Turbulent velocity profiles in sediment-laden flows

Profils de vitesse des écoulements turbulents avec transport de sédiment

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ABSTRACT

A theoretical analysis shows that velocity profiles in sediment-laden flows are similar to those in clear water. The modified log-wake law, which is developed for clear water by Guo, is also valid in sediment-laden flows. The analysis of the effects of sediment suspension on turbulent kinetic energy and turbulent diffusion shows that: (1) sediment suspension increases mean flow energy loss; (2) sediment suspension weakens turbulent diffusion in the vertical direction and then increases velocity gradient; and (3) sediment suspension affects velocity profile in two ways: average concentration and density gradient. The comparison with narrow-channel laboratory data confirms the theoretical analysis and shows that: (1) the modified log-wake law agrees well with experimental data for sediment-laden flows; (2) both average concentration and density gradient reduce the von Karman constant; and (3) for a given width-depth ratio, sediment concentration slightly increases the wake strength while density gradient has little effect on it. In addition, the modified log-wake law can reproduce experimental data where the maximum velocity occurs below the water surface.

RÉSUMÉ

Une analyse théorique démontre la similitude des profils de vitesse avec ou sans transport de sédiment. Les profils de vitesse logarithmiques avec coefficient de traînée sont donc applicables pour les écoulements chargés de sédiments. Cette loi permet de représenter les profils de vitesse avec maximum sous la surface libre. Les sédiments en suspension: (1) augmentent les pertes d'énergie; (2) affaiblissent la diffusion turbulente verticale et augmentent le gradient de vitesse; et (3) affectent les profils de vitesse en raison de la concentration moyenne et du gradient de densité. La comparaison avec des données de laboratoire en canal étroit confirme que: (1) la loi logarithmique avec coefficient de traînée s'applique bien aux données expérimentales; (2) la concentration moyenne et le gradient de densité diminuent la constante de von Kármán; et (3) pour un rapport largeur-profondeur donné, la concentration de sédiments augmente légèrement le coefficient de traînée tandis que le gradient de densité exerce peu d'influence sur ce coefficient.

1 Introduction

The study of turbulent velocity profiles in sediment-laden flows is one of the most important subjects in sediment transport and river mechanics. Vanoni (1946), Einstein and Chien (1955), Vanoni and Nomicos (1960), Elata and Ippen (1961), and many others examined the log law describing the variation of velocity with depth in sediment-laden flows. They concluded that the log law remains valid except that the von Karman constant decreases with sediment suspension. Einstein and Chien (1955) proposed a graphical relation to predict the von Karman constant κ based on an energy concept. They also pointed out that the main effect of sediment suspension occurs near the bed. Recently, Muste and Patel (1997) also studied experimentally the effect of sediment suspension on the log law. They concluded that small sediment concentrations have little effect on the log law near the bed.

Coleman (1981, 1986) introduced the log-wake law to open channels and studied the effect of sediment suspension on the von Karman constant κ and the wake strength Π. He argued that if the log-wake law is applied, κ remains the same as that in clear water, i.e., κ=0.4, but Π increases with the density gradient as described by Richardson number. He further pointed out that the previous conclusion, i.e., κ decreases with sediment suspension, was obtained by incorrectly extending the log law to the wake layer where the velocity deviates the log law systematically in clear water. Parker and Coleman (1986) and Cioffi and Gallerano (1991) supported Coleman's argument. Janin (1986) at CSU obtained a similar result in a large boundary layer wind tunnel. However, Lyn (1986,1988) found that the von Karman constant κ might decrease with sediment suspension even in the log-wake model. Kereselidze and Kutavaia (1995) deduced from their own experiments that both κ and Π vary with sediment suspension.

Guo (1998) pointed out that both the log law and the log-wake law cannot satisfy the outer boundary condition at the outer boundary. Furthermore, based on a similarity analysis, he developed a modified log-wake law:

$$\frac{\overline{u}_1}{u_*} = \frac{1}{\kappa}\ln\xi + \frac{\overline{u}_{1\max}}{u_*} - \Omega\cos^2\frac{\pi\xi}{2} + \left[\frac{1}{\kappa} - \frac{1}{u_*}\frac{d\overline{u}_1}{d\xi}\bigg|_{\xi=1}\right](1-\xi) \tag{1a}$$

or

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$$\frac{\bar{u}_{1\max}-\bar{u}_1}{u_*} = -\frac{1}{\kappa}\ln\xi + \Omega\cos^2\frac{\pi\xi}{2} - \left[\frac{1}{\kappa} - \frac{1}{u_*}\frac{d\bar{u}_1}{d\xi}\bigg|_{\xi=1}\right](1-\xi) \tag{1b}$$

in which $\bar{u}_{1\max}$ is the maximum velocity at the upper boundary edge; $\bar{u}_1$ is the local time-averaged velocity in the flow direction; u_* is the shear velocity; κ is the von Karman constant which is denoted as κ_0=0.406 for clear water (Guo, 1998); ξ is the normalized distance from the bed by the boundary layer thickness; Ω is the wake strength which is denoted as Ω_0 for clear water; and

$$\frac{d\bar{u}_1}{d\xi}\bigg|_{\xi=1}$$

is the velocity gradient at the outer boundary. The log term reflects the inertia effect near the wall; the cosine-square term expresses large-scale turbulent mixing; and the linear term reflects the effect of the upper boundary.

The objectives of this paper are: (1) to analyze the effects of sediment suspension on the governing equations and turbulent structures; (2) to show that the modified log-wake law is also valid in sediment-laden flows; and (3) to determine the effects of sediment suspension on the von Karman constant κ and the wake strength Ω in sediment-laden flows.

2 Governing equations for sediment-laden flows

2.1 Navier-Stokes equations for sediment-laden flows

The effects of sediment suspension on turbulence are examined in terms of continuity, momentum diffusion and density equations. For simplicity, the Boussinesq approximation (Kundu, 1990) on stratified flows is introduced, i.e., the effect of sediment concentration on the fluid density may be neglected in continuity and momentum equations except in the gravity term. The viscosity is assumed constant in this analysis.

2.1.1 Continuity equation

Based on the Boussinesq assumption, the continuity equation in sediment-laden flows is the same as that in clear water, i.e.

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

in which u_i is the mixture water velocity component in the x_i direction and x_i = 1, 2, and 3.

2.1.2 Momentum equation

The momentum equation in sediment-laden flows is written as

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = \frac{\rho}{\rho_m}g_i - \frac{1}{\rho_m}\frac{\partial p}{\partial x_i} + \nu_m\frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{3}$$

in which t is time; j is a dummy subscript; ρ is the local density and varies with sediment concentration; ρ_m is the space-averaged density of ρ, i.e.

$$\rho_m = \frac{1}{V}\int_V \rho dV$$

where V is a volume; g_i is the component of the gravitational acceleration in the x_i direction; p is pressure; and ν_m is the kinematic viscosity corresponding to ρ_m.

2.1.3 Sediment concentration equation

Conservation of mass applied to the sediment phase gives

$$\frac{\partial C}{\partial t} + u_j\frac{\partial C}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial C}{\partial x_j}\right) \tag{4}$$

in which C is the volumetric sediment concentration; the first term on the left-hand side is the concentration change with time; u_j is the convective velocity of sediment, i.e., not necessarily identical to u_j in (3); the second term on the left-hand side is the mass flux by advection; D is the molecular diffusion coefficient; and the right-hand side is the transport by molecular diffusion.

2.1.4 State equation or density equation

The density equation can be easily written as

$$\rho = \rho_0 + (\rho_s - \rho_0)C \tag{5}$$

in which ρ_0 is the clear water density; and ρ_s is the sediment density. The above equation set (2-5) is closed since one has 6 equations (1 continuity, 3 momentum, 1 concentration and 1 density equation) with 6 unknowns (3 velocity components u_i, 1 pressure p, 1 density ρ, and 1 concentration C). However, like any other turbulence, the above equations are very difficult to solve for large Reynolds number flows, i.e. turbulent flows. To study the mean turbulent velocity field, the Reynolds average method may be applied.

2.2 Reynolds average for turbulent sediment-laden flows

Following Reynolds averaging process, a variable is decomposed into a time-average component denoted with an overbar, and a turbulent component denoted with a prime, i.e.

$$u_i = \bar{u}_i + u_i' \qquad p = \bar{p} + p'$$
$$\rho = \bar{\rho} + \rho' \qquad C = \bar{C} + C' \tag{6}$$

Substituting (6) into (2-5) and introducing the Reynolds average method, one can get the motion equations for the mean flow and the turbulent flow, respectively.

2.2.1 Continuity equation

Substituting the expressions (6) into (2) and taking the Reynolds average, one has

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{7}$$

for the mean motion, and

$$\frac{\partial u_i'}{\partial x_i} = 0 \tag{8}$$

for the turbulent motion.
From the above two continuity equations, one can get the following two identities:

$$\overline{u}_j \frac{\partial f}{\partial x_j} = \frac{\partial(\overline{u}_j f)}{\partial x_j} \qquad u_j' \frac{\partial f}{\partial x_j} = \frac{\partial(u_j' f)}{\partial x_j} \tag{9}$$

in which f can be any variable. These two identities will be frequently used in the following derivations.

