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ABSTRACT
The influence of several parameters on the incipient motion of gravels has been carefully examined from theoretical and experimental standpoints. This analysis shows that the effects of the grain Reynolds number and relative submergence are negligible for turbulent flows over hydraulically rough boundaries. Laboratory experiments for homogeneous gravels on a plane bed provided detailed measurements of both mean flow velocity and velocity fluctuations for the analysis of the lift and drag coefficients. This analysis of the lift and drag coefficients enables a better evaluation of the Shields parameter for incipient motion as a function of the angle of repose tan\(\phi\). The theoretical developments presented in this paper are corroborated by the results of our laboratory experiments as well as many others available in the literature.

Keywords: Incipient motion; mathematical model; relative submergence; sediment transport; Shields parameter

1 Introduction
Incipient motion of grains (IMG) has been a frequently discussed topic (e.g. Buffington & Montgomery, 1997; Lamb, Dietrich, & Venditti, 2008) since the contribution of Shields (1936). The experiments that followed and the gradual specification of parameters for various flow conditions have led to new findings, revealing the complexity of the process. There have been many studies focused on the IMG for specific conditions but their results are often inconsistent. Some of the reported experiments (Bathurst, Graf, & Cao, 1987; Graf & Suszka, 1987; Lenzi, Mao, & Comiti, 2006; Mizuyama, 1977; Schvidchenko & Pender, 2000; Suszka, 1991) show dependence of the IMG on the relative submergence, while other experiments do not confirm it (Dey & Raju, 2002; Kanellopoulos, 1998; Neill, 1967a, 1967b). Some experimental studies reveal relationships between the IMG and the grain Reynolds number (Carling, 1983; Komar & Li, 1986); while others do not (Buffington & Montgomery, 1997; Yalin & Karahan, 1979). The results related to the effects of longitudinal bed slope and geometric properties of grains are not consistent either (Bathurst et al., 1987; Chiew & Parker, 1994; Fernandez Luque & van Beek, 1976; Komar & Li, 1986; Lamb et al., 2008; Mizuyama, 1977). In many cases, experiments are presented without information on grain properties, or do not take into account some effects without which their results are difficult to interpret and compare with those from other research work.

Theoretical and empirical relationships have been developed for determining the IMG, and they are constantly being refined (Armanini & Gregoretti, 2005; Chepil, 1959; Gregoretti, 2008; Lee & Balachandar, 2012; Miedema, 2012a; Valyrakis, Diplas, & Dancey, 2013; Vollmer & Kleinhans, 2007; Wiberg & Smith, 1987; Zanke, 2003). Such models are particularly focused on the description of forces and moments acting on a grain in a pre-defined position. An essential prerequisite for these models is to determine the coefficients contained in them so that they can be verified. Coefficients are determined from the measurements of grain properties (Brayshaw, Frostick, & Reid, 2005).
1983; Chepil, 1961; Loth, 2008) and flow parameters (Nikora, Goring, McEwan, & Griffiths, 2001; Nikora, Koll, McEwan, McLean, & Dittrich, 2004). Most of such mathematical models are based on micro (grain) scale analysis, although they are often used at the macro scale.

Due to the inconsistency in the results of previous experiments and analyses, there is a need for clarification of multiple effects on the IMG and this paper addresses this issue with a detailed experimental research programme. Based on the significance of these effects, the analytical derivation of a new formulation for the IMG is proposed. The study relates to a specific condition of submerged, approximately homogeneous (uniform size) non-cohesive grains (gravel) placed evenly (identical protrusion levels) on an approximately horizontal planar bed at fully turbulent uniform flow without self-aeration. Therefore, no consideration is given to the influences of different protrusion levels of grains caused by different grain exposure (Fenton & Abbott, 1977), sediment heterogeneity (Wiberg & Smith, 1987), transverse bed slope (Ikeda, 1982), bed forms and self-aeration of flow.

The objectives of this study are to examine the effects of various parameters on the IMG based on a review of the literature. A theoretical analysis of the lift and drag coefficients is then proposed. The new feature of this analysis is to include the effects of velocity fluctuations besides the effects of mean flow velocity. An experimental programme will then be presented for the estimation of the parameters based on the velocity profiles (mean and fluctuation). A comparison with laboratory experiments from the literature will also focus on the effects of grain Reynolds number, relative submergence, longitudinal bed slope and angle of repose. The proposed formulation for the effects of the angle of repose is compared with a wealth of laboratory measurements.

2 Parametric analysis

The incipient motion of submerged homogeneous gravel evenly placed on an approximately horizontal planar bed, depends on flow characteristics near the bed and on the placement of non-cohesive grains on the bed. The main parameters of the flow and the bed material include the longitudinal bed slope \( \sin \alpha \), the flow depth \( h \), the elevation above the bed \( z \), the density of water \( \rho \), the kinematic viscosity of water \( \nu \), the grain diameter \( d \) given the length \( a \), width \( b \) and thickness \( c \), the density of grains \( \rho_s \), the angle of repose specified by tan \( \phi \), and gravitational acceleration \( g \).

Dimensional analysis (Ettema, Arndt, Roberts, & Wahl, 2000; Novak & Čábelka, 1981; Yalin, 1972) shows that the IMG is determined by the following functional relationship:

\[
F_{sc} = f_1 \left( \Delta, R_s, \frac{h}{d}, \sin \alpha, \tan \phi \right)
\]

where \( F_{sc} = u_c/(gd)^{1/2} \) is the grain Froude number, \( u_c = (gh \sin \alpha)^{1/2} \) is the shear velocity, \( \Delta = (\rho_s - \rho)/\rho \) is the relative density, \( R_s = u_c d/\nu \) is the grain Reynolds number and \( h/d \) is the relative submergence. Introducing the critical Shields parameter for the IMG on a horizontal surface \( \theta_c = F_{sc}^2/\Delta \), Eq. (1) will be:

\[
\theta_c = \frac{u_c^2}{\Delta g d} = f_2 \left( R_s, \frac{h}{d}, \sin \alpha, \tan \phi \right)
\]

The following analysis examines successively the effects of each of the parameters \( R_s, h/d, \sin \alpha, \) and tan \( \phi \).

