = +
yc > y > y0
S2
dy/dx = +/− = −
yc > y0 > y
S3
dy/dx = −/− = +
Steep slope S0 > Sc
S0 > Sc
y > (y0 = yc)
y < (y0 = yc)
Horiz.
C1
C3
dy/dx = +/+ = +
dy/dx = −/− = +
y0 = yc
Critical slope S0 = Sc
S0 = Sc
H2
y0 = ∞
y > yc
y < yc
H3
dy/dx = −/+ = −
dy/dx = −/− = +
Horizontal bed S0 = 0
S0 = 0
A2
A3
dy/dx = −/+ = −
dy/dx = −/− = +
Adverse slope S0 < 0
S0 < 0

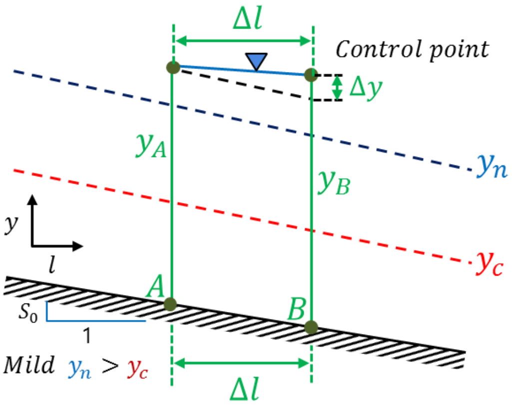
3.4. Backwater calculations

We are ready to calculate backwater curves

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{S_0\left[1-\left(\frac{y_n}{y}\right)^{\frac{10}{3}}\right]}{\left[1-\left(\frac{y_c}{y}\right)^3\right]}$$

First calculate y_c and y_n,

and compare with y



$$(Super-Critical) \quad y_B = y_A + \Delta y\,,\ upstream\ control$$

$$(Sub-Critical) \quad y_A = y_B - \Delta y\,,\ downstream\ control$$

There are two basic methods to calculate backwater profiles: (1) the standard step method uses a constant Δx; and (2) the direct step method uses a fixed Δy.

Standard-step method Fix Δx find Δy $$\Delta y = S_0 \Delta x \frac{\left[1-\left(\frac{y_n}{y}\right)^{\frac{10}{3}}\right]}{\left[1-\left(\frac{y_c}{y}\right)^3\right]}$$

Direct-step method Fix Δy find Δx $$\Delta x = \frac{\Delta y\left[1-\left(\frac{y_c}{y}\right)^3\right]}{S_0\left[1-\left(\frac{y_n}{y}\right)^{\frac{10}{3}}\right]}$$

Example 7: A smooth (n = 0.012) rectangular channel is 100 ft wide with a bed slope of 1:6,000 and a normal depth of 5 ft. A dam across the river raises the water surface to a depth of 9.8 ft upstream of the dam. Calculate the backwater profile from the standard step method.

$$B = 100\,ft,\ S_0 = \frac{1}{6{,}000},\ y_n = 5\,ft,\ y = 9.8\,ft,\ n = 0.012$$

$$A_n = By_n = 500\,ft^2, \qquad P_n = B + 2y_n = 110\,ft,$$

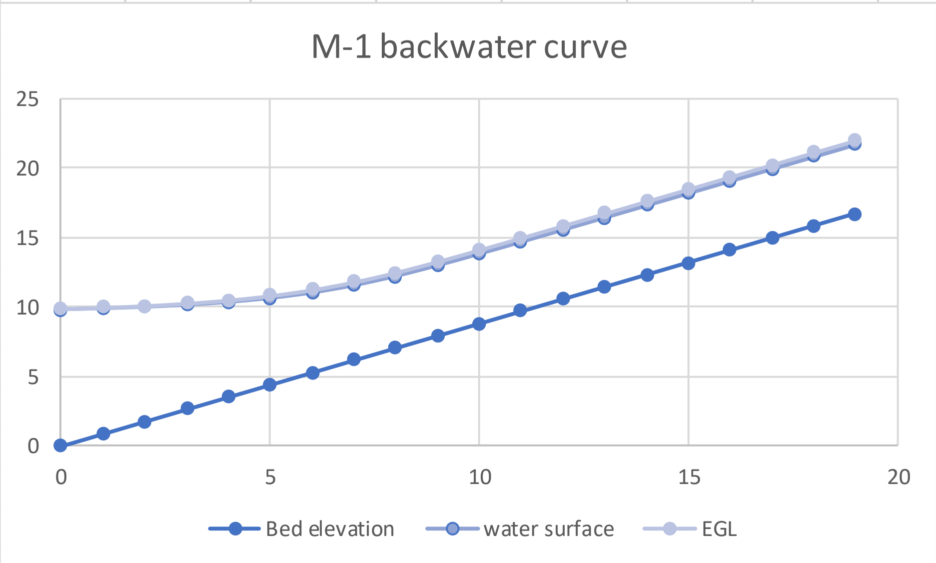
$$R_n = \frac{A_n}{P_n} = \frac{500}{110} = 4.54\,ft$$