2.2.2 Momentum equation
Substituting (6) into (3), one gets

$$\frac{\partial(\overline{u}_i + u_i')}{\partial t} + (\overline{u}_k + u_k')\frac{\partial(\overline{u}_i + u_i')}{\partial x_k} = \frac{\overline{\rho} + \rho'}{\rho_m} g_i - \frac{1}{\rho_m}\frac{\partial(\overline{p} + p')}{\partial x_i} + \nu_m \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u}_i + u_i')$$

in which both ρ_m and ν_m are time-space-averaged values which are constant for a given flow. Applying the identities (9) to the convective term and expanding it, one obtains

$$\frac{\partial(\overline{u}_i + u_i')}{\partial t} + \frac{\partial(\overline{u}_i\overline{u}_k + \overline{u}_i u_k' + u_i'\overline{u}_k + u_i'u_k')}{\partial x_k} = \frac{\overline{\rho} + \rho'}{\rho_m} g_i - \frac{1}{\rho_m}\frac{\partial(\overline{p} + p')}{\partial x_i} + \nu_m \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u}_i + u_i') \tag{10}$$

Taking the time-average over this equation and considering that the average of a fluctuating variable is zero, one has the following mean motion equation for sediment-laden flows

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial(\overline{u}_i\overline{u}_k + \overline{u_i'u_k'})}{\partial x_k} = \frac{\overline{\rho}}{\rho_m} g_i - \frac{1}{\rho_m}\frac{\partial \overline{p}}{\partial x_i} + \nu_m \frac{\partial^2 \overline{u}_i}{\partial x_k \partial x_k} \tag{11}$$

or

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_k \frac{\partial \overline{u}_i}{\partial x_k} = \frac{\overline{\rho}}{\rho_m} g_i - \frac{1}{\rho_m}\frac{\partial \overline{p}}{\partial x_i} + \nu_m \frac{\partial^2 \overline{u}_i}{\partial x_k \partial x_k} - \frac{\partial \overline{u_i'u_k'}}{\partial x_k} \tag{12}$$

in which $\overline{u_i'u_k'}$ is the second-order turbulence intensity. The product of ρ_m and $-\overline{u_i'u_k'}$ is the so-called turbulent stress or Reynolds stress. Equation (12) will be used to study the mean velocity profiles in sediment-laden flows.
The subtraction of the mean motion equation (11) from the total motion equation (10) gives the turbulent motion equation, i.e.

$$\frac{\partial u_i'}{\partial t} + \frac{\partial(\overline{u}_i u_k' + u_i'\overline{u}_k + u_i'u_k' - \overline{u_i'u_k'})}{\partial x_k} = \frac{\rho'}{\rho_m} g_i - \frac{1}{\rho_m}\frac{\partial p'}{\partial x_i} + \nu_m \frac{\partial^2 u_i'}{\partial x_k \partial x_k}$$

or

$$\frac{\partial u_i'}{\partial t} + u_k' \frac{\partial \overline{u}_i}{\partial x_k} + \overline{u}_k \frac{\partial u_i'}{\partial x_k} + \frac{\partial u_i'u_k'}{\partial x_k} - \frac{\partial \overline{u_i'u_k'}}{\partial x_k} = \frac{\rho'}{\rho_m} g_i - \frac{1}{\rho_m}\frac{\partial p'}{\partial x_i} + \nu_m \frac{\partial^2 u_i'}{\partial x_k \partial x_k} \tag{13}$$

This equation will serve to analyze the effects of sediment suspension on turbulence intensity.

2.2.3 Sediment concentration equation
Applying (6) to (4) yields

$$\frac{\partial(\overline{C} + C')}{\partial t} + (\overline{u}_j + u_j')\frac{\partial(\overline{C} + C')}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial(\overline{C} + C')}{\partial x_j}\right)$$

or

$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial C'}{\partial t} + \overline{u}_j \frac{\partial \overline{C}}{\partial x_j} + \overline{u}_j \frac{\partial C'}{\partial x_j} + u_j' \frac{\partial \overline{C}}{\partial x_j} + \frac{\partial u_j'C'}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial(\overline{C} + C')}{\partial x_j}\right) \tag{14}$$

Taking the time-average over this equation and rearranging it, one obtains the mean concentration equation:

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}_j \frac{\partial \overline{C}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial \overline{C}}{\partial x_j} - \overline{u_j'C'}\right) \tag{15}$$

Similarly, the subtraction of (15) from (14) gives the turbulent concentration equation:

$$\frac{\partial C'}{\partial t} + \overline{u}_j \frac{\partial C'}{\partial x_j} + u_j' \frac{\partial \overline{C}}{\partial x_j} + \frac{\partial u_j'C'}{\partial x_j} - \frac{\partial \overline{u_j'C'}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(D\frac{\partial C'}{\partial x_j}\right) \tag{16}$$

2.2.4 State equation or density equation
Applying (6) to (5) results in

$$\overline{\rho} + \rho' = \rho_0 + (\rho_s - \rho_0)(\overline{C} + C') \tag{17}$$

Taking the time-average gives the time-average density equation:

$$\overline{\rho} = \rho_0 + (\rho_s - \rho_0)\overline{C} \tag{18}$$

Note that this mean density varies with space and has the relation with the time-space-averaged mean ρ_m as:

$$\rho_m = \int_V \overline{\rho} dV \cdot$$

Similarly, one can obtain the turbulent density equation:

$$\rho' = (\rho_s - \rho_0)C' \tag{19}$$

In brief, the mean motion equations for sediment-laden flows can be summarized as (7), (12), (15) and (18). To solve this set of equations, one must make some assumptions about $u_i'u_k'$ and $u_j'C'$. This is known as the closure problem.

The turbulent motion equations are summarized as (8), (13), (16) and (19). Only (13) of this set will be used to study the effects of sediment suspension on turbulence intensity.

2.3 Sediment effects on momentum equations

For simplicity, this study assumes that the mean flow is two-dimensional, steady and uniform, as shown in Fig. 1. That is, steady:

$$\frac{\partial\overline{(\ )}}{\partial t} = 0 \tag{20}$$

uniform:

$$\frac{\partial\overline{(\ )}}{\partial x_1} = 0, \quad \frac{\partial\overline{(\ )}}{\partial x_2} = 0 \tag{21}$$

two-dimensional flow:

$$\overline{u}_1 = \overline{u}_1(x_3), \quad \overline{u}_2 = \overline{u}_3 = 0, \quad \overline{C} = \overline{C}(x_3) \tag{22}$$

in which t = time; $\overline{(\ )}$ denotes time averaging; $\overline{u}_1$ = time-averaged velocity in the flow direction x_1; $\overline{u}_2$ = time-averaged velocity in the lateral direction x_2, $\overline{u}_3$ = time-averaged velocity in the vertical (normal) direction x_3; and $\overline{C}$ = time-averaged sediment concentration.

Based on the above assumptions, the mean continuity equation is automatically satisfied. The momentum equations reduce to

x_1-direction:

$$\overline{\rho}g_1 + \mu_m\frac{\partial^2\overline{u}_1}{\partial x_3^2} - \rho_m\frac{\partial\overline{u_1'u_3'}}{\partial x_3} = 0 \tag{23}$$

x_2-direction:

$$\frac{\partial\overline{u_2'u_3'}}{\partial x_3} = 0 \tag{24}$$

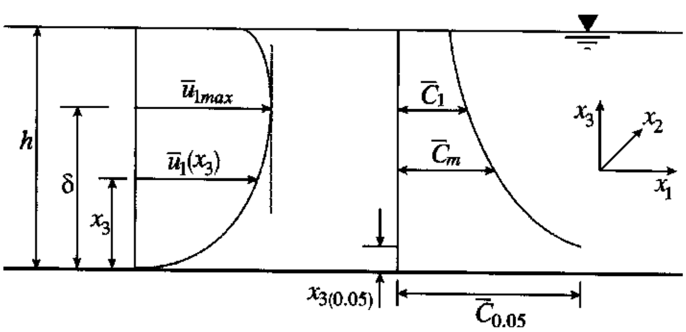

Fig. 1. Scheme of representative velocity and concentration profiles

x_3-direction:

$$\overline{\rho}g_3 - \frac{\partial\overline{p}}{\partial x_3} - \overline{\rho}_m\frac{\partial\overline{u_3'^2}}{\partial x_3} = 0 \tag{25}$$

Fortunately, (24) and (25) are not coupled with (23), then only (23) is used to find the velocity profile $\overline{u}_1(x_3)$. Furthermore, (23) can be written as

$$-\frac{\partial}{\partial x_3}\left(\mu_m\frac{\partial\overline{u}_1}{\partial x_3} - \rho_m\overline{u_1'u_3'}\right) = \overline{\rho}g_1 \tag{26}$$

in which $\mu_m = \rho_m\nu_m$ is the time-space-averaged mean dynamic viscosity of the mixture water. Consider that the shear stress at the outer boundary, $x_3 = \delta$, is

$$\left(\mu_m\frac{\partial\overline{u}_1}{\partial x_3} - \rho_m\overline{u_1'u_3'}\right)_{x_3=\delta} = \tau|_{\xi=1}$$

Substituting (18) into (26) and integrating yields

$$-\left(\mu_m\frac{\partial\overline{u}_1}{\partial x_3} - \rho_m\overline{u_1'u_3'}\right)\Big|_{x_3}^{\delta} = \int_{x_3}^{\delta}\left(\rho_0 + (\rho_s - \rho_0)\overline{C}\right)g_1 dx_3$$

or

$$\mu_m\frac{\partial\overline{u}_1}{\partial x_3} - \rho_m\overline{u_1'u_3'} - \tau|_{\xi=1} = \rho_0 g_1(\delta - x_3) + (\rho_s - \rho_0)g_1\int_{x_3}^{\delta}\overline{C}dx_3 \tag{27}$$

It is becoming clear that sediment suspension affects velocity profiles in three ways: (1) changing the fluid viscosity μ_m thus changing the viscous shear stress (lst term on the left-hand side); (2) changing the fluid density thus changing turbulence intensity and the turbulent shear stress (2nd term on the left-hand side); and (3) producing a density gradient thus increasing the gravitational component in the flow direction (2nd term on the right-hand side).

