

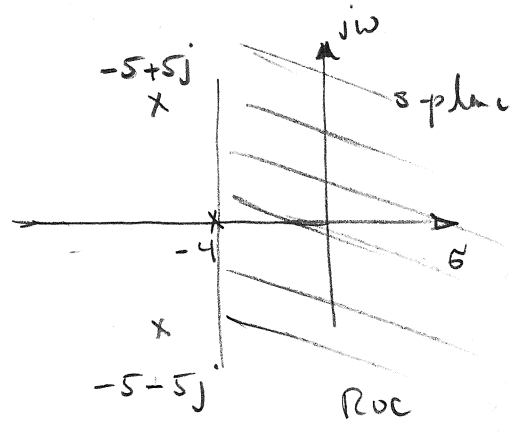
# ECE312: Homework 3 solutions

9.21

$$(b) \quad x(t) = e^{-4t} u(t) + e^{-5t} \sin(5t) u(t)$$

$$X(s) = \frac{1}{s+4} + \frac{5}{(s+5)^2 + 5^2}$$

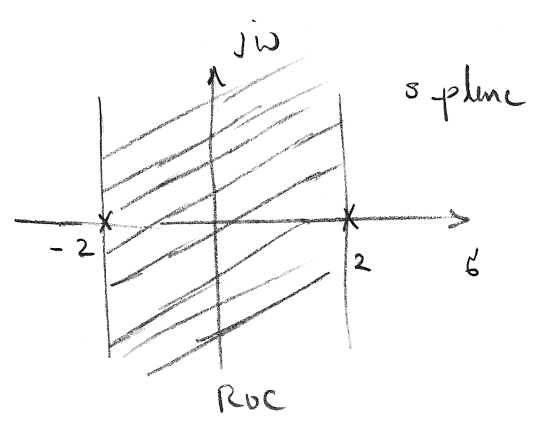
, Roc:  $\text{Re}\{s\} > -4$



$$(d) \quad x(t) = t e^{-2|t|}$$
$$= t e^{-2t} u(t) + t e^{2t} u(-t)$$

$$X(s) = \frac{1}{(s+2)^2} - \frac{1}{(s-2)^2}$$

,  $-2 < \text{Re}\{s\} < 2$



9.22

$$(a) X(s) = \frac{1}{s^2 + 9}$$

$$, \operatorname{Re}\{s\} > 0$$

$$= \frac{-1/6j}{s + 3j} + \frac{1/6j}{s - 3j}$$

$$, \operatorname{Re}\{s\} > 0$$

$$x(t) = \frac{-1}{6j} e^{-3jt} u(t) + \frac{1}{6j} e^{3jt} u(t)$$

$$= \frac{1}{3} \sin(3t) u(t)$$

$$(d) X(s) = \frac{s+2}{s^2 + 7s + 12}$$

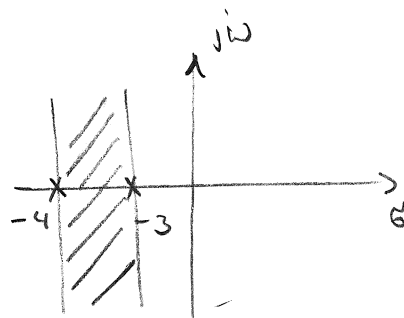
$$, -4 < \operatorname{Re}\{s\} < -3$$

$$= \frac{-1}{s+3} + \frac{2}{s+4}$$

$$\mathcal{L}^{-1} \downarrow \mathcal{L} s$$

$$\mathcal{R} s \downarrow \mathcal{L}^{-1}$$

$$x(t) = e^{-3t} u(-t) + 2e^{-4t} u(t)$$



$$(g) X(s) = \frac{s^2 - s + 1}{(s+1)^2}$$

$$= 1 + \frac{-3s}{(s+1)^2}$$

$$, \operatorname{Re}\{s\} > -1$$

$$= 1 + \frac{-3}{s+1} + \frac{3}{(s+1)^2}$$

$$, \operatorname{Re}\{s\} > -1$$

$$x(t) = \delta(t) - 3e^{-t} u(t) + 3te^{-t} u(t)$$

9.26

$$y(t) = x_1(t-2) * x_2(-t+3)$$

$$x_1(t) = e^{-2t} u(t) \quad \text{and} \quad x_2(t) = e^{-3t} u(t)$$

$$X_1(s) = \frac{1}{s+2}, \quad \text{Re}\{s\} > -2$$

$$X_2(s) = \frac{1}{s+3}, \quad \text{Re}\{s\} > -3$$