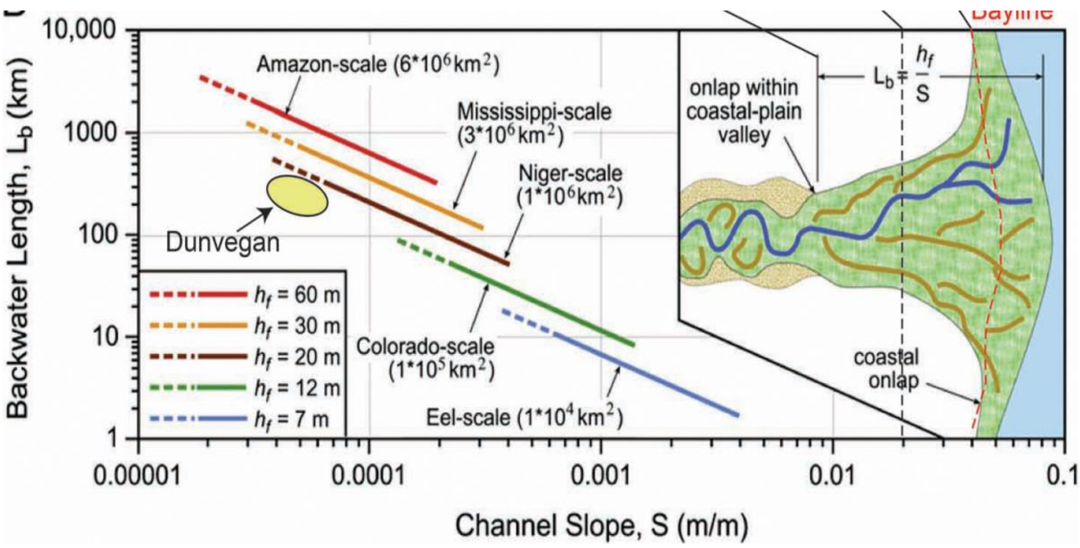
$$V_n = \frac{1.49}{n} R^{\frac{2}{3}} S_0^{\frac{1}{2}} = 4.39\,ft/s, \qquad q = V_n y_n = 21.97\,ft^2/s,$$

$$Q = A_0 V_0 = 2{,}200\,ft^3/s$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{21.97^2}{32.2}} = 2.46\,ft \quad \rightarrow \quad y_0 > y_c \quad \rightarrow \quad Mild\ Slope$$

	English Units					
	yn =	5	ft2/s	yc =	2.46	ft
	S0 =	0.000167		n =	0.012	
	q =	21.97	ft2/s	dx =	5,280	ft
x	x	y	dy	bed	HGL	EGL
ft	mi	ft	ft	ft	ft	ft
0	0	9.8	0.799253	0	9.8	9.878041
5280	1	9.000747	0.771747	0.88	9.880747	9.973263
10560	2	8.229	0.73237	1.76	9.989	10.09968
15840	3	7.496631	0.675759	2.64	10.13663	10.27
21120	4	6.820872	0.595383	3.52	10.34087	10.50197
26400	5	6.225489	0.486223	4.4	10.62549	10.81888
31680	6	5.739267	0.351992	5.28	11.01927	11.24681
36960	7	5.387275	0.214133	6.16	11.54728	11.80552
42240	8	5.173142	0.105774	7.04	12.21314	12.49321
47520	9	5.067367	0.043356	7.92	12.98737	13.27925
52800	10	5.024012	0.015796	8.8	13.82401	14.12095