The following order of magnitude analysis assumes

$$\overline{u}_1 \sim U, \quad x_3 \sim \delta \sim h, \quad \overline{u_1'u_3'} \sim u_*^2, \quad \overline{C} \sim \overline{C}_a, \quad \rho_m \sim$$

in which U is the depth-averaged velocity; u_* is the shear velocity; and $\overline{C}_a$ is a near bed concentration. Then the magnitude of each term in (27) is as follows:

$$\frac{\mu_m U}{h}, \quad \rho_m u_*^2, \quad \text{Keep}, \quad \rho_0 g_1 h, \quad \rho_0 g_1 h, \quad (\rho_s - \rho_0)g_1 h\overline{C}_a$$

Since $u_* \approx \sqrt{g\delta S} = \sqrt{g_1\delta}$ in which $S = \sin\theta$ is the channel slope, and $\rho_m \sim \rho_0$, one obtains after dividing by $\rho_m u_*^2$

$$\frac{1}{\mathrm{Re}}, \quad \left(\frac{u_*}{U}\right)^2, \quad \text{Keep}, \quad \left(\frac{u_*}{U}\right)^2, \quad \left(\frac{u_*}{U}\right)^2, \quad \frac{\rho_s - \rho_0}{\rho_m}\left(\frac{u_*}{U}\right)^2\overline{C}_a$$

In practice, one often has Re $>10^4$, $U/u_* \sim 10$, and let

$$\frac{\rho_s-\rho_0}{\rho_m}\bar{C}_a<0.1,$$

the above magnitude orders become

<0.01, 1, Keep, 1, 1, <0.1

This analysis shows that the viscous term and the effect of sediment on gravity may be neglected. Therefore, the momentum equation in steady uniform two-dimensional sediment-laden flows can be further simplified as

$$-\rho_m\overline{u_1'u_3'} - \tau|_{\xi=1} = \rho_0 g_1(\delta - x_3) \tag{28}$$

Note that $\tau|_{\xi=1}$ is kept in the above equation since it relates to the boundary condition at the outer boundary, and it may also be very small. The above equation is similar to that in clear water except that the turbulent shear stress may be modified by sediment suspension. Besides, since the viscous term is neglected, (28) is not valid very near the bed. Consequently, velocity profiles developed for clear water flows should also be valid in sediment-laden flows since their governing equations and boundary conditions are similar.

3 Sediment effects on turbulence intensity $\overline{u_i'u_j'}$

One may start with (13), the equation for the turbulent velocity component u_i'. One can also write the same equation for the velocity component u_j'. Multiplying the equation for u_i' by u_j' and the equation for u_j' by u_i', one gets

$$\begin{aligned}&u_j'\frac{\partial u_i'}{\partial t} + u_j'u_k'\frac{\partial \bar{u}_i}{\partial x_k} + u_j'\bar{u}_k\frac{\partial u_i'}{\partial x_k} + u_j'\frac{\partial(u_i'u_k' - \overline{u_i'u_k'})}{\partial x_k}\\ &= \frac{u_j'\rho'}{\rho_m}g_i - \frac{u_j'}{\rho_m}\frac{\partial p'}{\partial x_i} + \nu_m u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k}\end{aligned}$$

and

$$\begin{aligned}&u_i'\frac{\partial u_j'}{\partial t} + u_i'u_k'\frac{\partial \bar{u}_j}{\partial x_k} + u_i'\bar{u}_k\frac{\partial u_j'}{\partial x_k} + u_i'\frac{\partial(u_j'u_k' - \overline{u_j'u_k'})}{\partial x_k}\\ &= \frac{u_i'\rho'}{\rho_m}g_j - \frac{u_i'}{\rho_m}\frac{\partial p'}{\partial x_j} + \nu_m u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}\end{aligned}$$

Adding the above two equations gives

$$\begin{aligned}&\frac{\partial u_i'u_j'}{\partial t} + u_j'u_k'\frac{\partial \bar{u}_i}{\partial x_k} + u_i'u_k'\frac{\partial \bar{u}_j}{\partial x_k} + \bar{u}_k\frac{\partial u_i'u_j'}{\partial x_k}\\ &+ u_j'\frac{\partial(u_i'u_k' - \overline{u_i'u_k'})}{\partial x_k} + u_i'\frac{\partial(u_j'u_k' - \overline{u_j'u_k'})}{\partial x_k}\\ &= \frac{u_j'\rho'}{\rho_m}g_i + \frac{u_i'\rho'}{\rho_m}g_j - \frac{u_j'}{\rho_m}\frac{\partial p'}{\partial x_i} - \frac{u_i'}{\rho_m}\frac{\partial p'}{\partial x_j}\\ &+ \nu_m\left(u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k} + u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}\right)\end{aligned}$$

The time-average of this equation yields the turbulence intensity of $\overline{u_i'u_j'}$, i.e.

$$\begin{aligned}&\frac{\partial\overline{u_i'u_j'}}{\partial t} + \overline{u_j'u_k'}\frac{\partial \bar{u}_i}{\partial x_k} + \overline{u_i'u_k'}\frac{\partial \bar{u}_j}{\partial x_k} + \bar{u}_k\frac{\partial\overline{u_i'u_j'}}{\partial x_k}\\ &= -\left(\overline{u_j'\frac{\partial u_i'u_k'}{\partial x_k}} + \overline{u_i'\frac{\partial u_j'u_k'}{\partial x_k}}\right) + \frac{1}{\rho_m}\left(\overline{u_j'\rho'}g_i + \overline{u_i'\rho'}g_j\right)\\ &- \frac{1}{\rho_m}\left(\overline{u_j'\frac{\partial p'}{\partial x_i}} + \overline{u_i'\frac{\partial p'}{\partial x_j}}\right) + \nu_m\left(\overline{u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k}} + \overline{u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}}\right)\end{aligned} \tag{29}$$

The terms on the right-hand side of this equation may be transformed to measurable forms.

Applying the identities (9), the first term on the right-hand side of (29) becomes

$$\overline{u_j'\frac{\partial u_i'u_k'}{\partial x_k}} + \overline{u_i'\frac{\partial u_j'u_k'}{\partial x_k}} = \overline{u_j'u_k'\frac{\partial u_i'}{\partial x_k}} + \overline{u_i'\frac{\partial u_j'u_k'}{\partial x_k}} = \frac{\partial\overline{u_i'u_j'u_k'}}{\partial x_k} \tag{30}$$

Considering (19), the second term on the right-hand side of (29) becomes

$$\overline{u_j'\rho'}g_i + \overline{u_i'\rho'}g_j = (\rho_s - \rho_0)\left(\overline{u_j'C'}g_i + \overline{u_i'C'}g_j\right) \tag{31}$$

The third term on the right-hand side of (29) may be written as

$$\begin{aligned}&\overline{u_j'\frac{\partial p'}{\partial x_i}} + \overline{u_i'\frac{\partial p'}{\partial x_j}} = \frac{\partial\overline{p'u_j'}}{\partial x_i} - \overline{p'\frac{\partial u_j'}{\partial x_i}} + \frac{\partial\overline{p'u_i'}}{\partial x_j} - \overline{p'\frac{\partial u_i'}{\partial x_j}}\\ &= \left(\frac{\partial\overline{p'u_j'}}{\partial x_i} + \frac{\partial\overline{p'u_i'}}{\partial x_j}\right) - \overline{p'\left(\frac{\partial u_j'}{\partial x_i} + \frac{\partial u_i'}{\partial x_j}\right)}\end{aligned} \tag{32}$$

Since

$$\begin{aligned}&\frac{\partial^2}{\partial x_k\partial x_k}\overline{u_i'u_j'} = \frac{\partial}{\partial x_k}\overline{\left(u_i'\frac{\partial u_j'}{\partial x_k} + u_j'\frac{\partial u_i'}{\partial x_k}\right)}\\ &= \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} + \overline{u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}} + \overline{\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}} + \overline{u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k}}\\ &= \overline{u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}} + \overline{u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k}} + 2\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}\end{aligned}$$

one can get the fourth term on the right-hand side of (29) as

$$\overline{u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}} + \overline{u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k}} = \frac{\partial^2}{\partial x_k\partial x_k}\overline{u_i'u_j'} - 2\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} \tag{33}$$