$$\mathcal{L}\{x_2(-t+3)\} = \mathcal{L}\{x_2(at+3)\} \quad \text{for } a=-1$$

Using scaling & time shift property, we have

$$\mathcal{L}\{x_2(a(t-3))\} = e^{-3s} X_2(-s), \quad \text{Re}\{s\} < 3$$

$$\mathcal{L}\{x_1(t-2)\} = e^{-2s} X_1(s)$$

Therefore

$$Y(s) = e^{-2s} X_1(s) e^{-3s} X_2(-s)$$

$$= e^{-5s} X_1(s) X_2(-s)$$

$$= \frac{e^{-5s}}{(s+2)(-s+3)}, \quad -2 < \text{Re}\{s\} < 3$$

9.30

$$x_1(t) = u(t) \quad \longrightarrow \quad \boxed{h(t)} \quad \longrightarrow \quad y_1(t) = (1 - e^{-t} - t e^{-t}) u(t)$$

$$Y_1(s) = H(s) X_1(s)$$

$$Y_1(s) = \frac{1}{s} - \frac{1}{s+1} + \frac{1}{(s+1)^2}, \quad \text{Re}\{s\} > 0$$

$$X_1(s) = \frac{1}{s}, \quad \text{Re}\{s\} > 0$$

$$H(s) = \frac{Y_1(s)}{X_1(s)} = 2 - \frac{s+1/2}{(s+1)^2}$$

$$y_1(t) = (2 - 3e^{-t} + e^{-3t}) u(t)$$

$$\updownarrow \mathcal{L}$$

$$Y(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+3}, \quad \text{Re}\{s\} > 0$$

$$= \frac{6}{s(s+1)(s+3)}$$

$$X(s) = \frac{Y(s)}{H(s)} = \frac{3(s+1)}{s(s+1/2)(s+3)}, \quad \text{Re}\{s\} > 0$$

$$= \frac{2}{s} + \frac{-6/5}{s+1/2} + \frac{-4/5}{s+3}$$

$$x(t) = \left( 2 - \frac{6}{5} e^{-1/2 t} - \frac{4}{5} e^{-3t} \right) u(t)$$


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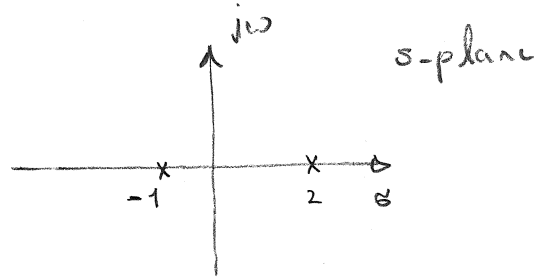
9.31

$$\frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

$$(a) \quad s^2 Y(s) - sY(s) - 2Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