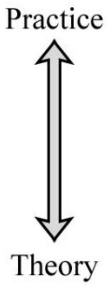


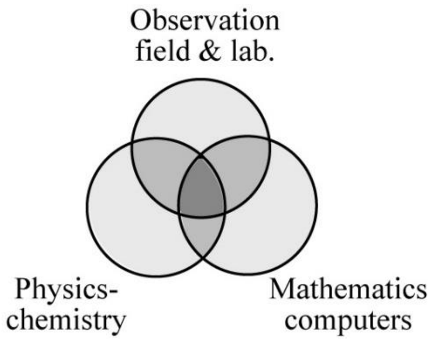


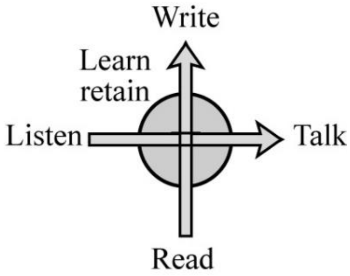
4. Society and Professional Obligations (*week 4*)

This section broadens the scope of the realm of hydraulic engineering beyond technical calculations and numerical models.

The attentive reader can already extract some key elements of hydraulic engineering.







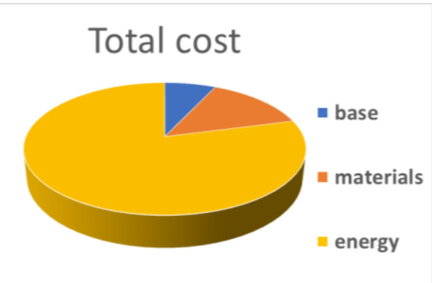
- There is a need to balance theory and practice. As sketched here, we readily understand that engineers seek practical applications but also need to understand the underlying theory and concepts behind their designs. New technical problems experienced in practice define the needs for additional research and better theoretical understanding of the underlying causes. Likewise, sound theoretical understanding can help the engineer solve problems outside the realm of standard methods.
- In the second diagram, there is triangular aspect to engineering in a counterclockwise loop starting with an observation of the facts. The second step requires physical understanding of the problem we are trying to solve. This step usually requires a combination of the first principles of conservation of mass, energy and momentum to define an analytical solution. And it is only when the problem is clearly understood that we can start to solve it mathematically. Numerical solutions are developed in terms of mathematical models which provide quantitative results. With a quantified solution at hand, we can then return where we started and compare the results of our analytical understanding and numerical models with the field observations. We can start with simple models and go around the cycle with added complexity until satisfactory results are obtained.
- The last diagram illustrates the importance of communication along two main poles. Verbal communication involves listening and talking. Written communication requires reading and writing skills. Written communication skills can be as short as a text message and as long as a thesis or dissertation.

4.1. Technology, the common good and sustainability

Hydraulic engineering is viewed here in a societal context with a discussion on the common good and our professional obligations. The development of technology involves the application of scientific knowledge to solve problems to the benefit of society. Civil engineers seek to raise the standards of living of their communities. This usually requires a direct interaction with the environment. Engineers often make recommendations (and even very important decisions) for the common good of the entire society. We also minimize the adverse impact of projects in terms of energy waste and pollution.

In the interaction with communities, engineers are seeking the best interest of the entire society, and not solely the interests of the majority, ethnic minorities, fringe groups, lobbying entities, let alone influential individuals. Some of the methods to seek the best interests of society can be quantified. For instance, a benefit-cost analysis can indicate which option in a design would be desirable. The lowest cost approach is a convincing tool to demonstrate that one alternative is preferable. For instance, in your pipe and pump problem, the determination of an optimal pump size and pipe diameter has been done in two different ways in Part II and Part III of this class. The lowest cost of materials was considered in Part II while you are now including the energy costs – does this change anything?

In order to cope with unexpected changes to your analysis, the following guidelines should be considered. When the solution can be quantified, it is important to find out which component contributes the major portion of your total cost. For the example shown here, the energy cost covers an overwhelming fraction of the total. Keeping a sense of proportions can help you in your analyses. For instance, the impact of doubling base construction costs would be small compared to doubling the energy costs.