Substituting (30-33) into (29) yields the turbulence intensity equation:

$$\begin{aligned}&\frac{\partial\overline{u_i'u_j'}}{\partial t} + \bar{u}_k\frac{\partial\overline{u_i'u_j'}}{\partial x_k} = -\left(\overline{u_j'u_k'}\frac{\partial \bar{u}_i}{\partial x_k} + \overline{u_i'u_k'}\frac{\partial \bar{u}_j}{\partial x_k}\right)\\ &+ \left[\frac{\rho_s-\rho_0}{\rho_m}\left(\overline{u_j'C'}g_i + \overline{u_i'C'}g_j\right)\right] - \frac{\partial\overline{u_i'u_j'u_k'}}{\partial x_k}\\ &- \frac{1}{\rho_m}\left(\frac{\partial\overline{p'u_j'}}{\partial x_i} + \frac{\partial\overline{p'u_i'}}{\partial x_j}\right) + \frac{1}{\rho_m}\overline{p'\left(\frac{\partial u_j'}{\partial x_i} + \frac{\partial u_i'}{\partial x_j}\right)}\\ &+ \nu_m\frac{\partial^2\overline{u_i'u_j'}}{\partial x_k\partial x_k} - 2\nu_m\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}\end{aligned} \tag{34}$$

This equation is the same as that in clear water (Hinze, 1975, p.324) except for the extra term in bracket [ ] that relates to sediment suspension. The above equation is the general one-point second-order turbulence intensity equation. It can be used to study $\overline{u_1'^2}$, $\overline{u_1'u_2'}$, $\overline{u_1'u_3'}$, $\overline{u_2'^2}$, $\overline{u_2'u_3'}$, $\overline{u_3'^2}$ and turbulent kinetic energy

$\frac{1}{2}\overline{u_i'u_j'}$.

In this study, only the turbulent kinetic energy

$\frac{1}{2}\overline{u_i'u_j'}$

and the vertical turbulence intensity $\overline{u_3'^2}$ are considered since they relate to the Richardson number and the vertical turbulent diffusion in sediment-laden flows.

3.1 Turbulent kinetic energy budget

To study the turbulent kinetic energy budget, let $i = j$ in (34), then one has

$$\frac{\partial \overline{u_i'u_i'}}{\partial t} = -2\overline{u_i'u_3'}\frac{\partial \overline{u}_i}{\partial x_3} + 2\frac{\rho_s - \rho_0}{\rho_m}\overline{u_i'C'}g_i - \frac{\partial \overline{u_i'u_i'u_3'}}{\partial x_3} - \frac{2}{\rho_m}\frac{\partial \overline{p'u_3'}}{\partial x_3} + \nu_m\frac{\partial^2 \overline{u_i'u_i'}}{\partial x_3 \partial x_3} - 2\nu_m\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}$$

or

$$\frac{\partial}{\partial t}\frac{\overline{q^2}}{2} = \underbrace{-\overline{u_1'u_3'}\frac{\partial \overline{u}_1}{\partial x_3}}_{\text{turb. production}} + \underbrace{\frac{\rho_s - \rho_0}{\rho_m}\overline{u_i'C'}g_i}_{\text{sediment suspension}} \underbrace{-\frac{\partial}{\partial x_3}\overline{\left(\frac{q^2}{2} + \frac{p'}{\rho_m}\right)u_3'}}_{\text{turbulent transport}} \underbrace{+\nu_m\frac{\partial^2}{\partial x_3^2}\left(\frac{\overline{q^2}}{2}\right)}_{\text{viscous transport}} - \underbrace{\nu_m\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}}_{\text{energy dissipation}} \tag{35}$$

in which $q^2 = \overline{u_i'u_i'}$. The transport by viscous diffusion is usually neglected. The turbulent transport may also be neglected if the turbulence intensity is not very strong. This is because $p' \propto q^2 > 0$, $\overline{p'u_3'} \propto \overline{q^2u_3'} \approx 0$. Thus, the main terms of (35) reduce to

$$\frac{\partial}{\partial t}\frac{\overline{q^2}}{2} = \underbrace{-\overline{u_1'u_3'}\frac{\partial \overline{u}_1}{\partial x_3}}_{\text{turb. production}} + \underbrace{\frac{\rho_s - \rho_0}{\rho_m}\overline{u_i'C'}g_i}_{\text{sediment suspension}} - \underbrace{\nu_m\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}}_{\text{energy dissipation}} \tag{36}$$

Experiments (Vanoni, 1946; Einstein and Chien, 1955; Elata and Ippen, 1961; and others) have shown that both $-\overline{u_1'u_3'}$ and

$\frac{\partial \overline{u}_1}{\partial x_3}$

are positive and increase with sediment suspension. This implies that the presence of sediment in suspension would increase turbulent energy production, beyond the clear water level.

Because the concentration field is homogeneous in the x_1 and the x_2 directions, the mean turbulent mixing fluxes in these directions must be zero, i.e.

$$\overline{u_1'C'} = \overline{u_2'C'} = 0 \tag{37}$$

Thus,

$$\frac{\rho_s - \rho_0}{\rho_m}\overline{u_i'C'}g_i = \frac{\rho_s - \rho_0}{\rho_m}\overline{u_3'C'}g_3$$

To balance sediment settling from upward, the turbulent mixing flux $\overline{u_3'C'}$ in the x_3 direction must be positive, i.e.

$$\overline{u_3'C'} > 0 \tag{38}$$

Considering $g_3 = -g\cos\theta \approx -g$ (in which θ is the angle between the channel bed and a datum), one has

$$\frac{\rho_s - \rho_0}{\rho_m}\overline{u_3'C'}g_3 < 0 \tag{39}$$

In conclusion, this implies that sediment suspension withdraws energy from the system and decreases the turbulent kinetic energy. In other words, the energy required to support suspended-load comes from the turbulent kinetic energy rather than the mean flow energy.

Since the presence of sediment increases the viscosity ν_m, the energy dissipation term is expected to increase in sediment-laden flows.

Of all three terms on the right-hand side in (36), the last two are negative (sediment suspension + energy dissipation) and the first term (turbulent production) is positive, the resultant of the right-hand side may increase or may decrease the turbulent kinetic energy. When the system reaches a new equilibrium state, the sum of the right-hand side must be zero. The sediment suspension always increases the mean flow energy loss because sediment suspension increases turbulent production that, in turn, comes from the mean flow energy.

The Richardson number R_i is defined as the ratio of the sediment suspension energy to the turbulent production in (36), i.e.

$$R_i = \frac{\frac{\rho_s - \rho_0}{\rho_m}\overline{u_3'C'}g}{-\overline{u_1'u_3'}\frac{d\overline{u}_1}{dx_3}} \tag{40}$$

Introducing

$$-\overline{u_1'u_3'} = \varepsilon_m\frac{d\overline{u}_1}{dx_3}$$

$$-\overline{u_3'C'} = \varepsilon_s\frac{d\overline{C}}{dx_3}$$

in which the turbulent sediment diffusion coefficient ε_s is proportional to the momentum eddy viscosity ε_m. Therefore,

$$R_i \propto -\frac{(\rho_s-\rho_0)g}{\rho_m}\frac{\dfrac{d\overline{C}}{dx_3}}{\left(\dfrac{d\overline{u}_1}{dx_3}\right)^2}$$

The global Richardson number is a function of maximum flow velocity $\overline{u}_{1\max}$, boundary layer thickness δ, concentration $\overline{C}_1$ at the outer boundary, and near bed concentration $\overline{C}_a$ at a distance a above the bed.

$$R_i \propto -\frac{(\rho_s-\rho_0)g}{\rho_m}\frac{\dfrac{\overline{C}_1-\overline{C}_{0.05}}{\delta}}{\left(\dfrac{\overline{u}_{1\max}}{\delta}\right)^2}$$

in which $a = 0.05$ is taken in this study. Considering $\overline{u}_{1\max} \propto u_*$, and $\rho_m = \rho_0 + (\rho_s - \rho_0)\overline{C}_m$ in which ρ_m is the time-depth-averaged concentration, the Richardson number R_i is defined as

$$R_i = \frac{g\delta}{u_*^2}\frac{\rho_s-\rho_0}{\rho_0}\frac{\overline{C}_{0.05}-\overline{C}_1}{1+\dfrac{\rho_s-\rho_0}{\rho_0}\overline{C}_m} \tag{41}$$

The Richardson number is important because it expresses the density gradient intensity in a sediment-laden flow. It vanishes in a neutral sediment-laden flow because $\overline{C}_{0.05} = \overline{C}_1$. One has $\overline{C}_{0.05} > \overline{C}_1$ in most sediment-laden flows, thus $R_i > 0$. The stronger the density gradient, the larger the Richardson number. In practice, the Richardson number can be determined from concentration measurements at $\overline{C}_{0.05}$ and $\overline{C}_1$.

Given $\overline{\rho} = \rho_0 + (\rho_s - \rho_0)\overline{C}$, the Richardson number can also be written as a function of mass density.

$$R_i = \frac{g\delta}{u_*^2}\frac{\overline{\rho}_{0.05}-\overline{\rho}_1}{\overline{\rho}_m} \tag{42}$$

which is the form used in Coleman (1981, 1986).

