Rheology of Hyperconcentrations

By Pierre Y. Julien[1] and Yongqiang Lan[2]

Abstract: A physically based quadratic rheological model for hyperconcentrated flows is tested with experimental data sets. The model includes components describing: (1) Cohesion between particles; (2) viscous friction between fluid and sediment particles; (3) impact of particles; and (4) turbulence. The resulting quadratic formulation of the shear stress is shown to be in excellent agreement with the experimental data sets of Bagnold, Savage and McKeown, and Govier et al. When the quadratic model is written in a linearized dimensionless form, the ratio D_v^* of dispersive to viscous stresses is shown to play a dominant role in the rheology of hyperconcentrations. The quadratic model is best suited when ($30 < D_v^* < 400$). At low values of D_v^*, the quadratic model reduces to the simple Bingham plastic model ($D_v^* < 30$), and at large values of D_v^*, a turbulent-dispersive model is indicated ($D_v^* > 400$).

Introduction

The rheology of highly concentrated sediment mixtures has been studied by various researchers including Bagnold (1954), Jeffrey and Acrivos (1976), Takahashi (1980), and Savage and McKeown (1983). Under high rates of shear, Bagnold proposed that the dominant shear stress can be attributed to interparticle friction and collisions. In this grain inertia region, both the normal and shear stresses depend on the second power of the shear rate. These results contrast with observations under low rates of shear (O'Brien and Julien 1988) because in the viscous region, the shear stress in excess of the yield stress increases linearly with the shear rate.

This study describes the rheological properties of hyperconcentrated sediment mixtures at shear rates ranging from the viscous region to the inertial region. It is proposed to test the quadratic rheological model suggested by O'Brien and Julien (1985) with existing data sets from Govier et al. (1957), Savage and McKeown (1983), and Bagnold (1954). This analysis points at the similarities and differences between these three data sets, which give quite different results when analyzed separately. As a second objective, the relative magnitude of the terms in the quadratic model is examined to define the conditions under which simplified rheological formulations can be applied.

Rheological Model Formulation

The shear stress encountered in fluids with large concentrations of sediments should include components to describe: (1) Cohesion between particles; (2) viscous interaction between sediment particles and the surrounding fluid; (3) impact of sediment particles; and (4) turbulence. After considering

[1]Assoc. Prof. of Civ. Engrg., Engrg. Res. Ctr., Colorado State Univ., Fort Collins, CO 80523.

[2]Res. Asst., Dept. of Civ. Engrg., Colorado State Univ., Fort Collins, CO.

Note. Discussion open until August 1, 1991. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on November 20, 1987. This paper is part of the *Journal of Hydraulic Engineering*, Vol. 117, No. 3, March, 1991. ©ASCE, ISSN 0733-9429/91/0003-0346/$1.00 + $.15 per page. Paper No. 25612.

that both turbulence and inertial impact of particles increase with the second power of the shear rate, O'Brien and Julien (1985) proposed the following quadratic rheological model:

$$\tau = \tau_y + \eta \frac{du}{dy} + \zeta \left(\frac{du}{dy}\right)^2 \quad (1)$$

where τ = the shear stress; τ_y = the yield shear stress, η = the dynamic viscosity, ζ = the turbulent-dispersive parameter; and du/dy = the velocity gradient normal to the flow direction. The physical reasoning behind this quadratic formulation (Eq. 1) is briefly summarized. The first term, τ_y, describes the yield stress due to cohesion between fine sediment particles. The yield strength is assumed to be a property of the material that does not depend on the rate of deformation. The second term describes the viscous stress of the fluid interacting with sediment particles. The third term referred to as the turbulent-dispersive stress combines the effects of turbulence and the effects of dispersive stress induced by the collisions between sediment particles. The conventional expression for the turbulent stress in sediment-laden flows merges with Bagnold's dispersive stress relationship because both stresses are proportional to the second power of the rate of shear. The purpose of combining these two terms stems from the concept that at large concentrations of coarse particles, the dispersive stress will be dominant, whereas at large concentrations of fine particles, the yield strength and the viscous stress will overcome the turbulent stress.

