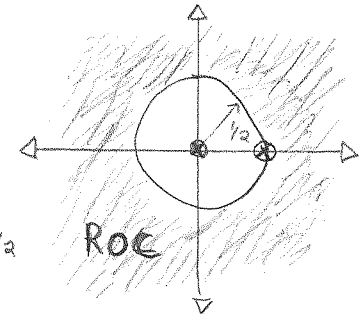
ECE 312 - HW #4 Solution Set

Problems Assigned: 10.22 (a, b, d), 10.25, 10.26, 10.30, 10.31

10.22) Determine the z-transform for each sequence. Sketch the pole-zero plot and indicate the ROC. Indicate whether or not the Fourier transform exists.

a) $\left(\frac{1}{2}\right)^n \{u[n+4] - u[n-5]\}$



$$x[n] = 2^4 \left(\frac{1}{2}\right)^{n+4} u[n+4] - \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{n-5} u[n-5]$$

$$X(z) = 16 z^4 \left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) - \frac{1}{32} z^{-5} \left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

$$X(z) = \left(\frac{2^9 z^9 - 1}{2^5 z^5}\right)\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

zeros: $z = 1/2$
poles: $z = 0, 1/2$

ROC includes unit circle $\Rightarrow \mathcal{F}\{x[n]\}$ exists

b) $n\left(\frac{1}{2}\right)^{|n|}$

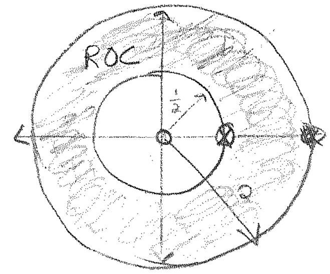
$$x[n] = n\left(\frac{1}{2}\right)^{-n} u[-n-1] + n\left(\frac{1}{2}\right)^n u[n]$$

let $x_1[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1] + \left(\frac{1}{2}\right)^n u[n]$

$$\Rightarrow X_1(z) = \frac{-1}{1-2z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$$

now see $n\,x[n]$ is differentiation in z-domain: $-z\frac{dX(z)}{dz}$

$$X(z) = -z\frac{dX_1(z)}{dz} = \left(\frac{-2z^{-1}}{(1-2z^{-1})^2} + \frac{1/2\, z^{-1}}{(1-\frac{1}{2}z^{-1})^2}\right)$$



$$X(z) = \frac{-2z^{-1}}{(1-2z^{-1})^2} + \frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$$

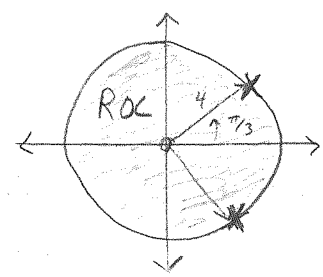
zeros: $z = 0, z = 1/2, z = 2$
poles: $z = 1/2, 2$

ROC does include unit circle so $\mathcal{F}\{x[n]\}$ exists

d) $4^n \cos\left[\frac{2\pi}{6}n + \frac{\pi}{4}\right] u[-n-1]$

$$x[n] = 4^n \left(\frac{1}{2}\left(e^{j\left(\frac{2\pi n}{6}+\frac{\pi}{4}\right)} + e^{-j\left(\frac{2\pi n}{6}+\frac{\pi}{4}\right)}\right)\right) u[-n-1] =$$

$$\frac{1}{2}e^{j\frac{\pi}{4}} 4^n e^{j\frac{2\pi n}{6}} u[-n-1] + \frac{1}{2}e^{-j\frac{\pi}{4}} 4^n e^{-j\frac{2\pi n}{6}} u[-n-1]$$



$$X(z) = \frac{1}{2}e^{j\pi/4}\left(\frac{-1}{1-4e^{j\pi/3}z^{-1}}\right) + \frac{1}{2}e^{-j\pi/4}\left(\frac{-1}{1-4e^{-j\pi/3}z^{-1}}\right)$$

poles at $z = 4e^{\pm j\pi/3}$, zero at $z = 0$

ROC includes unit circle so $\mathcal{F}\{x[n]\}$ exists

10.25) Consider a right-sided sequence $x[n]$ w/ z-transform

$$X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}.$$

a) Carry out PFE as a ratio of polynomials in z^{-1}. Find $x[n]$.

$$X(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-z^{-1}} = \frac{-1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-z^{-1}}$$

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] + 2u[n]$$

b) Rewrite $X(z)$ as a ratio of polynomials in z and repeat (a).

$$X(z) = \frac{z}{z-1/2}\left(\frac{z}{z-1}\right) = \frac{Az}{z-1/2} + \frac{Bz}{z-1} = \frac{-1z}{z-1/2} + \frac{2z}{z-1}$$

$X(z) = \frac{-1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-z^{-1}}$

$x[n] = -(\frac{1}{2})^n u[n] + 2u[n]$

same as part (a) :)

10.26) Consider a left-sided sequence $x[n]$ with z-transform

$$X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}$$

a) Write $X(z)$ as a ratio of polynomials in z rather than z^{-1}.

$$X(z) = \frac{z^2}{(z-\frac{1}{2})(z-1)}$$

b) Using PFE, express $X(z)$ as a sum of terms where each term represents a system pole.

$$X(z) = \frac{Az}{z-1/2} + \frac{Bz}{z-1} = \frac{-z}{z-1/2} + \frac{2z}{z-1}$$

c) Determine $x[n]$.

$$X(z) = \frac{-1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-z^{-1}} \iff x[n] = (\tfrac{1}{2})^n u[-n-1] - 2u[-n-1]$$

10.30) Consider a signal $y[n]$ related to two signals $x_1[n]$ and $x_2[n]$ by:

$$y[n] = x_1[n+3] * x_2[-n+1]$$

where $x_1[n] = (\frac{1}{2})^n u[n]$ and $x_2[n] = (\frac{1}{3})^n u[n]$. Determine $Y(z)$.

$$Y(z) = z^3 X_1(z)\, z^{-1} X_2(z^{-1}) =$$

$$z^3\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) z^{-1}\left(\frac{1}{1-\frac{1}{3}z}\right) =$$

$$\frac{6z}{-2+7z^{-1}-3z^{-2}}$$

10.31) Given the following five facts about $x[n]$ w/ z-transform $X(z)$

i) $x[n]$ is real and right-sided

ii) $X(z)$ has exactly two poles

iii) $X(z)$ has two zeros at the origin

iv) $X(z)$ has a pole at $z = \frac{1}{2}e^{j\pi/3}$

v) $X(1) = 8/3$

Determine $X(z)$ and specify its ROC.

pole at $z = \frac{1}{2}e^{j\pi/3} \Rightarrow$ cos-esque term so want pole at $z = \frac{1}{2}e^{-j\pi/3}$

- also ensures real $x[n]$

$$X(z) = \frac{Az^2}{(z-\frac{1}{2}e^{j\pi/3})(z-\frac{1}{2}e^{-j\pi/3})}$$

$$X(1) = \frac{A}{(3/4)} = 8/3 \Rightarrow A = 2$$

$$\therefore X(z) = \frac{2z^2}{(z-\frac{1}{2}e^{j\pi/3})(z-\frac{1}{2}e^{-j\pi/3})}, \quad \text{ROC } |z| > 1/2$$