3.2 Vertical turbulent diffusion

Vertical turbulent diffusion is usually expressed by an eddy viscosity ε_m, i.e.

$$\varepsilon_m \sim \delta u_3' \tag{43}$$

in which δ is the boundary layer thickness or the flow depth h for a two-dimensional open-channel flow. Therefore, to study the effect of sediment suspension on turbulent diffusion, one has to study the effects on u_3'. Let i = j = 3 in (34),

$$\begin{aligned}\frac{\partial\overline{u_3'^2}}{\partial t} + \overline{u}_k\frac{\partial\overline{u_3'^2}}{\partial x_k} &= -2\overline{u_3'u_k'}\frac{\partial\overline{u}_3}{\partial x_k} \\ &+ 2\frac{\rho_s-\rho_0}{\rho_m}\overline{u_3'C'}g_3 - \frac{\partial\overline{u_3'u_3'u_k'}}{\partial x_k} - \frac{2}{\rho_m}\frac{\partial\overline{p'u_3'}}{\partial x_3} \\ &+ \frac{2}{\rho_m}\overline{p'\frac{\partial u_3'}{\partial x_3}} + \nu_m\frac{\partial^2\overline{u_3'^2}}{\partial x_3^2} - 2\nu_m\overline{\frac{\partial u_3'}{\partial x_k}\frac{\partial u_3'}{\partial x_k}}\end{aligned} \tag{44}$$

Considering (21) and (22) gives that

$$\overline{u}_k\frac{\partial\overline{u_3'^2}}{\partial x_k} = 0 \tag{45}$$

Considering $\overline{u}_3 = 0$ gives that

$$\overline{u_3'u_k'}\frac{\partial\overline{u}_3}{\partial x_k} = 0 \tag{46}$$

Considering (21) gives that

$$\frac{\partial\overline{u_3'u_3'u_k'}}{\partial x_k} = \frac{\partial\overline{u_3'^3}}{\partial x_3} \tag{47}$$

$$\frac{\partial\overline{u_3'^2}}{\partial x_k^2} = \frac{\partial\overline{u_3'^2}}{\partial x_3^2} \tag{48}$$

Then, (44) becomes

$$\begin{aligned}\frac{\partial\overline{u_3'^2}}{\partial t} &= \underbrace{2\frac{\rho_s-\rho_0}{\rho_m}\overline{u_3'C'}g_3}_{\text{sediment suspension}} \\ &\underbrace{-\frac{\partial\overline{u_3'^3}}{\partial x_3} - \frac{2}{\rho_m}\frac{\partial\overline{p'u_3'}}{\partial x_3} + \frac{2}{\rho_m}\overline{p'\frac{\partial u_3'}{\partial x_3}}}_{\text{turbulent transport}} \\ &+\underbrace{\nu_m\frac{\partial^2\overline{u_3'^2}}{\partial x_3^2}}_{\text{vis. transp.}} - \underbrace{2\nu_m\overline{\frac{\partial u_3'}{\partial x_k}\frac{\partial u_3'}{\partial x_k}}}_{\text{energy dissipation}}\end{aligned} \tag{49}$$

In the following, assume clear water flowing in a flume, sediment is then added to the flow to see how u_3' adjusts according to the right-hand side terms.

Like those in (35), the viscous and the turbulent transport terms are comparatively small, the effects of sediment suspension on u_3' are examined through sediment suspension and energy dissipation.

As those in (36), both the sediment suspension term and the energy dissipation term are negative. They will dampen the turbulence intensity u_3'. Consequently, sediment suspension reduces the eddy viscosity in the vertical direction and weakens turbulent mixing. Therefore, the effect of sediment suspension increases the velocity gradient in a sediment-laden flow.

With reference to the modified log-wake velocity profile (1a), one has

$$\frac{1}{u_*}\frac{d\overline{u}_1}{d\xi} = \frac{1-\xi}{\kappa\xi} + \frac{\pi\Omega}{2}\sin\pi\xi + \frac{1}{u_*}\frac{d\overline{u}_1}{d\xi}\bigg|_{\xi=1} \tag{50}$$

In terms of κ and Ω, to get the velocity gradient in a sediment-laden flow greater than that in a clear water flow (κ_0 and Ω_0), there are three possibilities:

1) $\kappa < \kappa_0$ and $\Omega > \Omega_0$;
2) $\kappa < \kappa_0$ and $\Omega = \Omega_0$; and
3) $\kappa = \kappa_0$ and $\Omega > \Omega_0$.

Theoretically, one would thus expect $\kappa \le \kappa_0$ and $\Omega \ge \Omega_0$ in sediment-laden flows.

4 Test of the modified log-wake law in sediment-laden flows

Experiments by Wang and Qian (1989) and Coleman (1986) will be used to test the modified log-wake law. In both data sets, the width-depth ratios are less than 5, i.e., the maximum velocity occurs below the water surface. Thus, the boundary layer thickness is defined as the distance from the bed to the maximum velocity position, where the velocity gradient is zero. Finally, the velocity defect form of the modified log-wake law (lb) reduces to

$$\frac{\overline{u}_{1\max} - \overline{u}_1}{u_*} = -\frac{1}{\kappa}\ln\xi + \Omega\cos^2\frac{\pi\xi}{2} - \frac{1-\xi}{\kappa} \qquad (51)$$

in which κ and Ω are parameters in sediment-laden flows.
The purpose of this test is to determine: 1) whether or not the modified log-wake law (51) is valid in sediment-laden flows; and 2) how the model parameters κ and Ω vary with sediment suspension.

4.1 Effect of average concentration

Wang and Qian (1989) carried out three types of experiments: clear water and pure salt water, neutral sediment-laden flows (salt water + plastic particles), and density sediment-laden flows (clear water + plastic particles, clear water + natural sands). The specific gravity of plastic particles $G = 1.05$, the particle concentration distribution in clear water is close to uniform, so the clear water + plastic particle experiments can be regarded as quasi-neutral sediment-laden flows, i.e., the effect of density gradient may be neglected. Details of the experiments for three sediment sizes are given in Guo's (1998) Appendix C. The flume perimeters were kept the same (smooth boundary, flow depth $h = 8, 9, 10$cm, flume width $b = 30$cm, and bed slope $S = 0.01$). For a given flow depth, the differences among individual runs are only attributed to different concentrations. The maximum volumetric concentration was 20%.
For a clear water flow, the shear velocity u_* is determined by Clauser's method, i.e., taking the log law in $\xi \le 0.2$. Since the shear velocity is a kinematic parameter, it is independent of sediment suspension. Therefore, the shear velocity in a sediment-laden flow is the same as its counterpart in a clear water flow.
The kinematic molecular viscosity ν_m due to volumetric sediment concentration is calculated by (Coleman, 1986)

$$\nu_m = \frac{\mu\left(1 + 2.5\overline{C} + 6.25\overline{C}^2 + 15.62\overline{C}^3\right)}{\rho_0 + (\rho_s - \rho_0)\overline{C}} \qquad (52)$$

in which μ is the dynamic viscosity of water; and $\overline{C}$ is the volumetric sediment concentration. This effective viscosity is used to determine the lower limit of the log law or the modified log-wake law.
A least-squares method is used to analyze the velocity profiles and determine the parameters κ and Ω. The details can be found in Guo (1998). A representative velocity profile, along with the modified log-wake law, for neutrally-buoyant sediment-laden flows is shown in Fig. 2. All other profiles can be found in Guo's (1998) Appendix C. Four velocity profiles for fine sand (median size $d_{50} = 0.268$mm) with different concentrations are plotted in Fig. 3. From the above two figures, one sees that: 1) the modified log-wake law is valid in sediment-laden flows even beyond the boundary layer thickness; 2) the sediment concentration increases the thickness of the viscous layer (the viscous sublayer + the buffer layer); 3) the position of the maximum velocity moves closer to the water surface as the concentration increases; and 4) the von Karman constant κ decreases as sediment concentration increases.
The calculated results for all neutral and quasi-neutral particle experiments are shown in Tables 1 and 2 on page 10, respectively. A plot between $\overline{C}$ and κ, including clear water and pure saltwater experiments, is shown in Fig. 4. The von Karman con-