The combined turbulent-dispersive parameter ζ can be written as:

$$\zeta = \rho_m l_m^2 + a_1 \rho_s \lambda^2 d_s^2 \quad (2)$$

where ρ_m and l_m = the density and mixing length of the mixture, respectively; d_s = the diameter of sediment particles; a_1 = the empirical constant defined by Bagnold; and ρ_s = the density of sediment particles. The linear concentration λ defined by Bagnold depends on the volumetric sediment concentration C_v and the maximum volumetric sediment concentration C^* ($C^* \sim 0.615$):

$$\lambda = \left[\left(\frac{C^*}{C_v}\right)^{1/3} - 1\right]^{-1} \quad (3)$$

The density ρ_m of the mixture in Eq. 2 is calculated as follows from the volumetric sediment concentration C_v and the mass densities of the fluid, ρ, and of the solid particles, ρ_s:

$$\rho_m = \rho(1 - C_v) + \rho_s C_v \quad (4)$$

It can be seen from Eq. 2 that both sediment concentration and particle size play an important role in determining the magnitude of the turbulent-dispersive parameter ζ.

Experimental Test of the Quadratic Model

Three data sets from Govier et al. (1957), Savage and McKeown (1983), and Bagnold (1954) are examined to test the validity of the rheological model proposed in Eq. 1.

