ECE 312 - Homework #1: 5.21(b, g, k), 5.22(b, e, g), 5.27(a-ii, a-iv),
due Feb. 6, 2014 5.29(b) part i, 5.30(b-i, b-iii), 5.33(a, b-ii)

5.21) Compute the Fourier Transform of each of the following signals.

b) $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{-n} u[-n-1] e^{-j\omega n} =$$

$$\sum_{n=-\infty}^{-1} \left(\frac{1}{2} e^{j\omega}\right)^{-n} =$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2} e^{j\omega}\right)^{n} =$$

$$\left(\sum_{n=0}^{\infty} \left(\frac{1}{2} e^{j\omega}\right)^{n}\right) - 1 =$$

$$\frac{1}{1-\frac{1}{2}e^{j\omega}} - 1 =$$

$$e^{j\omega}/(2-e^{j\omega})$$

$$X(e^{j\omega}) = \frac{e^{j\omega}}{2-e^{j\omega}}$$

g) $x[n] = \sin\left(\frac{\pi}{2} n\right) + \cos(n)$

$$X(e^{j\omega}) = \frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \tfrac{\pi}{2} - 2\pi k) - \delta(\omega + \tfrac{\pi}{2} - 2\pi k)\} +$$

$$\pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - 1 - 2\pi k) + \delta(\omega + 1 - 2\pi k)\}$$

- transform directly from table, alternatively use periodicity:

$$X(e^{j\omega}) = \frac{\pi}{j}\{\delta(\omega - \tfrac{\pi}{2}) - \delta(\omega + \tfrac{\pi}{2})\} + \pi\{\delta(\omega - 1) + \delta(\omega + 1)\}, \ |\omega| \le \pi$$

k) $x[n] = \left(\frac{\sin(\pi n/5)}{\pi n}\right) \cos\left(\frac{7\pi}{2} n\right)$

- let $x_1[n] = \left(\frac{\sin(\pi n/5)}{\pi n}\right)$ and $x_2[n] = \cos\left(\frac{7\pi}{2} n\right)$, then note

20 pts. possible

$$X_1(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \frac{\pi}{5} \\ 0, & \frac{\pi}{5} < |\omega| \le \pi \end{cases}$$

$$X_2(e^{j\omega}) = \pi\{\delta(\omega - \tfrac{7\pi}{2}) + \delta(\omega + \tfrac{7\pi}{2})\} = \pi\{\delta(\omega + \tfrac{\pi}{2}) + \delta(\omega - \tfrac{\pi}{2})\}$$

where both signals are 2π periodic

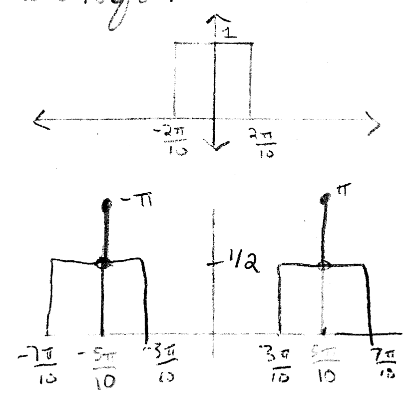
rely on convolution relationship and perform transform

$$X(e^{j\omega}) = X_1(e^{j\omega}) * X_2(e^{j\omega}) =$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) d\theta \Rightarrow$$

go to figures !!

$$X(e^{j\omega}) = \begin{cases} 1/2 & \frac{3\pi}{10} \le |\omega| \le \frac{7\pi}{10} \\ 0 & \text{else} \end{cases}$$



5.26) Determine the discrete time signal corresponding to each Fourier transform.

b) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

$x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10]$

20pts (c) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi}{2}k)$

note signal is periodic: $\omega_0 = \frac{\pi}{2} \Rightarrow T = 4$

$X(e^{j\omega}) = \sum_{k=0}^{3} (-1)^k \delta(\omega - \frac{\pi}{2}k) \Rightarrow$

$x[n] = \frac{1}{2\pi} \sum_{k=0}^{3} (-1)^k e^{jk\pi n/2} =$

$\frac{1}{2\pi}(1 - e^{j\pi n/2} + e^{j\pi n} - e^{j3\pi n/2})$

$x[n] = \frac{1}{2\pi}(1 - e^{j\pi n/2} + e^{j\pi n} - e^{j3\pi n/2})$

g) $X(e^{j\omega}) = (1 - \frac{1}{3}e^{-j\omega}) / (1 - \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega})$