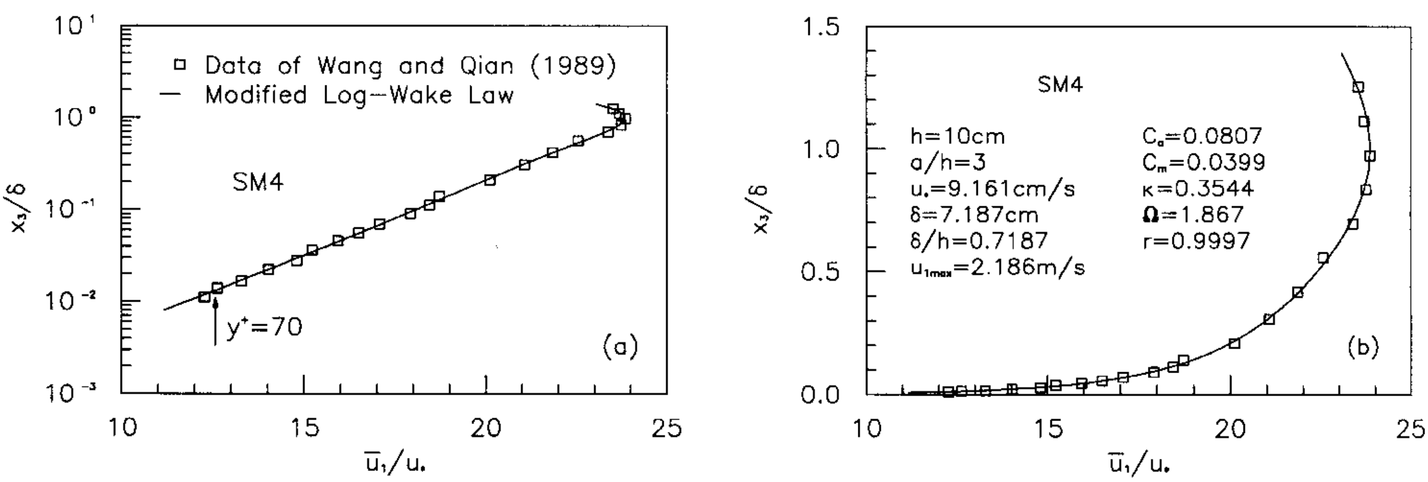

Fig. 2. A representative velocity profile of neutral sediment-laden flows in narrow channels [(a) semilog coordinates; (b) cartesian coordinates]

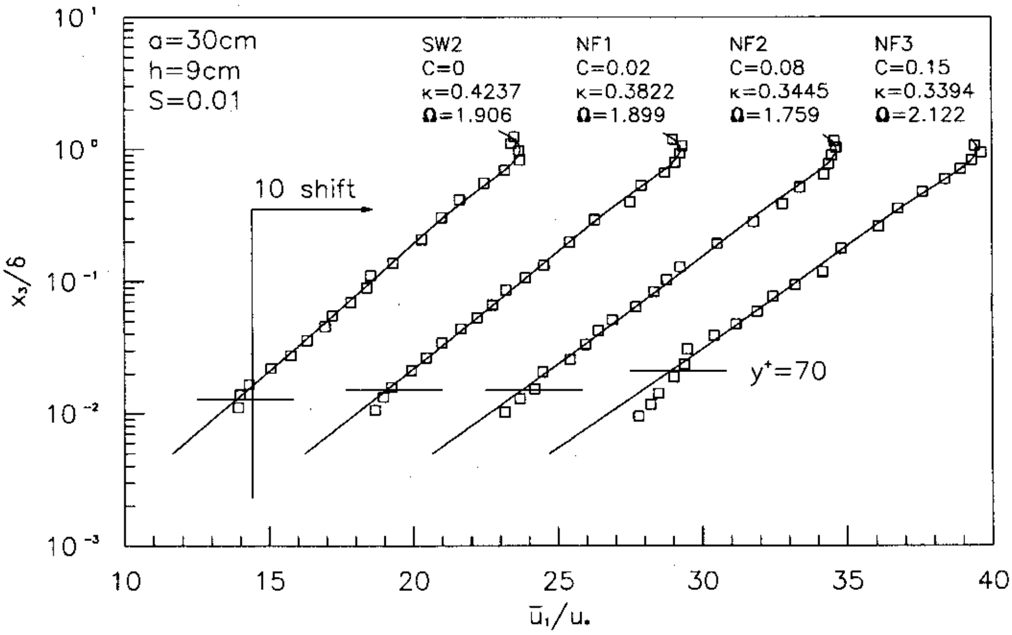

Fig. 3. Effect of average concentration on velocity profiles

stant κ decreases with sediment concentration $\overline{C}$. A linear relation between $\overline{C}$ and κ can be fitted to the data as

$$\frac{\kappa}{\kappa_0} = 1 - 0.92\overline{C} \tag{53}$$

in which the experimental constant κ_0 is determined to be 0.406, the value in clear water.

A plot of the volumetric sediment concentration $\overline{C}$ (the average values are taken for quasi-neutral particle experiments) versus the wake strength Ω is plotted in Fig. 5. It is shown that the wake strength Ω increases with sediment concentration (molecular viscosity). For Wang and Qian's (1989) experiments where $b/h \approx 3$, the following regression equation can be obtained:

$$\Omega = 1.65 + 3.71\overline{C} \tag{54}$$

When $\overline{C} = 0$, one has Ω = 1.65 which is compatible to the result in clear water (Guo, 1998).

In sediment-laden flows with density gradient ($R_i \neq 0$), $\overline{C}$ in (54) should be replaced by the water surface concentration $\overline{C}_1$. Fortunately, most sediment-laden flows in practice are density flows where $\overline{C}_1$ is usually very small. Therefore, the effect of sediment concentration on the wake strength Ω may be neglected in most practical situations. Moreover in wide channels where $b/h \geq 5$, the wake strength Ω should be neglected (Guo, 1998).

$$\Omega_0 = \begin{cases} -0.75b/h + 3.75 & \text{if } b/h < 5 \\ 0 & \text{if } b/h \geq 5 \end{cases} \tag{55}$$

4.2 *Effect of density gradient*

Coleman's (1986) data set contains all necessary information to test the modified log-wake law in sediment-laden flows. The flow

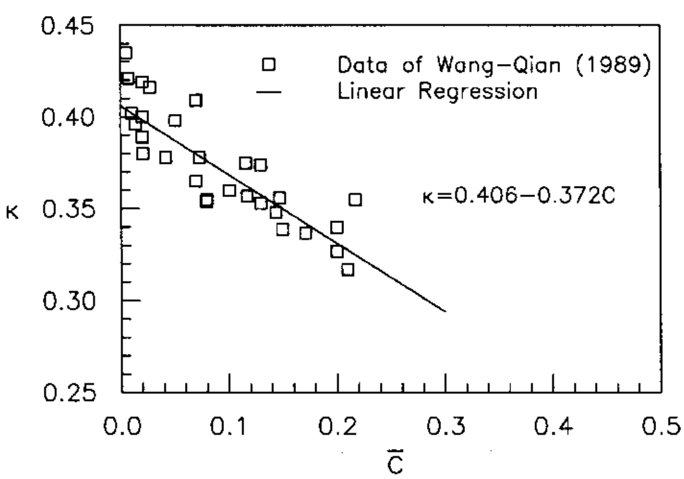

Fig. 4. Average concentration effect on the von Karman constant

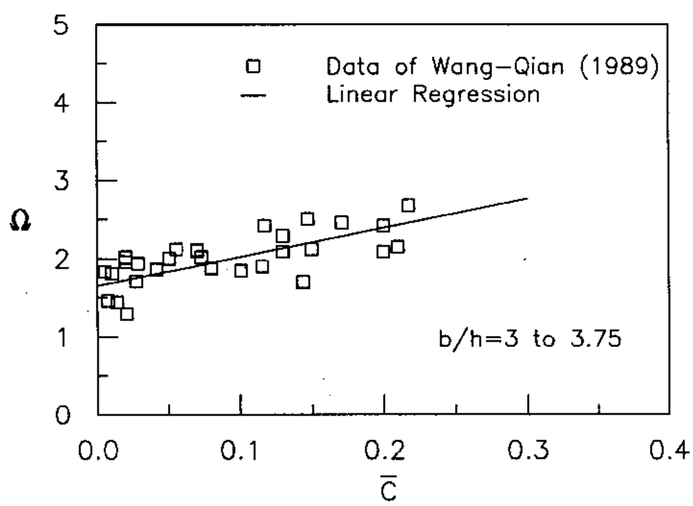

Fig. 5. Average concentration effect on the wake strength

Table 1: Calculated results of Wang-Qian's neutral particle experimental data

	h (cm)	b/h	u_* (cm/s)	δ (cm)	$\bar{u}_{1\max}$ (m/s)	$\bar{C}$	κ	Ω	r
NF1	9	3.33	8.81	6.77	2.14	0.02	0.389	1.96	0.999
NF2	9	3.33	8.81	7.00	2.17	0.08	0.355	1.88	0.999
NF3	9	3.33	8.81	7.61	2.16	0.15	0.339	2.12	0.998
NM1	10	3.00	9.16	6.79	2.08	0.02	0.419	2.01	0.999
NM2	10	3.00	9.16	7.04	2.12	0.07	0.365	2.10	0.999
NM3	10	3.00	9.16	7.65	2.13	0.13	0.374	2.29	0.998
NM4	10	3.00	9.16	7.94	2.12	0.20	0.327	2.09	0.999
NC1	10	3.00	9.16	6.33	2.10	0.02	0.400	2.02	0.999
NC2	10	3.00	9.16	7.44	2.10	0.07	0.409	2.11	0.997
NC3	10	3.00	9.16	6.94	2.11	0.13	0.353	2.09	0.998
NC4	10	3.00	9.16	7.16	2.12	0.20	0.340	2.42	0.998
For all runs, the slope $S = 0.01$									

Table 2: Calculated resul;ts of Wng-Qian's quasi-neutral particle experimental data