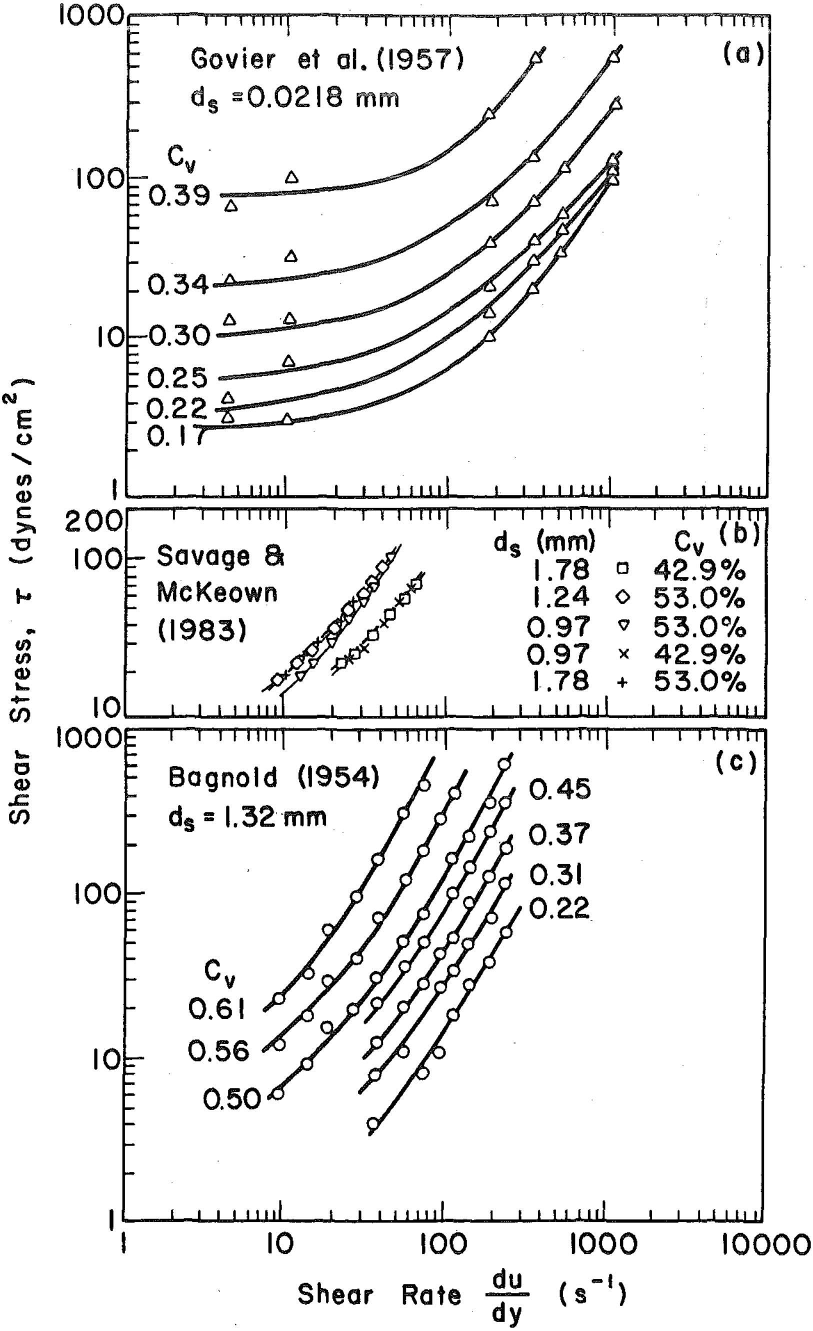

FIG. 1. Rheograms for Three Data Sets: Govier et al. (1957); Savage and McKeown (1983); and Bagnold (1954)

TABLE 1. Coefficients τ_y, η, and ζ for Three Data Sets

d_s (mm) (1)	C_v (%) (2)	τ_y (dynes/cm^2) (3)	η poises (4)	ζ (g/cm) (5)
(*a*) Govier et al. (1957)				
0.0218	39.7	78.4	0.351	3.15×10^{-3}
0.0218	34.1	20.7	0.29	2.4×10^{-4}
0.0218	30.3	9.84	0.137	1.28×10^{-4}
0.0218	24.9	5.0	0.093	3.10×10^{-5}
0.0218	21.8	3.2	0.0670	3.8×10^{-5}
0.0218	16.8	2.61	0.0315	6.34×10^{-5}
(*b*) Savage and McKeown (1983)				
0.97	42.9	1.96	0.715	5.8×10^{-3}
0.97	53.0	1.75	0.975	2.55×10^{-3}
1.78	53.0	3.59	1.34	1.88×10^{-2}
1.78	42.9	0.14	0.882	2.72×10^{-3}
1.24	53.4	6.61	0.983	2.63×10^{-2}
(*c*) Bagnold (1954)				
1.32	60.6	8.15	0.75	0.0342
1.32	55.5	6.72	0.485	0.0224
1.32	49.5	3.0	0.3	0.0088
1.32	44.5	4.2	0.185	0.0048
1.32	37.3	2.93	0.126	0.0025
1.32	30.8	2.18	0.083	0.00144
1.32	22.2	0.0	0.067	0.00064

In Govier's experiments, the gap size ΔR of the rotational viscometer was small (ΔR = 1.17 mm), and fine galena particles of medium silt size (d_s = 0.0218 mm) were sheared at rates varying from 5 s^{-1} to 1,000 s^{-1}. In both Bagnold, and Savage and McKeown's experiments, neutrally buoyant particles of coarse sand size (d_s = 0.5 ~ 2 mm) were sheared in rotational viscometers with about the same gap size (ΔR = 1.08 cm and 1.43 cm, respectively). Two types of coaxial rotational viscometers have been used for the rheological measurements: (1) Bagnold and Govier et al. rotated the outer cylinder; and (2) Savage and McKeown rotated the inner cylinder. The shear rates in Bagnold's experiments (5 s^{-1}–250 s^{-1}) were larger than those of Savage and McKeown (10 s^{-1}–60 s^{-1}). Therefore, larger turbulent-dispersive stresses are expected in Bagnold's experiments.