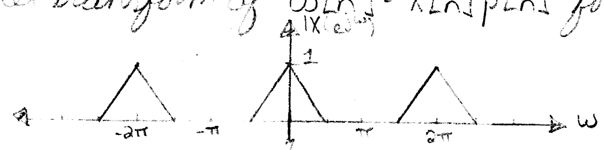
$X(e^{j\omega}) = \frac{1 - \frac{1}{3}e^{-j\omega}}{(1 - \frac{1}{2}e^{-j\omega})(1 + \frac{1}{4}e^{-j\omega})} = \frac{2/9}{(1 - \frac{1}{2}e^{-j\omega})} + \frac{7/9}{(1 + \frac{1}{4}e^{-j\omega})}$

used partial fraction expansion so matches Table 5.2

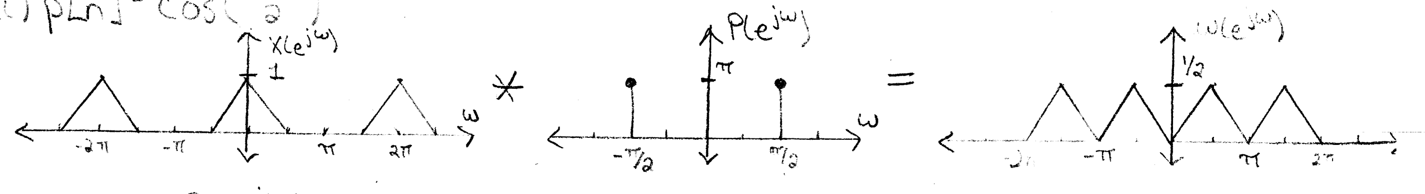
$x[n] = \frac{2}{9}\left(\frac{n!}{n!0!}(\frac{1}{2})^n u[n]\right) + \frac{7}{9}(-\frac{1}{4})^n u[n]$

$x[n] = \frac{1}{9}[2(\frac{1}{2})^n + 7(-\frac{1}{4})^n] u[n]$

5.27) Let $x[n]$ be a discrete-time signal w/ Fourier transform $X(e^{j\omega})$ shown. Sketch the Fourier transform of $w[n] = x[n]p[n]$ for the following signals, $p[n]$.

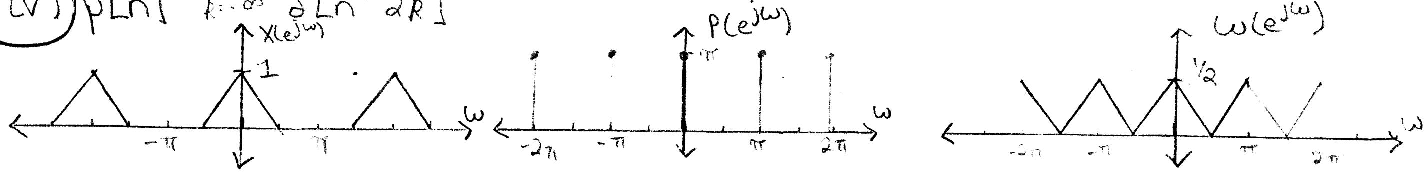


ii) $p[n] = \cos(\frac{\pi n}{2})$



$P(e^{j\omega}) = \pi\{\delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2})\}, \ |\omega| < \pi$

20pts (iv) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n - 2k]$



$P(e^{j\omega}) = \frac{2\pi}{2} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{2})$

5.29) Suppose $h[n] = [(\frac{1}{2})^n \cos(\frac{\pi n}{2})]u[n]$. Use Fourier transforms to determine the response to each of the following inputs:

i) $x[n] = (\frac{1}{2})^n u[n]$

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$, $y[n] = x[n] * h[n]$ or $Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$

$h[n] = [(\frac{1}{2})^n \cos(\frac{\pi n}{2})] u[n] =$ (use Eulers)

$[(\frac{1}{2})^n (\frac{1}{2}\{e^{j\pi n/2} + e^{-j\pi n/2}\})] u[n] =$

$\frac{1}{2}(\frac{1}{2}e^{j\pi/2})^n u[n] + \frac{1}{2}(\frac{1}{2}e^{-j\pi/2})^n u[n]$

$\therefore H(e^{j\omega}) = \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}e^{j\pi/2}e^{-j\omega}} + \frac{1}{1-\frac{1}{2}e^{-j\pi/2}e^{-j\omega}}\right]$, $X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega}) =$

$\left(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right)\left(\frac{1}{2-e^{j\pi/2}e^{-j\omega}} + \frac{1}{2-e^{-j\pi/2}e^{-j\omega}}\right) =$

$\left[\frac{1}{(1-\frac{1}{2}e^{-j\omega})(2-e^{j\pi/2}e^{-j\omega})} + \frac{1}{(1-\frac{1}{2}e^{-j\omega})(2-e^{-j\pi/2}e^{-j\omega})}\right] =$

$\left[\frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-je^{-j\omega})(1+je^{-j\omega})}\right] =$