RUN	h (cm)	b/h	u_* (cm/s)	δ (cm)	$\bar{u}_{1\max}$ (m/s)	$\bar{C}_{0.05}$ (%)	$\bar{C}_m$ (%)	κ	Ω	r
SF1	10	3.00	9.16	6.66	2.12	0.53	0.41	0.435	1.83	0.999
SF2	10	3.00	9.16	6.6	2.09	1.39	1.02	0.396	1.44	0.999
SF3	10	3.00	9.16	6.57	2.07	2.86	2.28	0.465	1.94	0.998
SF4	10	3.00	9.16	7.44	2.08	5.55	4.6	0.449	2.12	0.999
SF5	8	3.75	8.4	5.29	1.96	10.1	9.06	0.360	1.85	0.998
SF6	9	3.49	8.65	9.01	2.16	14.7	13.3	0.356	2.51	0.998
SM1	10	3.00	9.16	7.06	2.11	0.74	0.42	0.421	1.46	0.999
SM2	10	3.00	9.16	6.55	2.15	2.74	1.20	0.416	1.71	0.999
SM3	10	3.00	9.16	7.07	2.16	5.07	2.38	0.398	2.00	1.000
SM4	10	3.00	9.16	7.19	2.19	7.99	3.99	0.354	1.87	1.000
SM5	10	3.00	9.16	8.83	2.2	11.6	6.23	0.375	1.90	0.999
SM6	10	3.00	9.16	9.4	2.21	14.4	7.54	0.348	1.70	0.995
SM7	10	3.00	9.16	8.68	2.23	21.7	13.7	0.355	2.68	1.000
SC1	10	3.00	9.16	6.43	2.12	1.04	0.43	0.402	1.81	0.999
SC2	10	3.00	9.16	6.79	2.10	2.06	0.85	0.38	1.29	1.000
SC3	10	3.00	9.16	6.64	2.11	4.18	1.98	0.378	1.86	1.000
SC4	10	3.00	9.16	7.19	2.13	7.31	3.40	0.378	2.02	0.999
SC5	10	3.00	9.16	7.35	2.15	11.7	6.51	0.357	2.42	0.999
SC6	10	3.00	9.16	7.54	2.17	17.1	9.37	0.337	2.46	0.999
SC7	10	3.00	9.16	7.73	2.16	21.0	12.3	0.317	2.15	0.999
For all runs, the slope $S = 0.01$										

conditions (smooth boundary, $h \approx 170$mm, $b = 356$mm, $S = 0.002$) were kept constant in all runs. The maximum local volumetric concentration is 2.3%. Hence, the differences of the velocity profiles among individual runs are attributed to the density gradient. The shear velocity u_*, like that for neutral particle-laden flows, is determined to be 0.041 m/s by Clauser's method.

Fig. 6 shows a representative velocity profile of Coleman's (1986) measurements compared with the modified log-wake law. Fig. 7 shows a comparison of 5 velocity profiles with different Richardson number R_i. The von Karman constant κ decreases with R_i while the variation of the wake strength Ω is not clear at this moment.

From (41), one sees that the estimation of R_i requires the values of $\bar{C}_{0.05}$, $\bar{C}_1$, and $\bar{C}_m$, all measured in Coleman's (1986) experiments. The calculated values of R_i for Coleman's (1986) experimental profiles are shown in Table 3 and plotted versus the von Karman constant κ in Fig. 8. It can be seen that the density gradient (the Richardson number R_i) has a significant effect on the von Karman constant κ. The stronger the density gradient, the smaller the von Karman constant. An exponential relation between κ and R_i has been fitted to the data as

$$\frac{\kappa}{\kappa_0} = \exp\left\{-0.065 R_i^{0.716}\right\} \tag{56}$$

in which κ_0 is the von Karman constant 0.406 for clear water flows. The general correlation coefficient is 0.89.

The relation between the wake strength Ω and the Richardson number R_i is plotted in Fig. 9, where the density gradient does not influence the wake strength Ω. The wake strength Ω may be related to large-scale turbulent mixing (secondary flows).

4.3 Combined effects of average concentration and density gradient

A composite expression for the effects of average concentration and Richardson number on the von Karman constant may be expressed as

$$\frac{\kappa}{\kappa_0} = \exp\left\{-0.065 R_i^{0.716}\right\} - 0.92\bar{C}_{0.05} \tag{57}$$

Table 3: Calculated results of Coleman's experimental data

RUN	δ (cm)	$\bar{u}_{1\max}$ (m/s)	$\bar{C}_{0.05}$ (10^{-3})	$\bar{C}_1$ (10^{-3})	$\bar{\bar{C}}_m$ (10^{-3})	R_i	κ	Ω	r
1	13.26	1.054	0.00	0.00	0.00	0.00	0.370	2.71	1.000
2	12.59	1.045	0.84	0.09	0.25	0.90	0.351	2.48	0.999
3	12.70	1.043	1.65	0.16	0.45	1.81	0.334	2.76	0.998
4	12.88	1.046	2.70	0.23	0.67	3.04	0.319	2.72	0.999
5	12.86	1.046	3.82	0.34	0.92	4.28	0.322	3.06	0.998
6	12.73	1.052	4.89	0.42	1.12	5.47	0.341	3.70	0.998
7	12.81	1.053	5.95	0.48	1.40	6.71	0.295	3.19	0.998
8	13.29	1.044	7.21	0.53	1.58	8.49	0.299	3.21	0.999
9	13.22	1.051	8.53	0.52	1.79	10.10	0.289	3.34	0.997
10	13.12	1.063	10.31	0.64	2.06	12.13	0.270	3.24	0.999
11	13.16	1.081	11.35	0.62	2.25	13.55	0.261	3.41	0.998
12	13.74	1.049	11.86	0.58	2.24	14.75	0.253	2.79	0.997
13	12.74	1.065	13.53	0.79	2.71	15.5	0.252	3.37	0.998
14	13.09	1.065	14.26	0.77	2.84	16.85	0.245	3.10	0.998
15	12.82	1.075	16.18	0.99	3.04	18.58	0.239	3.31	0.999
16	12.76	1.073	17.27	0.94	3.18	19.89	0.228	3.11	0.998
17	14.02	1.065	16.68	0.77	2.77	21.29	0.222	2.3	0.996
18	12.91	1.053	17.95	0.93	3.12	20.92	0.244	3.00	0.999
19	12.92	1.072	19.79	0.95	3.27	23.28	0.228	3.21	0.998
20	12.91	1.070	21.48	1.05	3.37	25.21	0.217	2.89	0.998
21	12.61	1.048	0.00	0.00	0.00	0.00	0.400	2.60	0.999
22	12.72	1.027	0.95	0.07	0.20	1.07	0.508	3.10	0.998
23	12.46	1.047	2.04	0.13	0.39	2.28	0.431	3.38	0.999
24	12.74	1.050	3.21	0.21	0.58	3.68	0.345	3.01	0.997
25	12.49	1.069	4.63	0.26	0.78	5.27	0.295	3.09	0.999
26	13.01	1.045	5.08	0.31	0.96	5.95	0.324	3.29	0.999
27	12.74	1.069	6.33	0.33	1.15	7.36	0.319	3.72	0.997
28	12.91	1.063	7.59	0.34	1.29	8.98	0.304	3.55	0.999
29	13.01	1.083	8.84	0.46	1.46	10.5	0.287	3.65	0.998
30	13.06	1.092	10.2	0.49	1.58	12.2	0.295	3.79	0.997
31	13.25	1.060	10.9	0.47	1.62	13.2	0.270	3.53	0.999
32	12.88	1.025	0.00	0.00	0.00	0.00	0.432	3.36	1.000
33	13.08	1.036	0.25	0.02	0.04	0.28	0.389	3.07	0.996
34	12.70	1.044	0.48	0.03	0.07	0.54	0.457	3.83	0.997
35	13.06	1.055	0.84	0.05	0.11	0.98	0.375	3.52	0.998
36	13.02	1.082	1.52	0.08	0.18	1.80	0.342	3.54	0.997
37	12.96	1.081	1.72	0.11	0.22	2.02	0.328	3.77	0.998
38	13.05	1.114	1.7	0.11	0.22	1.83	0.385	4.27	0.995
39	13.15	1.096	2.39	0.15	0.30	2.57	0.358	3.91	0.998
40	13.21	1.101	2.3	0.16	0.31	2.71	0.318	4.06	0.996
For all runs, $S \approx 0.002$, $h \approx 17$ cm, $b/h \approx 2$									