2.1 Effect of grain Reynolds number \( R_s \)

The critical value of the Shields parameter \( \theta_c \) changes primarily with the material properties, the type of grain motion, i.e. sliding, rolling, lifting (Ling, 1995), and the method of determining the IMG (Buffington & Montgomery, 1997). The independence of the IMG from \( R_s \) is based on the self-similarity of the velocity profile near the planar bed in the case of uniform fully turbulent flow \( R_s > 70 \) with \( h/d > 3 \) to 4. Due to the heterogeneity of bed roughness formed by natural grains, it is appropriate to express the velocity profile using the time- and area-averaged component of velocity \( \bar{u}_c \) (in the direction of the axis \( x \)):

\[
\frac{\bar{u}_c}{u_c} = \frac{1}{\kappa} \ln \left( \frac{z}{\kappa a} \right) + C
\]

Where the overbar defines time and area averaging, \( \kappa \) is the von Kármán’s constant and \( \kappa a \) is Nikuradse’s equivalent sand roughness height and is expressed by the size of grain \( d \). For the Nikuradse case, \( \kappa = 0.4, \kappa d = d, C = 8.5 \) (Keulegan, 1938; Schlichting, 1979). A constant value of the von Kármán’s number \( \kappa a = u'/u \) (Novak & Čábelka, 1981) \( (u' \) is the fluctuation of the velocity) and anisotropic turbulence (Okazaki, 2004) are considered. For the IMG, only the macro-turbulent eddies from the spectrum of velocity fluctuations should be considered because only these have enough energy to move a grain (Valyrakis et al., 2013).

For the case of very porous bed material (gravel), the flow velocity in the near-bed layer (Fig. 1) is described by Eq. (3)
while the flow velocity in the subsurface layer is constant. However, surface grains are lying in the interfacial layer between the surface and the subsurface layers. Therefore, in the interfacial layer, a continuous transition occurs between these velocities (Gregoretti, 2008; Lee & Ferguson, 2002; Nikora et al., 2004). Numerical simulations show that the velocity profile does not change with the value of $R_e$, even in the interfacial layer when using the same material (Chen, Huang, Leu, & Lai, 2007). In the case of grains equally placed on a planar bed and in fully turbulent flow, the velocity profile is self-similar in that the velocity profile from Eq. (3) is only function of $z/d$. Therefore, if the velocity in the sub-surface layer is neglected, the $R_e$ has no effect on the IMG, as confirmed experimentally (Yalin & Karahan, 1979).

### 2.2 Effect of relative submergence $h/d$

The effect of $h/d$ on $\theta_*$ has been dealt with by a relatively large number of authors, but with inconsistent conclusions. Table 1 gives a research summary with the range of selected variables, which was carried out in fully turbulent flow $R_e \geq 400$ and with $h/d \leq 16$. Shields (1936) states that the effect of $h/d$ is shown when values of $R/d < 25$, where $R$ is the hydraulic radius. Data obtained from laboratory experiments carried out by Meyer-Peter and Müller (1948) do not confirm this relationship. Neill (1967a, 1967b) assumes a relationship between $\theta_*$ and $h/d$, but his measurements do not confirm it either. Kališ (1970) mentions the effect of $h/d$ in the sense of the self-aeration of flow and highlights the uncertainty involved in determining the surface and bed levels. Yalin (1972) does not consider the effect of $h/d$ because he assumes that the structure of flow close to the bed does not change with $h/d$. Fenton and Abbott (1977) accentuate the significance of the shape of the bed surface from the point of view of grain exposure. They present three main surface types for three characteristic materials: some grains almost completely exposed — fine sand, grains over-riding others — coarse sand, co-planar bed — gravel. For a co-planar bed and grains with zero exposure, there is no relationship on $h/d$.

Mizuyama (1977) determines a significant relationship to $h/d$ and attributes it to a change in the velocity field caused by a change in friction. Graf and Suszka (1987) do not mention a relationship to $h/d$, but their data confirm a relationship. Schvidchenko and Pender (2000) determine the IMG from values obtained for the dimensionless discharge of bedload and give a relationship to $h/d$ for a selected value. Papanicolaou (1997) determined the effect of the packing of grains in the zone above the bed on the IMG; his measurements also show that $h/d$ has no effect on the IMG. Kanellopoulos (1998) found out on the basis of measurements with fully exposed grains that for $h/d \leq 4$ there is an effect on the IMG. However, he did not state a reason for this because he had not detected a change in the velocity field close to the bed when $h/d$ changed. Gregoretti (2000) carried out measurements to determine the incipient of scour formation. It is not possible to determine the effect of the $h/d$ (of submerged grains on incipient motion) from the measured data because the flow was self-aerated and some grains protruded from water surface. Recking (2006) carried out measurements and determined the IMG with the value of the dimensionless bedload discharge. The values determined in this way show that $h/d$ with values of $R/d \leq 20$ has an effect on the IMG. Based on a theoretical analysis, Lamb et al. (2008) state that exposure is important, but $h/d$ has no effect on the