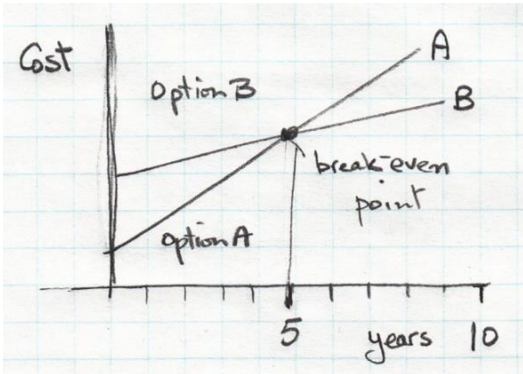


Going back to your pump and pipe problem, can you examine the distribution of costs in your project (pump, pipes, energy…)? In the analysis of unexpected circumstances, you may be able to associate a cost increment ΔC, which is usually perceived as a source of conflict. However, several solutions can be explored in terms of seeking ways to reduce this added cost increment ΔC.

Example 8: You designed a project assuming a pump efficiency of 100% while the pump that you installed has a 50% efficiency, your client is not be happy to cover energy costs twice what you had anticipated. Does the energy constitute a main component of the total cost? Well, even when it does, the incremental cost still allows you to consider the possibility of reducing ΔC. In this case, replacing the pump with a more efficient one could significantly reduce ΔC.

In terms of long-term sustainability, the pump and pipe system is also a very good example where you can evaluate how you plan to dispose of the pumps and the pipes at the end of their service life. Are there contaminants involved in your project? Do you need to replace the materials? Also, now that you are including the energy consumption costs to your pipe and pump problem, can you consider the amount of energy that would only contribute to global warming. For instance, if you had selected a pump with a 65% efficiency, can you figure out what fraction of this energy (and associated wasted cost) would only contribute to global warming? Can you eliminate waste and achieve long-term sustainability?

Most engineering situations involve interaction with others. Even when the problems can be quantified, the perception for different groups and individuals can be completely different from your own. For instance, if you look at the two options shown in this sketch, it is quite obvious that option B has the lowest long-term cost and is preferable for the community. But what if the decision is made by an executive who has been appointed for 3 years and believes that option A is preferable because it has the lowest cost during his mandate? Here, there is a need to separate the interests of the community and the interest of the individual.



Some unexpected circumstances cannot be quantified and communication skills become of foremost importance. The ability to place yourself in someone else skin may help you understand their situation and their interests. Not that you want to fulfill their own wishes and expectations, but it will help you better understand a problematic in a broader perspective. The ability to explain your design in simple terms that can relate to the entire public is very important for engineers. As readily mentioned, communication skills can be developed along two main axes: (1) the ability to read and write; and (2) the ability to listen and speak. These skills can sometimes become as important as the ability to calculate correctly and to solve difficult technical problems. Like any other skill, things get better with practice. While students typically develop the ability to listen and read, they are encouraged to take every opportunity to write and speak. In learning to speak publicly, do not be afraid to look bad. The benefits from developing your ability to speak openly in public is way more important than the fear of saying something inaccurate. The next section covers a lot more information regarding the specific obligations of engineers.

4.2. Professional engineering obligations

From the National Society of Professional Engineers.

https://www.nspe.org/resources/ethics/code-ethics

1. **Engineers shall be guided in all their relations by the highest standards of *honesty and integrity*.**
   a. Engineers shall acknowledge their errors and shall not distort or alter the facts.
   b. Engineers shall advise their clients or employers when they believe a project will not be successful.
   c. Engineers shall not accept outside employment to the detriment of their regular work or interest. Before accepting any outside engineering employment, they will notify their employers.
   d. Engineers shall not attempt to attract an engineer from another employer by false or misleading pretenses.
   e. Engineers shall not promote their own interest at the expense of the dignity and integrity of the profession.

2. **Engineers shall at all times strive to serve the public interest.**
   a. Engineers are encouraged to participate in civic affairs; career guidance for youths; and work for the advancement of the safety, health, and well-being of their community.
   b. Engineers shall not complete, sign, or seal plans and/or specifications that are not in conformity with applicable engineering standards. If the client or employer insists on such unprofessional conduct, they shall notify the proper authorities and withdraw from further service on the project.
   c. Engineers are encouraged to extend public knowledge and appreciation of engineering and its achievements.
   d. Engineers are encouraged to adhere to the principles of sustainable development in order to protect the environment for future generations.