In spite of the similarities in sediment sizes and rates of shear, the experimental measurements from all three data sets (Fig. 1) yield fairly different results. For example, in Govier et al. (1957) data set, the values of shear stress remain relatively constant at rates of shear smaller than about 70 s^{-1}. Beyond this value, shear stress increases by about one order of magnitude as the rate of shear increases from 100 s^{-1} to 1,000 s^{-1}. Under similar rates of shear, the results of Bagnold, and Savage and McKeown are strikingly different. For example, the shear stress increases by about two orders of magnitude in Bagnold's experiment as the rate of shear varies from 10 s^{-1} to 250 s^{-1}.

The best-fitted quadratic curves obtained by regression analysis are com-

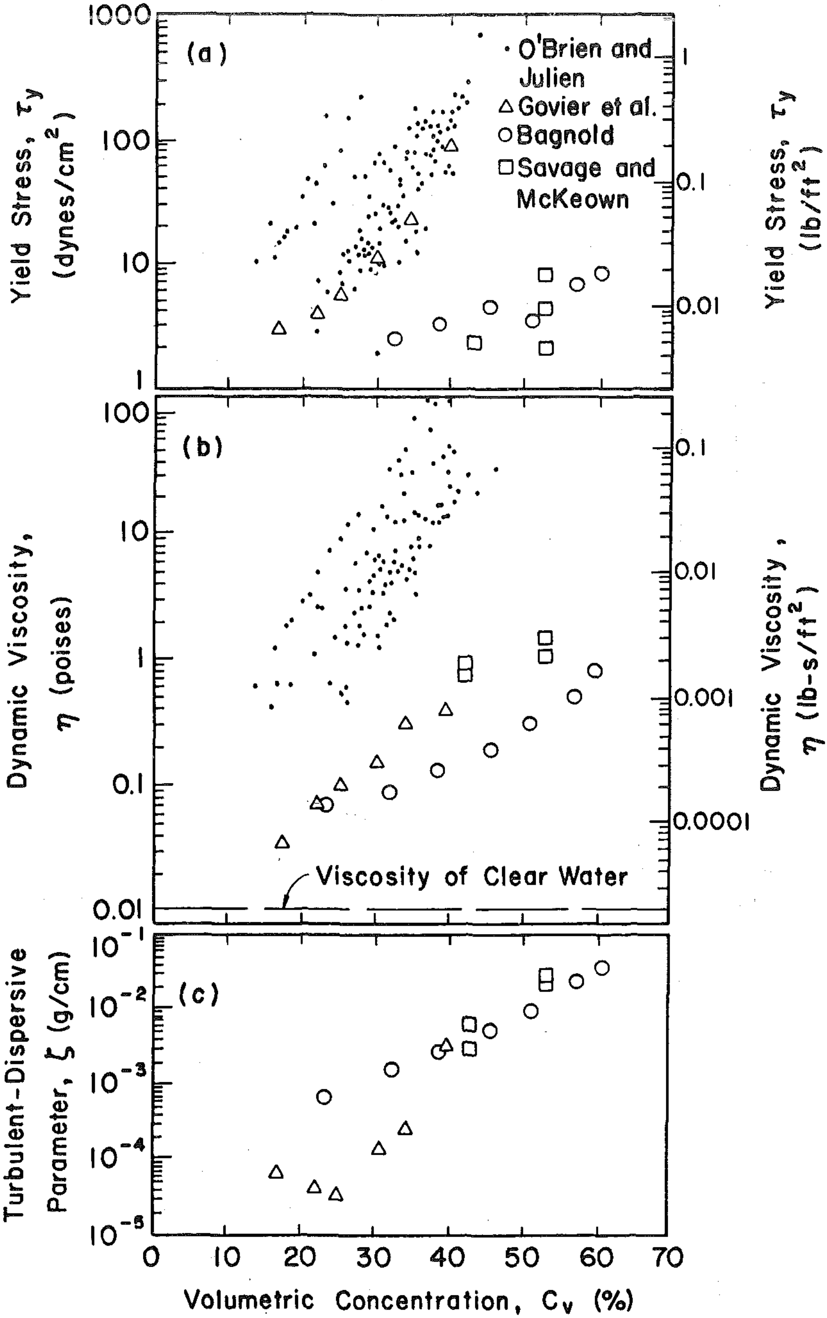

FIG. 2. Yield Stress τ_y, Dynamic Viscosity η, and Turbulent-Dispersive Parameter ζ for Hyperconcentrations