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>$\phi$ ($\phi = f (d)$) (%$)$</th>
<th>$\sin \alpha$ (%)</th>
<th>$h/d$ ($R/d$) (–)</th>
<th>$R_e$ (–)</th>
<th>$\theta_*$ (–)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyer-Peter and Müller (1948) (L)</td>
<td>(36)</td>
<td>0.49–1.05</td>
<td>9.7–18.1</td>
<td>575–686</td>
<td>0.037–0.050</td>
</tr>
<tr>
<td>Neill (1967a, 1967b) (L)</td>
<td>(35)</td>
<td>0.85–2.7</td>
<td>4.9–14.3</td>
<td>596–1987</td>
<td>0.029–0.041</td>
</tr>
<tr>
<td>Kališ (1970) (L)</td>
<td>45</td>
<td>5.9–9.2</td>
<td>0.94–1.47</td>
<td>10,889–17,963</td>
<td>0.032–0.059</td>
</tr>
<tr>
<td>Fenton and Abbott (1977) — series B (L)</td>
<td>(35.3; 38)</td>
<td>0.52–1.90</td>
<td>5.45–7.50</td>
<td>405–820</td>
<td>0.080–0.267</td>
</tr>
<tr>
<td>Fenton and Abbott (1977) — series C (L)</td>
<td>(35.3)</td>
<td>0.5–1.6</td>
<td>2.8–4.4</td>
<td>1690–3280</td>
<td>0.009–0.012</td>
</tr>
<tr>
<td>Mizuyama (1977) (L)</td>
<td>52.4; 45</td>
<td>1–20</td>
<td>0.6–8.5</td>
<td>463–3558</td>
<td>0.047–0.099</td>
</tr>
<tr>
<td>Bathurst, Li, and Simons (1979) (L)</td>
<td>(35)</td>
<td>2–8</td>
<td>1.5–5.3</td>
<td>881–6198</td>
<td>0.090–0.193</td>
</tr>
<tr>
<td>Cao (1985) (L)</td>
<td>(35)</td>
<td>1–9</td>
<td>1.2–8.6</td>
<td>3008–9567</td>
<td>0.053–0.088</td>
</tr>
<tr>
<td>Bathurst et al. (1987) (L)</td>
<td>35; 40.5; 40</td>
<td>0.5–9</td>
<td>1.3–11.9</td>
<td>1059–11,296</td>
<td>0.036–0.099</td>
</tr>
<tr>
<td>Graf and Suszka (1987) (L)</td>
<td>35; 40.5</td>
<td>0.5–2.5</td>
<td>4.0–13.4</td>
<td>929–3816</td>
<td>0.034–0.063</td>
</tr>
<tr>
<td>Suszka (1991) (L)</td>
<td>35</td>
<td>0.5–2.5</td>
<td>4.1–13.5</td>
<td>1086–3577</td>
<td>0.041–0.064</td>
</tr>
<tr>
<td>Papanicolaou (1997) (L)</td>
<td>(35.3)</td>
<td>0.8–1.2</td>
<td>7.13–9.50</td>
<td>535–757</td>
<td>0.037–0.075</td>
</tr>
<tr>
<td>Schvidchenko and Pender (2000) (L)</td>
<td>(35)</td>
<td>0.65–2.87</td>
<td>(3.6–11.9)</td>
<td>421–1514</td>
<td>0.045–0.068</td>
</tr>
<tr>
<td>Gregoretti (2000) (L)</td>
<td>50.2; 51.3; 47.7</td>
<td>21–36</td>
<td>0.47–1.19</td>
<td>4616–8963</td>
<td>0.131–0.236</td>
</tr>
<tr>
<td>Dey and Raju (2002) (L)</td>
<td>(35)</td>
<td>0.73–1.85</td>
<td>1.4–5.7</td>
<td>400–1977</td>
<td>0.026–0.097</td>
</tr>
<tr>
<td>Mueller, Pitlick, and Nelson (2005) (F)</td>
<td>(35)</td>
<td>0.21–5.09</td>
<td>3.2–14.9</td>
<td>5582–96,865</td>
<td>0.011–0.134</td>
</tr>
<tr>
<td>Lenzi et al. (2006) (F)</td>
<td>(41.5)</td>
<td>13.6</td>
<td>1.6–3.8</td>
<td>28,684–95,349</td>
<td>0.037–0.406</td>
</tr>
<tr>
<td>Recking (2006) (L)</td>
<td>(35)</td>
<td>5–9</td>
<td>2.20–3.14</td>
<td>409–1808</td>
<td>0.077–0.153</td>
</tr>
<tr>
<td>Present study (L)</td>
<td>35.3–41.5</td>
<td>1–7.5</td>
<td>0.76–7.16</td>
<td>523–4519</td>
<td>0.035–0.054</td>
</tr>
</tbody>
</table>

Abbreviations: L, laboratory data; F, field measurements
IMG. The evaluation of field measurements by Mueller et al. (2005) does not confirm a relationship. Lenzi et al. (2006) show a distinct relationship for all data measured by them in a steep mountain stream with heterogeneous material. Consequently, it is not possible to clearly determine a general relationship for \( \Theta_c = f(h/d) \) from the literature and it may be necessary for us to carry out additional measurements.

2.3 Effect of longitudinal bed slope \( \sin \alpha \)

For the case with a longitudinal bed slope \( \sin \alpha \) (angle \( \alpha \) is positive when the bed elevation decreases in the downstream direction), the change in the applied forces has been considered by many investigators (Bathurst et al., 1987; Chiew & Parker, 1994; Fernandez Luque, 1974; Fernandez Luque & van Beek, 1976; Lamb et al., 2008; Lau & Engel, 1999; Mizuyama, 1977; Zanke, 2003). Incipient motion on a sloping surface can be defined with the parameter \( \Theta_c \). The relation, expressed from moment equilibrium (Bormann & Julien, 1991; Fernandez Luque, 1974) and also from force equilibrium (Chiew & Parker, 1994), or using dimensional analysis (Lau & Engel, 1999) and verified by measurements (Chiew & Parker, 1994; Dey & Debnath, 2000; Fernandez Luque, 1974), is:

\[
\Theta_c = \Theta_{c0} \frac{\sin \phi}{\sin(\phi - \alpha)}
\]

Using Eq. (4), the identified Shields parameter for the given bed slope \( \Theta_{c0} \) can be converted to the Shields parameter for the horizontal bed \( \Theta_c \).

