Error Probability Bounds for Binary Relay Trees with Unreliable Communication Links

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***Abstract*—We study the decentralized detection problem in the context of balanced binary relay trees. We assume that the communication links in the tree network fail with certain probabilities. Not surprisingly, the step-wise reduction of the total detection error probability is slower than the case where the network has no communication link failures. We show that, under the assumption of identical communication link failure probability in the tree, the exponent of the total error probability at the fusion center is $o(\sqrt{N})$ in the asymptotic regime. In addition, if the given communication link failure probabilities decrease to 0 as communications get closer to the fusion center, then the decay exponent of the total error probability is $\Theta(\sqrt{N})$, provided that the decay of the failure probabilities is sufficiently fast.**

I. Introduction

Consider the *decentralized detection* problem: each sensor makes a measurement and sends a summary of this measurement to a fusion center, which makes the overall detection decision based on the summarized messages. This problem has been widely studied in the context of various architectures, including the parallel configuration [1]–[15], [33], tandem networks [16]–[20], [33], and bounded-height trees [21]–[29], [33]. In this paper, we consider the decentralized detection problem in the context of balanced binary relay trees [30] (height is unbounded). The objective of the network is to make a decision between two hypotheses H_0 and H_1. As shown in Fig. 1, leaf nodes are sensors undertaking initial and independent detections of the same event in a scene. These measurements are summarized into binary messages and then these messages are forwarded to the nodes at the next level. Each non-leaf node—except the root, which is the fusion center—is a relay node, which fuses binary messages it receives (if any, and at most two) into one new binary message and forwards the new binary message to its parent node. This process takes place at each intermediate node culminating in the fusion center, at which the final decision is made based on the information received.

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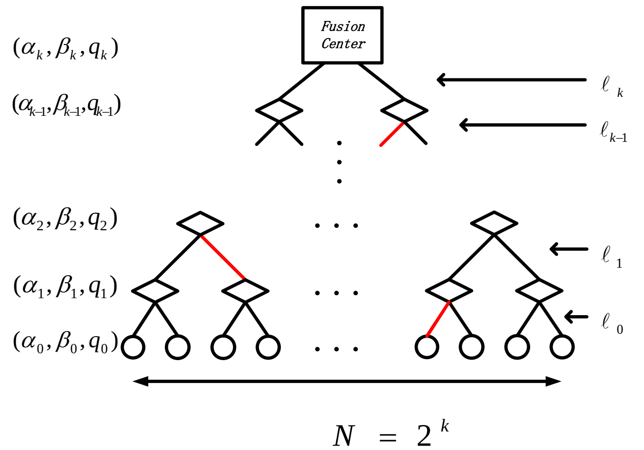

Fig. 1. A balanced binary relay tree with height k. Circles represent sensors making measurements. Diamonds represent relay nodes which fuse binary messages. The rectangle at the root represents the fusion center making an overall decision.

In [31], which studied the balanced binary relay tree with no sensor failures and no communication link failures, we provided upper and lower bounds for the total error probability at the fusion center as functions of N. These bounds reveal that the convergence of the total error probability at the fusion center is sub-exponential with an exponent $\sqrt{N}$. In [32], we studied the case where sensors fail with certain probabilities. We showed that the convergence of the total error probability at the fusion center is sub-exponential with the same exponent $\sqrt{N}$.

In this paper, we assume that the communication links between nodes in the network fail with certain probabilities. We derive upper and lower bounds for the total error probability at the fusion center as functions of N. Not surprisingly, we find that the decay of the total error probability for each step is worse than the case where there is no sensor failures and no communication failures. But this decay of the total error probability is still sub-exponential with the same exponent $\sqrt{N}$ in the asymptotic regime, provided the decay of failure probabilities is sufficiently fast.

II. Problem Formulation

We consider the problem of binary hypothesis testing between H_0 and H_1 in a balanced binary relay tree with failure-prone communication links. We assume that all sensors are independent given each hypothesis, and that all sensors have identical Type I error probability α_0 (also known as probability of false alarm) and identical Type II error probability β_0 (also known as probability of missed detection). In addition, we denote the Type I and Type II error probabilities for the nodes at height k by α_k and β_k, respectively. Moreover, we assume that all communication links between nodes at height k and height $k+1$ have identical failure probability ℓ_k. As a result of the communication failure, with a certain probability each node at level k in the tree does not have any data, which we denote by p_k. Assuming equal prior probabilities, we use the likelihood-ratio test [34] with unit threshold when fusing binary messages at the relay nodes and the fusion center.