pared with the experimental data points in Fig. 1, and the parameters τ_y, η, and ζ for each data set are compiled in Table 1. Yield stress values from Table 1 are plotted against sediment concentration in Fig. 2(*a*) for comparison with measurements for natural mudflow matrices from O'Brien and Julien (1988). It is found that Govier's measurements for silt size galena particles compare with mudflow matrices comprising silt and clay particles, whereas, the neutrally buoyant sand size particles of Bagnold, and Savage and McKeown give lower values of yield stress. For all three data sets, the dynamic viscosity measurements shown in Fig. 2(*b*) range between those of clear water (η = 0.01 poise) and the mudflow matrices (silt and clay mixtures). The interesting Fig. 2(*c*) shows reasonable agreement among all values of the combined turbulent-dispersive parameter, ζ. For the three data sets, the values of ζ are found to increase very rapidly with volumetric sediment concentration.

Dimensionless Formulation of the Rheological Model

The relative magnitude of the terms in Eq. 1 is examined to seek possible reduction of the three data sets into a dimensionless rheological model, and also to better define the conditions under which simplified models can be applied.

A dimensionless rheological model can be obtained after rewriting Eqs. 1 and 2 in the following form:

$$\tau^* = 1 + (1 + T_d^*)a_1 D_v^* \qquad (5)$$

in which the three dimensionless parameters are defined as:

1. Dimensionless excess shear stress

$$\tau^* = \frac{\tau - \tau_y}{\eta \dfrac{du}{dy}} \qquad (6)$$

2. Dimensionless dispersive-viscous ratio

$$D_v^* = \frac{\rho_s \lambda^2 d_s^2}{\eta}\left(\frac{du}{dy}\right) \qquad (7)$$

3. Dimensionless turbulent-dispersive ratio

$$T_d^* = \frac{\rho_m l_m^2}{a_1 \rho_s \lambda^2 d_s^2} \qquad (8)$$

This linearized formulation of the rheological model (Eq. 5) for hyperconcentrated water-sediment mixtures has three major advantages over previous formulations:

1. It involves three dimensionless parameters τ^*, D_v^*, and T_d^*.
2. The second term on the right-hand side of Eq. 5 reflects the deviation from the Bingham plastic model.

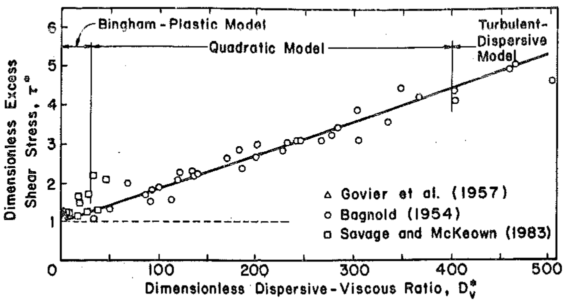

FIG. 3. Comparison of Dimensionless Model with Experimental Data

3. Newtonian and non-Newtonian fluids can be modeled depending on the relative magnitude of the parameters D_v^*, T_d^*, and τ_y.

The usefulness of the dimensionless rheological model is demonstrated in Fig. 3 where τ^* is plotted versus D_v^*. When Eq. 5 is fitted to the experimental data sets of Govier et al. (1957), Savage and McKeown (1983), and Bagnold (1954), it is found that $a_1(1 + T_d^*) = 0.0087$. It has not been possible to evaluate the parameter T_d^* because the mixing length l_m from existing experiments is not available. It is interesting to notice, however, that the value $a_1 = 0.01$ suggested by Bagnold is comparable to the value $a_1\ (1 + T_d^*) = 0.0087$ obtained from Eq. 5 when assuming that the turbulent stress is negligible compared to the dispersive stress ($T_d^* << 1$).