2.4 Effect of angle of repose \( \phi \)

The geometric properties of grains, such as surface texture (Shields, 1936), roundness (Simons, 1957), shape (Komar & Li, 1986), and size (Julien, 2010) exert an effect on \( R_s \) and \( \tan \phi \). Other parameters include the abundance of grain sizes (Wiberg & Smith, 1987) and the compaction and placement of grains (Fenton & Abbott, 1977; Miedema, 2012b; Papanicolaou, 1997). Nevertheless, such variables are difficult to determine in general. Therefore, many authors use \( \tan \phi \) to express these effects collectively when developing IMG models, whether they are looking at a regression analysis of dimensionless parameters collectively when developing IMG models, whether they are looking at a regression analysis of dimensionless parameters involving both time fluctuation at a given point and the fluctuation within an averaging area (Nikora et al., 2001; Yalin, 1972):

\[
F = \mathcal{F} + F'
\]

Lift and drag forces are decomposed into the time- and area-averaged component \( \mathcal{F} \) (for planes parallel to the plane of the grain surface) and the fluctuation component \( F' \) involving both time fluctuation at a given point and the fluctuation within an averaging area (Nikora et al., 2001; Yalin, 1972):

\[
\mathcal{F} = \mathcal{F}_L + \mathcal{F}_D + \mathcal{F}_r
\]

The lift force \( \mathcal{F}_L \) acts in the direction perpendicular to the plane. It is caused by pressure on the grain surface, and is derived from the Bernoulli equation for a linear velocity gradient along the vertical (\( du/dz = \) constant) (Wiberg & Smith, 1987). It is given as:

\[
\mathcal{F}_L = 0.5C_L \rho A_{xy} (u_{r,m}^2 - u_{r,d}^2)
\]
downstream \( x \) direction. The overbar indicates time- and area-averaged values, the prime indicates fluctuations, the subscript \( u \) is at the top of the grain, the subscript \( d \) is at the bottom of the grain, and the subscript \( c \) is at the centre of the grain.

The fluctuation component of the drag force \( F_{Dx} \) acts in the direction perpendicular to the \( xy \) plane, which is caused by the velocity fluctuation in same direction (the average velocity fluctuation is zero, but the average of its square value is not):

\[
F'_{Dx} = 0.5C_{Dx} \rho A_{xy} u'^2_{x,c}
\]  
(10)

where \( u_{tc} \) is the flow velocity perpendicular to the \( xy \) plane. The drag force \( F_{Dx} \) acts in the direction parallel to the \( xy \) plane. Its time- and area-averaged component is:

\[
\bar{F}_{Dx} = 0.5C_{Dx} \rho A_{xy} \bar{u}^2_{x,c}
\]  
(11)

and its fluctuation component is:

\[
F'_{Dx} = 0.5C_{Dx} \rho A_{xy} u'^2_{x,c}
\]  
(12)

where \( A_{xy} \) is the projection of the grain surface onto a plane perpendicular to the direction of flow.

The ratio of the parallel forces to the perpendicular forces with the \( xy \) plane governs the tangent \( \phi \):

\[
\frac{\bar{F}_{Dx} + F'_{Dx}}{F_g - F_h - F'_{L} - F'_{Dx}} = \tan \phi
\]  
(13)

The variability of the tangent \( \phi \) of individual grains in the surface layer governs the amount of grains in motion (expressed by the dimensionless discharge of bedload) at a given \( \phi \) value. Every grain in motion introduces another force from the interaction of grains into the force equilibrium, and therefore only the absolute IMG is considered.

After substituting Eqs (6)–(12) in Eq. (13), it yields:

\[
\frac{1}{\theta_c} = \frac{3}{4} \left[ \frac{C_{Dx} \rho A_{xy} (\bar{u}^2_{x,c} + u'^2_{x,c})}{\tan \phi} \right] + C_L \left[ \frac{\bar{u}^2_{x,w}}{u^2_{x}} + \frac{u'^2_{x,d}}{u^2_{x}} - \frac{u'^2_{x,c}}{u^2_{x}} \right]
\]  
(14)

and the expression of \( \theta_c \) from Eq. (2) results in the following general form:

\[
\frac{1}{\theta_c} = \frac{d}{2F} \left[ \frac{C_{Dx} A_{xy} (\bar{u}^2_{x,c} + u'^2_{x,c})}{\tan \phi} \right] + C_L \left[ \frac{\bar{u}^2_{x,w}}{u^2_{x}} + \frac{u'^2_{x,d}}{u^2_{x}} + \frac{u'^2_{x,c}}{u^2_{x}} \right] + C_D A_{xy} \frac{u'^2_{x,c}}{u^2_{x}}
\]  
(15)

In order for the equation to be usable in practice, it is necessary to know the grain shape and to have accurate velocity measurements.

In the case of a spherical grain, \( A_{xy} = A_{yz} = A, V = 2/3dA \), \( C_{Dx} = C_{Dz} = C_D \). Eq. (15) takes the form:

\[
\frac{1}{\theta_c} = \frac{3}{4} \left[ \frac{C_D}{\tan \phi} \left( \frac{\bar{u}^2_{x,c} + u'^2_{x,c}}{u^2_{x}} \right) \right] + C_L \left[ \frac{\bar{u}^2_{x,w}}{u^2_{x}} + \frac{u'^2_{x,d}}{u^2_{x}} - \frac{u'^2_{x,c}}{u^2_{x}} \right] + C_D \frac{u'^2_{x,c}}{u^2_{x}}
\]  
(16)

This equation can be solved using the coefficients \( C_D \) and \( C_L \), and the velocity measurements in relation to \( u \).

For an ellipsoid, which approximately corresponds with the shape of natural gravel grain placed on a bed in the most stable position, i.e. \( A_{xy} = \pi ab/4, A_{yz} = \pi ac/4, V = \pi abc/6 \), the equation becomes:

\[
\frac{1}{\theta_c} = \frac{3}{4} \left[ \frac{C_{Dx}}{\tan \phi} \left( \frac{\bar{u}^2_{x,c} + u'^2_{x,c}}{u^2_{x}} \right) \right] + C_L \frac{d}{c} \left[ \frac{\bar{u}^2_{x,w}}{u^2_{x}} + \frac{u'^2_{x,d}}{u^2_{x}} - \frac{u'^2_{x,c}}{u^2_{x}} \right] + C_D \frac{d}{c} \frac{u'^2_{x,c}}{u^2_{x}}
\]  
(17)

To solve Eq. (17) it is necessary to know the dimensions of the grains \( \{a,b,c\} \) where \( d = (abc)^{1/3} \) and the coefficients \( C_{Dx}, C_L, C_{Dz} = C_{Dab} \), which can be found from careful experiments.