Consider the simple problem of fusing binary messages passed to a node by its two immediate child nodes. Assume that the two child nodes have identical Type I error probability α, identical Type II error probability β, and identical node failure probability p. Moreover, assume that the two communication links connecting the child nodes to the parent node fail with identical probability ℓ. We can show that for each child node, the parent node will not receive any data from it with a certain probability, which we denote by q (henceforth, we call this the *node failure probability*), and given by:

$$q = p + (1-p)\ell.$$

Denote the Type I and Type II error probability after the fusion by α' and β'. The probability that the parent node does not have any data is

$$p' = (p + (1-p)\ell)^2 = q^2.$$

If the parent node receives data from only one of the child nodes, then the Type I and Type II error probabilities do not change since the parent node receives only one binary message. The probability of this event is $2q(1-q)$, in which case we have

$$(\alpha', \beta') = (\alpha, \beta).$$

If the parent node receives messages from both child nodes, then the scenario is the same as that in [30] and [31]. The probability of this event is $(1-q)^2$, in which case we have

$$(\alpha', \beta') = \begin{cases} (1-(1-\alpha)^2, \beta^2), & \alpha \le \beta, \\ \\ (\alpha^2, 1-(1-\beta)^2), & \alpha > \beta. \end{cases}$$

Let $\bar{\alpha}'$ and $\bar{\beta}'$ be the mean Type I and Type II error probabilities conditioned on the event that the parent node receives at least one message from its child nodes, i.e., the parent node has data. We have

$$\begin{aligned} &(\bar{\alpha}', \bar{\beta}', q') = f(\alpha, \beta, q) \\ &= \begin{cases} \left(\frac{(1-q)(2\alpha-\alpha^2)+2q\alpha}{1+q}, \frac{(1-q)\beta^2+2q\beta}{1+q}, q^2 + (1-q^2)\ell'\right), \\ \qquad \text{if } \alpha \le \beta, \\ \\ \left(\frac{(1-q)\alpha^2+2q\alpha}{1+q}, \frac{(1-q)(2\beta-\beta^2)+2q\beta}{1+q}, q^2 + (1-q^2)\ell'\right), \\ \qquad \text{if } \alpha > \beta. \end{cases} \end{aligned}$$

Our assumption is that all sensors have the same error probabilities (α_0, β_0, q_0). Therefore by the above recursion, all relay nodes at level 1 will have the same *error probability triplet* $(\alpha_1, \beta_1, q_1) = f(\alpha_0, \beta_0, q_0)$ (where α_1 and β_1 are the conditional mean error probabilities). Similarly we can calculate error probability triplets for nodes at all other levels. We have

$$(\alpha_{k+1}, \beta_{k+1}, q_{k+1}) = f(\alpha_k, \beta_k, q_k), \quad k = 1, 2, \dots, \quad (1)$$

where (α_k, β_k, q_k) is the error probability triplet of nodes at the k-th level of the tree.

The relation (1) allows us to consider (α_k, β_k, q_k) as a discrete dynamic system. For the case where $\ell_k = 0$ for all k, we have studied (see [31]) the precise evolution of the sequence $\{(\alpha_k, \beta_k)\}$, derived total error probability bounds as functions of N, and established asymptotic decay rates. In this paper, we study the case where $\ell_k \neq 0$. We derive total error probability bounds and determine the decay rate of the total error probability.

We start by looking at the single trajectory shown in Fig. 2(a), with the communication failure probabilities given by $\ell_{k+1} = \ell_k^2$. We observe that q_k decreases very quickly to 0. In addition, as shown in Fig. 2(b), the trajectory approaches $\beta = \alpha$ at the beginning. After (α_k, β_k) gets too close to $\beta = \alpha$, the next pair $(\alpha_{k+1}, \beta_{k+1})$ will be repelled toward the other side of the line $\beta = \alpha$. This behavior is similar to the non-failure scenario, in which case there exists an invariant region in the sense that the system stays in the invariant region once the system enters it [31]. Is there an invariant region for the case where $q \neq 0$? We answer this question affirmatively by precisely describing this invariant region in $\mathbb{R}^3$.