It is found in this analysis that not only the linearized dimensionless model (Eq. 5) is applicable to all three data sets, but the parameter D_v^* can be used to delineate particular cases of the quadratic model. The results shown in Fig. 3 indicate that τ^* is sufficiently close to unity when $D_v^* < 30$ to justify the use of a Bingham plastic model. On the other hand, τ^* exceeds 4 when D_v^* is roughly larger than 400, which indicates that in this region the turbulent-dispersive stress is dominant.

Summary and Conclusion

We concur with Savage and McKeown's (1983) conclusion that the rheology of hyperconcentrations is somewhat more complex than originally pictured by Bagnold (1954). The quadratic model describing the rheology of hyperconcentrated sediment flows is well suited to the experimental data sets of Govier et al. (1957), Savage and McKeown (1983), and Bagnold (1954). This analysis illustrates the benefits of combining the turbulent stress with the dispersive stress. When the quadratic model is written in a linearized dimensionless form (Eq. 5), the ratio of dispersive to viscous stresses D_v^* becomes of foremost importance in selecting appropriate rheological models.

It can be concluded that the quadratic model is valid for all values of the parameter D_v^*, and reduces to the Bingham plastic model when $D_v^* < 30$. Turbulent-dispersive formulations may be useful in the inertial region when $D_v^* > 400$.

Acknowledgments

The writers are grateful to E. V. Richardson, who demonstrated great enthusiasm in this work and also supported the second writer during the course of his Ph.D. studies at Colorado State University.

Appendix I. References

Bagnold, R. A. (1954). "Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear." *Proc. of the Royal Soc. of London,* 225, 49–63.

Govier, G. W., Shook, C. A., and Lilge, E. O. (1957). "The properties of water suspension of finely subdivided magnetite, galena, and ferrosilicon," *Trans. Can. Inst. Mining and Met.*, 60, 147–154.

Jeffrey, D. J., and Acrivos, A. (1976). "The rheological properties of suspension of rigid particles." *Am. Inst. of Chem. Engrg. J.*, 22, 417–432.

O'Brien, J. S., and Julien, P. Y. (1985). "Physical properties and mechanics of hyperconcentrated sediment flows." *Proc. of the Specialty Conference on Delineation of Landslides, Flash Flood and Debris Flow Hazards in Utah,* Utah Water Research Laboratory, 260–279.

O'Brien, J. S., and Julien, P. Y. (1988), "Laboratory Analysis of Mudflow Properties," *J. Hydr. Engrg.*, ASCE, 114(8), 877–887.

Savage, S. B., and McKeown, S. (1983). "Shear stresses developed during rapid shear of concentrated suspension of large spherical particles between concentric cylinders." *J. Fluid Mech.*, 127, 453–472.

Takahashi, T. (1980). "Debris flow on prismatic open channel." *J. Hydr. Div.*, ASCE, 106(3), 381–396.

Appendix II. Notation

The following symbols are used in this paper:

a_1 = dispersive stress coefficient;
C_v = volumetric sediment concentration;
C^* = maximum volumetric sediment concentration ($C^* \sim 0.615$);
D_v^* = dimensionless dispersive-viscous ratio;
d_s = average diameter of sediment particles;
l_m = mixing length of mixture;
T_d^* = dimensionless turbulent-dispersive ratio;
ΔR = gap size of viscometer;
ζ = turbulent-dispersive parameter;
η = dynamic viscosity of mixture;
λ = linear concentration of sediments;
ρ, ρ_s, ρ_m = densities of fluid, particles, and sediment-water mixture, respectively;
τ = shear stress;
τ_y = yield shear stress; and
τ^* = dimensionless excess shear stress.