4 Experimental procedure and measurements

Experiments were carried out in the Laboratory of Water Management Research at the Institute of Water Structures at the Faculty of Civil Engineering of Brno University of Technology in the Czech Republic. A 6 m long, 0.5 m wide and 0.5 m deep flume shown in Fig. 2 was used with a tilt range from 0% to 7% and side walls made of Perspex. The flume bed was roughened by concrete cross sills that are 8 mm high, 8 mm wide and spaced 8 mm apart from each other. The sills eliminate the undesirable sliding of the grain layer along the smooth flume bed. Grains of a certain fraction (material) were placed on the roughened surface, levelled and compacted (Robar, 2014) with a plate compactor. The thickness of the granular layer was at least 3d in order to meet grain placement conditions (Salem, 2013), and at most 5d to eliminate the effect of subsurface discharge (Gregoretti, 2000). The surface grains were placed in a co-planar bed (Fenton & Abbott, 1977).

The inlet and outlet parts of the flume were adapted so that steady uniform flow was ensured at the point of measurement (at 2/3 of the length of the flume from the inlet), with a fully developed velocity profile. The range of discharge was from 0.004 m³ s⁻¹ to 0.036 m³ s⁻¹ (Table 2). The influence of the side walls (\( B/h \geq 5 \), where \( B \) is the flume width) and the influence of surface tension (\( h \geq 0.015 \) m) (Novak & Čabalka, 1981) were negligible. A height-adjustable sill was placed at the end of the flume that stabilized the layer of the bed material. The
The depth of water \( h \) was measured using a length gauge (error \( \pm 1 \text{ mm} \)) and was determined as a perpendicular distance between the time- and area-averaged water surface level and the area-averaged grain surface level. The bed level was determined to be \( 0.2b \) below the grains tops on the basis of the velocity profile (Dwivedi, 2010; Grass, 1971; Papanicolaou, 1997). The discharge was determined using an electromagnetic flow meter with an uncertainty of 1%. Point velocities were measured through the water surface by an ultrasonic velocity profile monitor (model XW-PSi shown in Fig. 2) using 4 MHz probes at the absolute IMG (Zachoval, Pařílková, Roušar, & Roháčová, 2011). These measured velocities were used to determine the patterns of time- and area-averaged velocities \( \bar{u}_z \) and their fluctuations along vertical profiles from \( z/d = 0.0 \) to a level of 0.015 m below the water surface (a limitation of the measuring instrument).

The experimental procedure involved gradually changing the flume bottom slope (0.5% intervals) during constant discharge. The absolute IMG was determined visually as the first grain motion in the control section of the flume. For all fractions, two

---

### Table 2 Range of experimental hydraulic and sediment parameters

<table>
<thead>
<tr>
<th>Fraction (mm)</th>
<th>( h ) (mm)</th>
<th>( Q ) (ls(^{-1}))</th>
<th>( \sin \alpha ) (%)</th>
<th>( h/d ) (–)</th>
<th>( R_e ) (–)</th>
<th>( \theta_c ) (–)</th>
<th>Number of measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>6–8</td>
<td>30–58</td>
<td>7.9–19.8</td>
<td>1.0–2.0</td>
<td>3.70–7.16</td>
<td>523–603</td>
<td>0.038–0.050</td>
<td>6</td>
</tr>
<tr>
<td>8–10</td>
<td>19–62</td>
<td>5.0–23.5</td>
<td>1.0–3.5</td>
<td>1.95–6.25</td>
<td>699–823</td>
<td>0.036–0.049</td>
<td>8</td>
</tr>
<tr>
<td>10–16</td>
<td>23–72</td>
<td>7.3–32.9</td>
<td>1.5–4.0</td>
<td>1.68–4.97</td>
<td>1286–1504</td>
<td>0.036–0.047</td>
<td>10</td>
</tr>
<tr>
<td>16–20</td>
<td>21–85</td>
<td>4.5–32.9</td>
<td>1.5–6.0</td>
<td>1.05–4.18</td>
<td>1976–2327</td>
<td>0.041–0.054</td>
<td>11</td>
</tr>
<tr>
<td>20–25</td>
<td>32–77</td>
<td>9.8–36.0</td>
<td>2.5–5.0</td>
<td>1.23–3.05</td>
<td>2899–3250</td>
<td>0.038–0.048</td>
<td>10</td>
</tr>
<tr>
<td>25–31.5</td>
<td>24–74</td>
<td>8.1–36.0</td>
<td>3.0–7.5</td>
<td>0.76–2.35</td>
<td>3658–4519</td>
<td>0.035–0.050</td>
<td>14</td>
</tr>
</tbody>
</table>

---

This text describes the experimental setup and measurements for a hydraulic study. It includes details on the water depth measurement, discharge determination, and velocity profile monitoring. The table provides a range of experimental hydraulic and sediment parameters, including the fraction size, water depth, discharge, and various dimensionless parameters. The text also outlines the experimental procedure, which involves gradually changing the flume bottom slope while maintaining a constant discharge to study the flow characteristics.
series of measurements were carried out with practically identical results. In total 59 measurements were performed. In order to determine the velocity $\bar{u}_s$ in the subsurface layer $z/d \leq -0.8$ (Fig. 1), separate measurements were carried out using a permeameter with a range of slopes of the hydraulic grade line $\sin \alpha$ at the IMG (Table 2). The height of the permeameter was 0.22 m and the diameter was 0.123 m.