3. **Engineers shall avoid all conduct or practice that deceives the public.**
   a. Engineers shall avoid the use of statements containing a material misrepresentation of fact or omitting a material fact.
   b. Consistent with the foregoing, engineers may advertise for recruitment of personnel.
   c. Consistent with the foregoing, engineers may prepare articles for the lay or technical press, but such articles shall not imply credit to the author for work performed by others.

4. **Engineers shall not disclose, without consent, confidential information** concerning the business affairs or technical processes of any present or former client or employer, or public body on which they serve.
   a. Engineers shall not, without the consent of all interested parties, promote or arrange for new employment or practice in connection with a specific project for which the engineer has gained particular and specialized knowledge.
   b. Engineers shall not, without the consent of all interested parties, participate in or represent an adversary interest in connection with a specific project or proceeding in which the engineer has gained particular specialized knowledge on behalf of a former client or employer.

5. **Engineers shall not be influenced in their professional duties by conflicting interests.**
   a. Engineers shall not accept financial or other considerations, including free engineering designs, from material or equipment suppliers for specifying their product.
   b. Engineers shall not accept commissions or allowances, directly or indirectly, from contractors or other parties dealing with clients or employers of the engineer in connection with work for which the engineer is responsible.

6. **Engineers shall not attempt to obtain employment or advancement or professional engagements by untruthfully criticizing other engineers, or by other improper or questionable methods.**
   a. Engineers shall not request, propose, or accept a commission on a contingent basis under circumstances in which their judgment may be compromised.
   b. Engineers in salaried positions shall accept part-time engineering work only to the extent consistent with policies of the employer and in accordance with ethical considerations.
   c. Engineers shall not, without consent, use equipment, supplies, laboratory, or office facilities of an employer to carry on outside private practice.

7. **Engineers shall not attempt to injure, maliciously or falsely, directly or indirectly, the professional reputation, prospects, practice, or employment of other engineers.** Engineers who believe others are guilty of unethical or illegal practice shall present such information to the proper authority for action.

a. Engineers in private practice shall not review the work of another engineer for the same client, except with the knowledge of such engineer, or unless the connection of such engineer with the work has been terminated.
b. Engineers in governmental, industrial, or educational employ are entitled to review and evaluate the work of other engineers when so required by their employment duties.
c. Engineers in sales or industrial employ are entitled to make engineering comparisons of represented products with products of other suppliers.

8. **Engineers shall accept personal responsibility for their professional activities,** provided, however, that engineers may seek indemnification for services arising out of their practice for other than gross negligence, where the engineer's interests cannot otherwise be protected.
   a. Engineers shall conform with state registration laws in the practice of engineering.
   b. Engineers shall not use association with a non-engineer, a corporation, or partnership as a "cloak" for unethical acts.

9. **Engineers shall give credit for engineering work to those to whom credit is due,** and will recognize the proprietary interests of others.
   a. Engineers shall, whenever possible, name the person or persons who may be individually responsible for designs, inventions, writings, or other accomplishments.
   b. Engineers using designs supplied by a client recognize that the designs remain the property of the client and may not be duplicated by the engineer for others without express permission.
   c. Engineers, before undertaking work for others in connection with which the engineer may make improvements, plans, designs, inventions, or other records that may justify copyrights or patents, should enter into a positive agreement regarding ownership.
   d. Engineers' designs, data, records, and notes referring exclusively to an employer's work are the employer's property. The employer should indemnify the engineer for use of the information for any purpose other than the original purpose.
   e. Engineers shall continue their professional development throughout their careers and should keep current in their specialty fields by engaging in professional practice, participating in continuing education courses, reading in the technical literature, and attending professional meetings and seminars.

Example 9: In the pump and pipe optimization problem, you gave a preliminary cost estimate including only the cost of materials. Perhaps you did not mention this in your earlier communication. Since the energy costs could be significant, it could have been perceived as an attempt to deceive the public. Since it was preliminary estimate, you can still use your communication skills to clarify the situation.