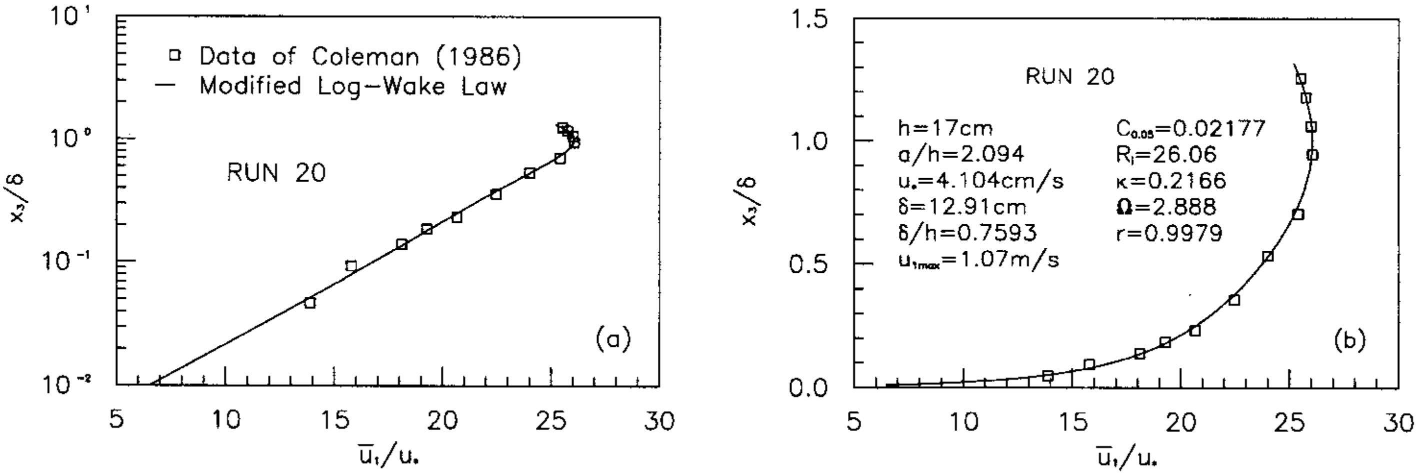

Fig. 6. A representative velocity profile for sediment-laden flows in narrow channels [(a) semilog coordinates; (b) cartesian coordinates]

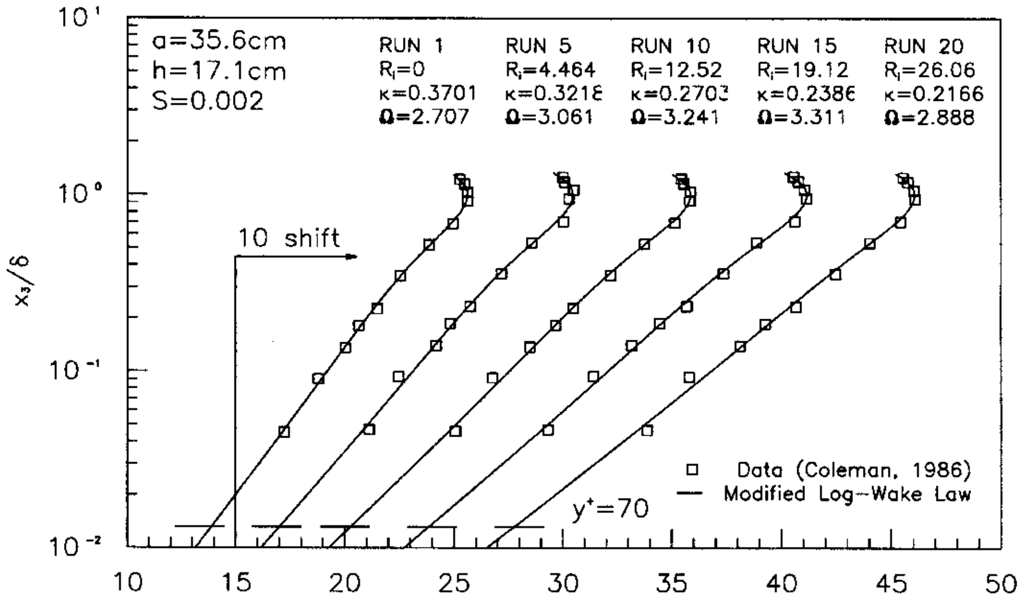

Fig. 7. Density gradient effect on velocity profiles

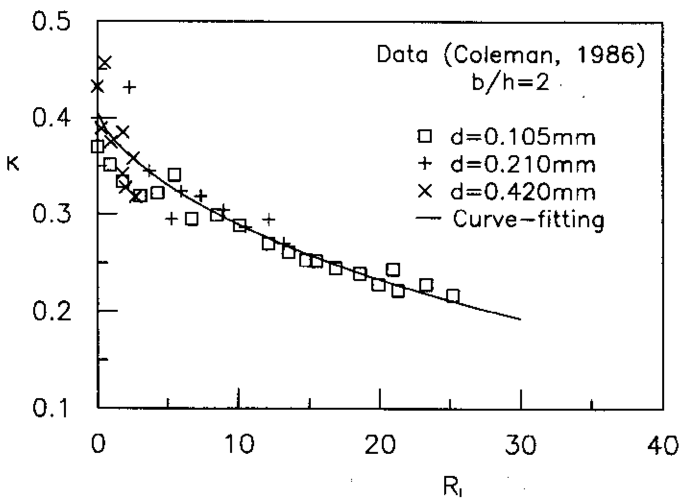

Fig. 8. Density gradient effect on the von Karman constant

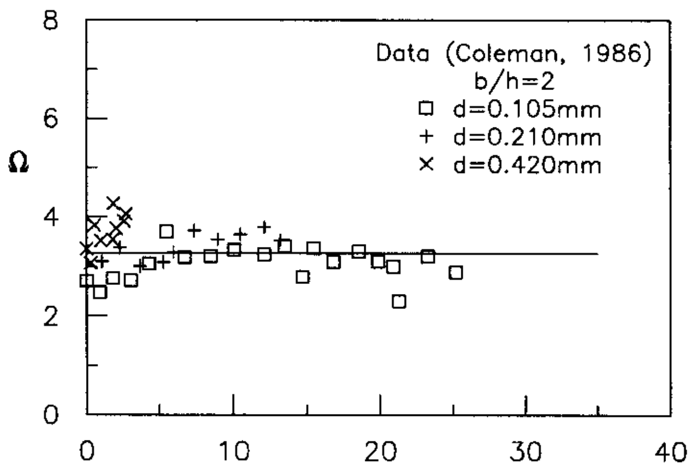

Fig. 9. Density gradient effect on the wake strength

5 Conclusions

The influences of sediment suspension on the governing equations, turbulence intensity and turbulent velocity profiles are investigated theoretically and experimentally. The theoretical analysis shows that: (1) the modified log-wake law developed in clear water flows is also valid for sediment-laden flows; (2) sediment suspension increases mean flow energy loss and decreases turbulence intensity in the vertical direction; (3) sediment suspension affects the modified log-wake law in two ways: average concentration and density gradient (Richardson number); and (4) in sediment-laden flows, the von Karman constant becomes smaller than that in clear water, and the wake strength becomes slightly larger. The comparison with Wang and Qian's (1989) and Coleman's (1986) experimental data confirms the analysis of the effects on velocity profiles. That is, (1) the modified log-wake law agrees well with flume experimental data; (2) both average concentration and density gradient (Richardson number) reduce the von Karman constant in sediment-laden flows; and (3) the average concentration slightly increases the wake strength while the density gradient has little effect on the wake strength, given a width-depth ratio. In practice, the effects of sediment concentration on the wake strength should be negligible.

6 Symbols

The following symbols are used in this paper:

a	relative distance from the bed
b	channel width
C	instantaneous volumetric sediment concentration
$\overline{C}$	local time-average volumetric sediment concentration
$\overline{C}_a$	a reference volumetric sediment concentration
$\overline{C}_1$	time-averaged sediment concentration at $\xi = 1$
C'	prime for sediment concentration fluctuation around the mean
D	molecular sediment diffusion coefficient in (4)
d	particle diameter
d_{50}	median particle diameter
f	a functional symbol
G	$(= \rho_s / \rho_0)$ specific gravity
g	gravitational acceleration
g_i	gravitational acceleration component along x_i
h	flow depth
i, j, k	subscripts in tensor variables
m	subscript for water-sediment mixture
p	pressure
$\overline{p}$	time-averaged pressure
p'	prime for pressure fluctuation around the mean
q	$(= \overline{u_i' u_i'})$ turbulent kinetic energy
Re	$(= Vh/\nu)$ Reynolds number
R_i	Richardson number in (40)
r	correlation coefficient
S	energy or bed slope
t	time
U	depth-averaged velocity
u_i	instantaneous velocity in direction i, and $i = 1, 2, 3$. "1" denotes the flow direction; "2" denotes the lateral direction; and "3" denotes the vertical direction
$\overline{u}$	overbar for time-averaged value
u_i'	prime for turbulent velocity fluctuation around the mean
u_*	shear velocity
$\overline{u}_{1\max}$	maximum velocity at the boundary layer, i.e. at $\xi = 1$
$\overline{u_i' u_j'}$	turbulence intensity
V	volume
x_i	coordinates
y^+	normalized distance by viscous length scale
δ	boundary layer thickness, which is defined as the distance from the bed to the position of maximum velocity
ε_m	eddy viscosity in sediment-laden flows
ε_s	sediment diffusivity coefficient
κ	von Karman constant in sediment-laden flows ($\kappa_0 = 0.406$ in clear water)
μ	dynamic viscosity for clear water
μ_m	dynamic viscosity for water-sediment mixture
ν	kinematic viscosity of clear water
ν_m	kinematic viscosity of water-sediment mixture
ρ	mass density of water-sediment mixture
ρ_0	mass density of water
ρ_m	time-space-averaged value of mass density of water-sediment mixture
ρ_s	mass density of sediment
ξ	$(= x_3 / \delta)$ distance from the bed normalized by the boundary layer thickness
Π	Coles wake strength
Ω	wake strength
Ω_0	wake strength for clear water
θ	angle between the channel bed and a horizontal datum

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