Six fractions of extracted natural grains were used: 6–8, 8–10, 10–16, 16–20, 20–25, 25–31.5 (mm). A sample of 100 grains was collected from each fraction to measure the length a, width b, and thickness c with a digital calliper (error ±0.02 mm). The values were used to calculate the average values for each dimension, as well as the Corey shape factor $C_\text{ab} = c/(ab)^{1/2}$ (Corey, 1949). The density of grains $\rho_s$ was determined by the underwater weighing method, the angle of repose $\tan \phi$ from the measured slope, and the porosity $n$ from the volume of water in a unit volume of material (Table 3). The determination of the drag coefficient $C_{\text{Dab}} = 4(\rho_s - \rho)gc/(3\rho w^2)$ perpendicular to the largest grain area (dimensions a, b) was made on the basis of the terminal settling velocity of grains w in a tank with standing water (Table 3). The flow in the pores was turbulent; therefore the hydraulic conductivity for sub-surface flow was expressed as $k_v = \bar{u}_s n/\sin \alpha (\text{mm})/\text{s}$ (Table 3).

5 Results and discussion

5.1 Velocity profiles and grain Reynolds number $R_e$

The time- and area- averaged velocity ratio $\bar{u}_s/u_s$ was plotted in relation to the relative height $z/d$ for all measured cases of the absolute IMG (see Fig. 3). For $z/d > 0.2$ (the surface layer of flow) and under the prerequisite that $k_v = d/b$ the average value $\kappa = 0.4$ was determined. The $C$ value varied from 7.5 to 9.8 for $z/d = 1$, with its average value being approximately $C = 8.5$. It is found that the values of the parameters of Eq. (4) are in agreement with those obtained by many authors (Keulegan, 1938; Schlichting, 1979). Figure 3 clearly shows that the $h/d$ has no effect on the time- and area-averaged velocity profile and also that it is not influenced by relative height for $z/d > 0.2$. The values of the velocities in the subsurface layer of flow (Fig. 3) are significantly smaller (by at least 12 times) relative to the velocity values on the surfaces of grains, and can be considered negligible. Measurements in the range of $0.0 < z/d < 0.2$ must be considered approximate due to possible reflections of the ultrasound beam. Fig. 3 also shows that the boundary between the surface and the interfacial layer of flow is in the range of $0.2 < z/d < 0.4$, for further we considered $z/d = 0.2$. The velocity profile between the surface and subsurface layers of flow was determined approximately by linear extrapolation of the measured $\bar{u}_s/u_s$ values in the range of $0.0 < z/d < 0.2$ up to the value of the flow velocity in the subsurface layer. The thickness of the interfacial layer was approximately the same as the grain thickness c (Fig. 3). The coefficient $C_\zeta = 6.7$ in the ratio $\bar{u}_s/u_s = C_\zeta z/d$ in the interfacial layer lies within the range of values presented by other authors (Nikora et al., 2001; Shimizu, Tsujimoto, & Nakagawa, 1990).

The normalized standard deviation of the velocity component in $x$ direction $\sigma_3/\bar{u}_s$, where $\sigma_3$ is standard deviation of the velocity $u$, was also evaluated along the $z/d$ for all measured cases (Fig. 4). The maximum value is 2.6 (1.7 to 3.5) and occurs approximately at $z/d = 0.2$, which is in agreement with measurements made by others (Papanicolaou, 1997; Pokrajac & Manes, 2009). The measurements of velocity in the interfacial layer must be taken as approximate due to the method of measurement, yet they are in agreement with measurements made by other authors (e.g. Pokrajac & Manes, 2009).

The velocity profiles do not change with $R_e$ and thus have no effect on the IMG. This conclusion is confirmed by Fig. 5, where a relationship between $\theta_0$ and $R_e$ is shown. In the case of the grains used in our experiment, it is possible for IMG to take the mean value $\theta_0 = 0.043$ (Fig. 5).

5.2 Relative submergence $h/d$

The values of $\theta_0$, in relation to the value $h/d$ from our experiments and from other experiments are plotted in Fig. 6. The experiments show that the following features have an effect on the IMG with small values of $h/d$: the position of grains relative to the bed (grain exposure to the flow), the relative degree of submergence of the grains (grains protruding above the water surface) and self-aeration of flow. Note that the data with self-aerated flows were not used.

If the IMG was determined by the chosen non-zero value of dimensionless volume bedload discharge $q_b = q_b/\left(Agd^3\right)^{1/2}$ (Bathurst et al., 1987; Suszka, 1991), where $q_b$ is the specific volume bedload discharge, the relationship between $\theta_0$ and $h/d$ exist (Fig. 6). However, for a different chosen value of $q_b$ is
Figure 3 Dependence of $\bar{u}_i/u_*$ on the distance from the bed $z/d$ (top of the grains at 0.2 $z/d$)
Figure 4  Dependence of $\sigma_x/u_*$ on the distance from the bed $z/d$ (top of the grains at 0.2 $z/d$)
different relationship between $\theta_c$ and $h/d$. Therefore, these data were also not used.

The relationship between $\theta_c$ and $h/d$ at the absolute IMG was plotted in Fig. 7 for the data fulfilling the requirements: planar bed with homogeneous roughness, submerged grains (Dey & Raju, 2002; Fenton & Abbott, 1977; Kališ, 1970; Mizuyama, 1977; Neill, 1967a, 1967b). Figure 7 shows the values $\theta_c$ relative to the value at the largest measured relative depth $\theta_{c, h/d=\text{max}}$ as a function of $h/d$. Under these conditions for determining the IMG, there is no relationship between $\theta_c$ and $h/d$.

5.3 Longitudinal bed slope sin $\alpha$

The effect of longitudinal bed slope sin $\alpha$ on the IMG was defined by Eq. (4). Figure 8 shows the comparison of our
measured data with other laboratory data (Chiew & Parker, 1994; Dey & Debnath, 2000; Fernandez Luque, 1974). All measured data confirm that the longitudinal bed slope has an influence on incipient motion and that Eq. (4) is quite satisfactory.