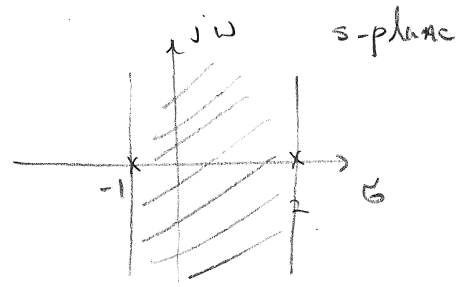
$$s^2 - s - 2 = 0 \quad \text{Poles} \quad \begin{matrix} s = -1 \\ s = 2 \end{matrix}$$



$$(b) \quad H(s) = \frac{-1/3}{s+1} + \frac{1/3}{s-2}$$

1. stable inversion: ROC must include  $j\omega$ -axis

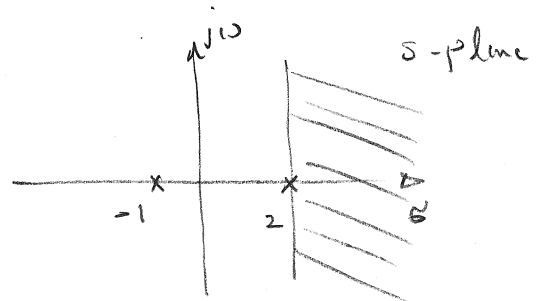
$$h(t) = -\frac{1}{3} e^{-t} u(t) - \frac{1}{3} e^{2t} u(-t)$$



$$\text{ROC: } -1 < \text{Re}\{s\} < 2$$

2. Causal inversion: ROC must be to the right of right-most pole

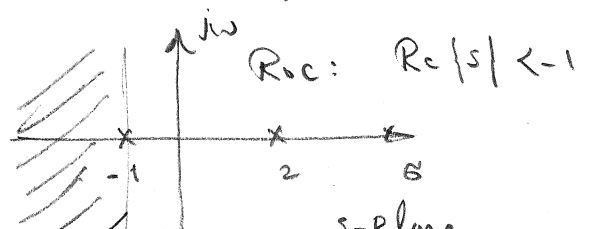
$$h(t) = -\frac{1}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$



$$\text{ROC: } \text{Re}\{s\} > 2$$

3. Neither causal, nor stable

$$h(t) = \frac{1}{3} e^{-t} u(-t) - \frac{1}{3} e^{2t} u(-t)$$



9.36

System S:

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}$$

System  $S_1$ :

$$H(s) = \frac{1}{s^2 + 3s + 2}$$

$$(a) \quad \frac{Y(s)}{Y_1(s)} = 2s^2 + 4s - 6$$

$$y(t) = 2 \frac{d^2 y_1(t)}{dt^2} + 4 \frac{dy_1(t)}{dt} - 6y_1(t)$$

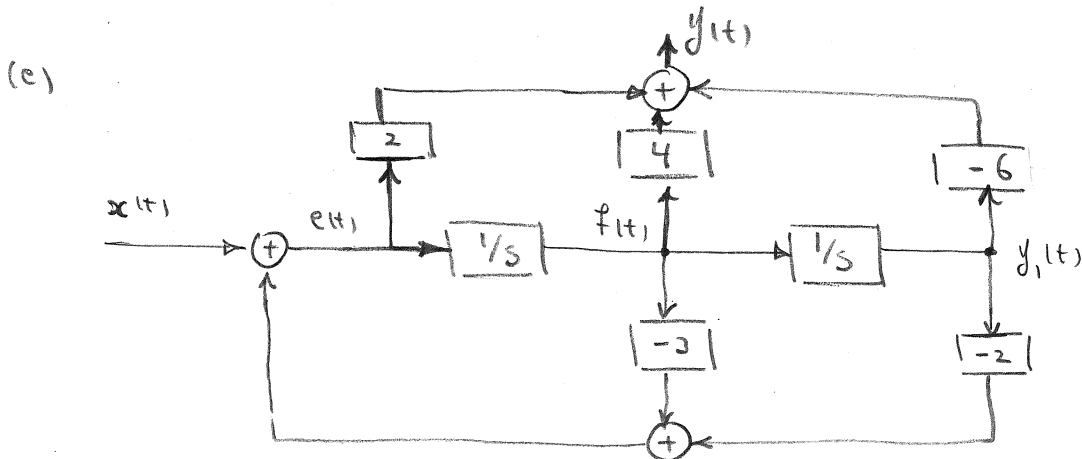
$$(b) \quad \frac{Y_1(s)}{F(s)} = \frac{1}{s}$$