$\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right) + \frac{j}{j-1}\left(\frac{1}{1-je^{-j\omega}}\right) + \frac{j}{j+1}\left(\frac{1}{1+je^{-j\omega}}\right)$

$\therefore y[n] = \{(\frac{1}{2})^{n+1} + \frac{j}{j-1}(j)^{n+1} + \frac{j}{j+1}(-j)^n\} u[n]$

$y[n] = \{(1/2)^{n+1} + \frac{j}{j-1}(j)^{n+1} + \frac{j}{j+1}(-j)^n\} u[n]$

ii) $x[n] = \cos(\frac{\pi n}{2})$

$X(e^{j\omega}) = \pi\{\delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2})\}$, $|\omega| \leq \pi$

$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega}) =$

$\pi\{\delta(\omega-\frac{\pi}{2}) + \delta(\omega+\frac{\pi}{2})\}\left(\frac{1}{(1-\frac{1}{2}e^{-j\pi/2}e^{-j\omega})(1-\frac{1}{2}e^{j\pi/2}e^{-j\omega})}\right) =$

$\pi\{\delta(\omega-\frac{\pi}{2}) + \delta(\omega+\frac{\pi}{2})\}\left[\frac{4}{4+e^{-2j\omega}}\right] =$ ω $\pi/2$ else fcn 0

$\pi\{\delta(\omega-\frac{\pi}{2}) + \delta(\omega+\frac{\pi}{2})\}\left(\frac{4}{4+e^{-j\pi}}\right) =$

$\frac{4\pi}{3}\{\delta(\omega-\frac{\pi}{2}) + \delta(\omega+\frac{\pi}{2})\}$

$y[n] = 4/3 \cos(\frac{\pi n}{2})$

$y[n] = \frac{4}{3}\cos(\frac{\pi n}{2})$

5.30) b) Consider the signal $x[n] = \sin(\frac{\pi n}{8}) - 2\cos(\frac{\pi n}{4})$. Suppose $x[n]$ is the input to LTI systems w/ the following impulse responses. Determine the output

i) $h[n] = \sin(\frac{\pi n}{6})/(\pi n)$

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \pi/6 \\ 0 & \text{else} \end{cases}$$

$$X(e^{j\omega}) = \frac{\pi}{j}\{\delta(\omega - \frac{\pi}{8}) - \delta(\omega + \frac{\pi}{8})\} - 2\pi\{\delta(\omega - \frac{\pi}{4}) + \delta(\omega + \frac{\pi}{4})\}, \ |\omega| \le \pi$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{\pi}{j}\{\delta(\omega - \frac{\pi}{8}) - \delta(\omega + \frac{\pi}{8})\}$$

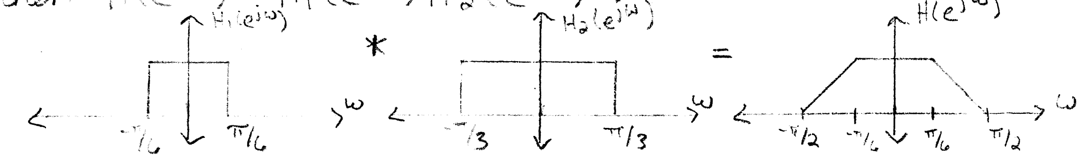
$$y[n] = \sin(\pi n/8)$$

$$y[n] = \sin(\frac{\pi n}{8})$$

iii) $h[n] = \sin(\frac{\pi n}{6})\sin(\frac{\pi n}{3})/(\pi^2 n^2)$

let $h_1[n] = \sin(\frac{\pi n}{6})/(\pi n)$ $\quad h_2[n] = \sin(\frac{\pi n}{3})/(\pi n)$

then $H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})$



$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{\pi}{j6}\{\delta(\omega - \frac{\pi}{8}) - \delta(\omega + \frac{\pi}{8})\} - \frac{\pi}{2}\{\delta(\omega - \frac{\pi}{4}) + \delta(\omega + \frac{\pi}{4})\}$$

$$y[n] = \frac{1}{6}\sin(\frac{\pi n}{8}) - \frac{1}{4}\cos(\frac{\pi n}{4})$$

$$y[n] = \frac{1}{6}\sin(\frac{\pi n}{8}) - \frac{1}{4}\cos(\frac{\pi n}{4})$$

5.32 Consider a causal LTI system described by:

20 pts $\quad y[n] + \frac{1}{2}y[n-1] = x[n]$

a) Determine the frequency response of the system, $H(e^{j\omega})$.

$$Y(e^{j\omega}) + \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega})$$

$$H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

b-ii) What is the response of the system to $x[n] = (-\frac{1}{2})^n u[n]$?

$$X(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2}$$

$$y[n] = (n+1)(-\frac{1}{2})^n u[n]$$

$$y[n] = (n+1)(-\frac{1}{2})^n u[n]$$