5.4 Angle of repose $\phi$

The $\theta_c$ values obtained from our measurements at the absolute IMG and values from other authors satisfying $h/d \geq 1$ and $R_\theta \geq 100$ are shown in Fig. 9. Overall, our experiments showed a range $0.03 < \theta_c < 0.06$ and demonstrate a relationship to $\tan \phi$. Our measurements are compared with the values obtained by the authors who considered a spectrum of conditions ranging from the absolute IMG to occasional motion (Chiew & Parker, 1994; Fernandez Luque & van Beek, 1976; Graf & Suszka, 1987; Kališ, 1970; Mizuyama, 1977). The scattering of values in this zone is caused in particular by the method of determining the IMG. Kališ (1970), Fernandez Luque and van Beek (1976) and Mizuyama (1977) determined the absolute IMG. The $\theta_c$ values obtained from the measurements of Graf and Suszka (1987) ranged from the absolute IMG to those determining occasional motion. The $\theta_c$ values obtained by Chiew and Parker (1994) determined occasional motion. Other $\theta_c$ values outside the given range include conditions at a chosen bedload discharge (Dey & Debnath, 2000), extrapolation to a chosen dimensionless bedload discharge (Bathurst et al., 1987) or grains exposed above the bed (Kanellopoulos, 1998), or for self-aerated flow (Gregoretti, 2000).

From Fig. 9, the proposed relationships vary quite significantly, and only match the data for which the relevant model was determined (Dey, 1999). In most cases, experiments confirm the
character of the relationship with an increase in \( \tan \phi \) resulting in an increase in \( \theta_c \). The relationship in Eq. (5) is difficult to plot because the coefficients \( C_p, C_D \) and \( C_L \) have different values for different material and experiments (Chiew & Parker, 1994; Fernandez Luque & van Beek, 1976; Graf & Suszka, 1987; Mizzymana, 1977). It is necessary to note that Eq. (5) only takes into account time- and area-averaged values of velocity and does not fully reflect the character of the velocity fluctuations.

Our experimental measurements including velocity fluctuations enable us to use the more detailed model of the lift and drag coefficients developed in section 3. It becomes possible to define the parameters from Eq. (17) based on the mean flow velocity measurements in Fig. 3 and the velocity fluctuations specified by standard deviation of velocity in Fig. 4. The time- and area-averaged velocity in the direction of axis \( x \) on the grain surface \( z/d = 0.2 \) is determined by the logarithmic velocity profile (Eq. (3)), which yields \( \bar{u}_{x,0}^{2}/\bar{u}_x^{2} = 20 \) for \( \kappa = 0.4, k_x = d, C = 8.5 \) (from Fig. 3). In the case of gravels and sands, the velocity of water can be neglected in the subsurface layer of flow \( \bar{u}_{z,d}^{2}/\bar{u}_z^{2} = 0 \) (Fig. 3). The velocity profile in the interfacial layer of flow can be replaced with a linear profile (Nikora et al., 2001), after which the mean velocity in the direction of axis \( x \) for the centre of the grain is \( \bar{u}_{x,c}^{2}/\bar{u}_x^{2} = (1/2)^{2}\bar{u}_{x,0}^{2}/\bar{u}_x^{2} = 5 \) (Fig. 3).

The normalized velocity fluctuation in the \( x \) direction at the top of the grain \( \bar{u}_{x,0}^{2}/\bar{u}_x^{2} \) was replaced in Eq. (17) by normalized standard deviation of velocity from our measurement \( \sigma_{z,0}^{2} / \bar{u}_z^{2} = 6.25 \) (Fig. 4), which corresponds with the values measured by many other authors (e.g. Grass, 1971; Papanicolaou, 1997; Pokrajac & Manes, 2009). A similar procedure of normalized velocity fluctuation in other positions was used below.

Considering that the velocity fluctuation at the centre of the grain is half the value as at the top of the grain, \( \bar{u}_{x,c}^{2}/\bar{u}_x^{2} = (1/2)^{2}\bar{u}_{x,0}^{2}/\bar{u}_x^{2} = 1.56 \). The velocity fluctuations in the vertical direction were not measured, but if the agreement of the measurement with Papanicolaou (1997) holds true, \( \bar{u}_{z,0}^{2}/\bar{u}_z^{2} = 16 \). For the boundary of the subsurface layer, it can be considered that \( \bar{u}_{z,d}^{2}/\bar{u}_z^{2} = 0 \). The vertical velocity fluctuation at the grain centre is similarly obtained as \( \bar{u}_{z,c}^{2}/\bar{u}_z^{2} = (1/2)^{2}\bar{u}_{z,0}^{2}/\bar{u}_z^{2} = 4 \).

In the case of the ellipsoid, where \( d = b, \) and for the above-described characteristics of the velocity field, Eq. (17) takes the form:

\[
\frac{1}{\theta_c^2} = \frac{3}{4} \left[ 6.56 \frac{C_{Dx}}{\tan \phi} + 26.3 \frac{C_L}{c} + 4C_{Dx} \frac{d}{c} \right]
\]  

Further evaluation of the parameters in Eq. (18) is indicated for practical applications. The average measured value of the drag coefficient was \( C_{Dx} = 1.30 \). The average ratio of the dimensions of the grain was \( d/c = 1.49 \). The coefficient \( C_{Lx} \) can only be obtained precisely by measurement, but this was not done. Instead it was determined using the ratio \( C_{Dx}/C_{Dz} \), which is known for the relevant shape of the ellipsoid of the same volume, defined by the ratio of its axes in a wide range of \( R_e \). For the average ratio \( b/c = 1.49, C_{Dx}/C_{Dz} = 0.69 \) for the nominal diameter of the circular area of grain (index \( n \)) (Richter & Nikrityuk, 2012). More specifically, for elliptical areas, \( C_{Dx} = C_{Dx} \frac{b}{c} C_{Dz}/C_{Dz} = 1.34 \). The lift coefficient \( C_L \) was not measured in the given case either, but the measurement made by Schmeeckle, Nelson, and Shreve (2007), i.e. \( C_L = 0.28 \) should be considered. This also corresponds with...
values measured by other authors (Brayshaw et al., 1983; Chepil, 1959) and with values obtained from numerical simulations (Lee & Balachandar, 2012). Accordingly, Eq. (18) results in the following approximation:

\[
\theta_\epsilon = \frac{\tan \phi}{6.5 + 14 \tan \phi}
\]

Equation (19) is depicted in Fig. 9. It practically agrees with the values determined from our own measurements, but is also validated by the values obtained from other measurements under similar conditions (Chiew & Parker, 1994; Fernandez Luque & van Beek, 1976; Graf & Suszka, 1987; Kaliş, 1970; Mizuyama, 1977).