$$\frac{dy_1(t)}{dt} = f(t)$$

$$(c) \quad \frac{Y_1(s)}{E(s)} = \frac{1}{s^2}$$

$$\frac{d^2 y_1(t)}{dt^2} = e(t)$$

$$(d) \quad y(t) = 2e(t) + 4f(t) - 6y_1(t)$$



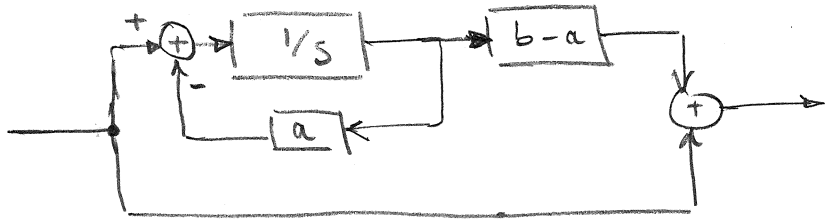
9.36 - Cont.

(f)

$$H(s) = \left( \frac{2(s-1)}{s+2} \right) \left( \frac{s+3}{s+1} \right)$$

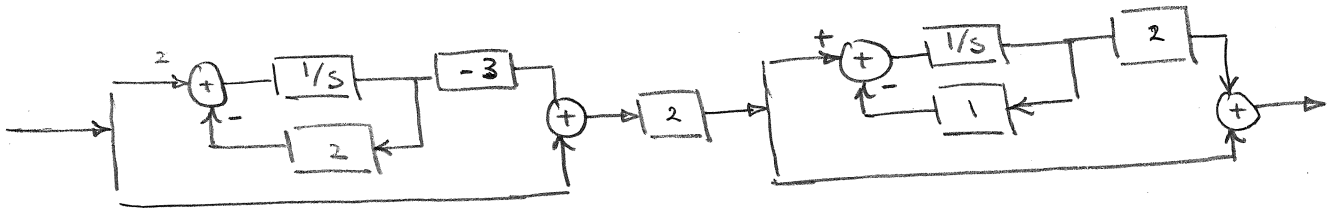
Block diagram for

$$G_1(s) = \frac{s+3}{s+1}$$

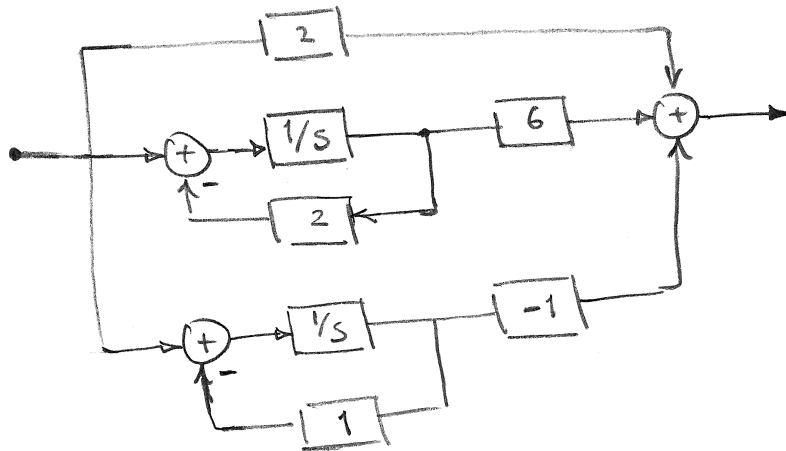


Block diagram for  $H(s) = G_1(s) G_2(s)$ , where

$$G_1(s) = 2 \frac{s-1}{s+2} \quad \text{and} \quad G_2(s) = \frac{s+3}{s+1}$$



(g)



ECE 312: Homework 3 solutions

9.46

Solved in class on Thursday Apr. 3, 2014.