In the case of spheres, higher values of the Shields parameter \( \theta_\epsilon \) can be obtained from Eq. (17). For instance, \( \theta_\epsilon = \tan \phi / (2.3 + 6.9 \tan \phi) \) can be obtained with \( C_D = 0.47 \) and \( C_L = 0.28 \). These higher values for non-protruding spheres also confirm the measurements made by Fenton and Abbott (1977), who obtained \( \theta_\epsilon = 0.16 \) (0.1 to 0.24), and by Dwivedi (2010), obtaining \( \theta_\epsilon = 0.14 \).

### 6 Conclusions

The influence of several parameters on the incipient motion of grains (IMG) has been carefully examined from a theoretical and experimental standpoint. Dimensional analysis was used to define the main parameters for the IMG expressed by the Shields parameter \( \theta_\epsilon \). The main parameters examined include the grain Reynolds number \( R_\epsilon \), relative submergence \( h/d \), the longitudinal bed slope sin \( \alpha \), and the angle of repose \( \tan \phi \). Laboratory experiments were performed for fully submerged non-cohesive homogeneous gravels placed on a planar bed in fully turbulent flow without self-aeration. Both the mean flow velocity and the velocity fluctuations were measured for a more detailed analysis of the lift and drag coefficients.

From this theoretical and experimental study, it is concluded that the effect of the grain Reynolds number \( R_\epsilon \) and the relative submergence \( h/d \) are negligible. The effect of the longitudinal bed slope sin \( \alpha \) is more significant and is well described by Eq. (3). The effect of the angle of repose with \( \tan \phi \) is also thoroughly discussed in this paper. One of the main contributions is the detailed analysis of the lift and drag coefficients as a function of both mean velocity components and velocity fluctuations in Eq. (17). Moreover, the laboratory experiments enabled the evaluation of several factors leading to Eq. (18), which can reduce further to Eq. (19) based on other laboratory investigations found in the literature. The effect of \( \tan \phi \) is very significant and the theoretical developments presented in this paper are corroborated by the results of our laboratory experiments as well as many others found in the literature.

### Funding

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### Notation

- \( a \) = grain length (m, in tables mm)
- \( A \) = area (m²)
- \( b \) = grain width (m, in tables mm)
- \( B \) = flume width (m)
- \( c \) = grain thickness (m, in tables mm)
- \( C \) = velocity profile constant (–)
- \( C_D \) = drag coefficient (–)
- \( C_{Dab} \) = drag coefficient (–)
- \( C_L \) = lift coefficient (–)
- \( C_\phi \) = proportionality coefficient (–)
- \( C_i \) = interfacial flow coefficient (–)
- \( d \) = grain size (m in tables mm)
- \( D_B \) = buoyancy force (kg m s⁻²)
- \( F_D \) = drag force (kg m s⁻²)
- \( F_g \) = gravitational force (kg m s⁻²)
- \( F_L \) = hydrodynamic lift force (kg m s⁻²)
- \( F_{nc} \) = grain Froude number (–)
- \( g \) = gravitational acceleration (m s⁻²)
- \( h \) = flow depth (m)
- \( K_a \) = von Kármán’s number (–)
- \( k_s \) = Nikuradse’s equivalent sand roughness height (m)
- \( k_t \) = turbulent hydraulic conductivity (m s⁻¹)
- \( n \) = sediment porosity (–)
- \( q_b \) = specific volume bedload discharge (m² s⁻¹)
- \( q_{se} \) = dimensionless volume bedload discharge (–)
- \( Q \) = flow discharge (m³ s⁻¹ in tables l s⁻¹)
- \( R \) = hydraulic radius (m)
- \( R_\epsilon \) = grain Reynolds number (–)
- \( u \) = instantaneous velocity (m s⁻¹)
- \( u_{se} \) = shear velocity (m s⁻¹)
- \( V \) = grain volume (m³)
- \( w \) = terminal settling velocity (m s⁻¹)
- \( x \) = downstream coordinate axis (m)
- \( y \) = lateral coordinate axis (m)
- \( z \) = perpendicular coordinate axis to bed plane (m)
- \( \alpha \) = longitudinal bed slope angle (°)
- \( \Delta \) = relative sediment density (–)
- \( \theta_\epsilon \) = critical Shields parameter for a planar horizontal bed (–)
- \( \theta_{cor} \) = critical Shields parameter for a planar non-horizontal bed (–)
\[ \theta_{c,h/d={\text{max}}} = \text{critical Shields parameter for a planar horizontal bed at the highest measured relative submergence (}) \]

\[ \kappa = \text{von Kármán’s constant (}) \]

\[ \rho, \rho_s = \text{fluid and sediment densities, respectively (kg m}{^{-3})} \]

\[ \sigma = \text{standard deviation} \]

\[ \nu = \text{kinematic viscosity (m}{^{2}}\text{s}^{-1}) \]

\[ \phi = \text{angle of repose (°)} \]

Indexes

\[ c = \text{centre of the grain} \]

\[ d = \text{bottom of the grain} \]

\[ n = \text{nominal} \]

\[ u = \text{top of the grain} \]

\[ x = \text{component in the direction of axis } x \]

\[ y = \text{component in the direction of axis } y \]

\[ z = \text{component in the direction of axis } z \]

\[ ' = \text{fluctuation} \]

\[ = \text{time- and area-averaged} \]

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