III. The Evolution of Type I, Type II, and Node Failure Probabilities

The relation (1) is symmetric about the hyperplanes $\alpha + \beta = 1$ and $\beta = \alpha$. Thus, it suffices to study the evolution of the dynamic system only in the region bounded by $\alpha + \beta < 1$, $\beta \ge \alpha$, and $0 \le q \le 1$. Let

$$\mathcal{U} := \{(\alpha, \beta) \ge 0 | \alpha + \beta < 1, \beta \ge \alpha, \text{ and } \ 0 \le q \le 1\}$$

be this triangular prism. Similarly, define the complementary triangular prism

$$\mathcal{L} := \{(\alpha, \beta) \ge 0 | \alpha + \beta < 1, \beta < \alpha, \text{ and } \ 0 \le q \le 1\}.$$

First, we introduce the following region:

$$\begin{aligned} B_1 := \{&(\alpha, \beta, q) \in \mathcal{U} | \beta \le -q/(1-q) + \\ &\sqrt{q^2 + (1-q)^2(2\alpha - \alpha^2) + 2q(1-q)\alpha}/(1-q)\}. \end{aligned}$$

If $(\alpha_k, \beta_k, q_k) \in B_1$, then the next triplet $(\alpha_{k+1}, \beta_{k+1}, q_{k+1})$ jumps across the plane $\beta = \alpha$ away from (α_k, β_k, q_k). More precisely, if $(\alpha_k, \beta_k, q_k) \in \mathcal{U}$, then $(\alpha_k, \beta_k, q_k) \in B_1$ if and only if $(\alpha_{k+1}, \beta_{k+1}, q_{k+1}) \in \mathcal{L}$. In other words, B_1 is the *inverse image* of $\mathcal{L}$ in $\mathcal{U}$ under mapping f.

It is easy to see if we start with $(\alpha_0, \beta_0, q_0) \in \mathcal{U} \setminus B_1$, then before the system enters B_1, we have $\alpha_{k+1} > \alpha_k$ and

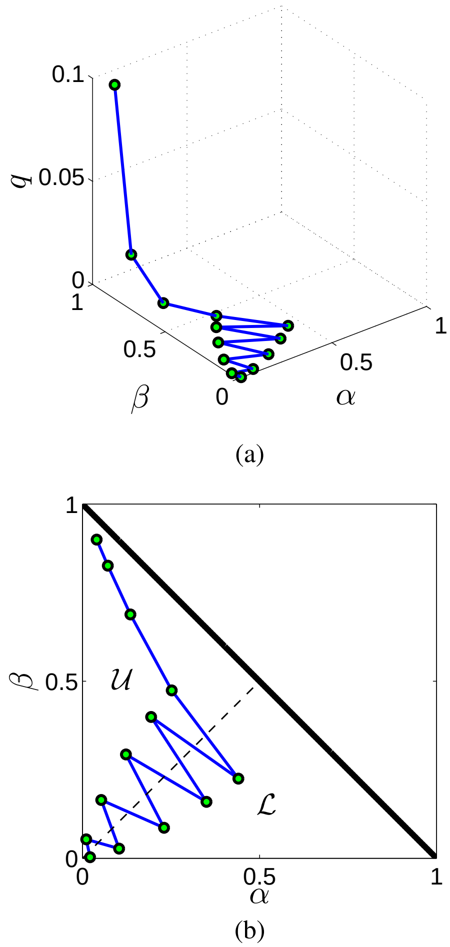

Fig. 2. (a) A typical trajectory of (α_k, β_k, q_k) in the (α, β, q) coordinates. (b) The trajectory in (a) projected onto the (α, β) plane.

$\beta_{k+1} < \beta_k$. Thus, the system moves towards the $\beta = \alpha$ plane. Therefore, if the number of sensors N is sufficiently large, then the system is guaranteed to enter B_1.

Next we consider the behavior of the system after it enters B_1. If $(\alpha_k, \beta_k, q_k) \in B_1$, we consider the position of the next pair $(\alpha_{k+1}, \beta_{k+1}, q_{k+1})$, i.e., consider the *image* of B_1 under f, denoted by $R_{\mathcal{L}}$. Similarly we denote the reflection of $R_{\mathcal{L}}$ with respect to $\beta = \alpha$ by $R_{\mathcal{U}}$. We find that

$$R_{\mathcal{U}} := \{(\alpha, \beta, q) \in \mathcal{U} | \beta \leq -\alpha + 2(\sqrt{q^2 + (1-q^2)\alpha} - q)/(1-q)\}.$$

The sets $R_{\mathcal{U}}$ and B_1 have some interesting properties. We denote the projection of the upper boundary of $R_{\mathcal{U}}$ and B_1 onto the (α, β) plane for a fixed q by $R_{\mathcal{U}}^q$ and B_1^q, respectively. It is easy to see that if $q_1 \leq q_2$, then $R_{\mathcal{U}}^{q_1}$ lies above $R_{\mathcal{U}}^{q_2}$ in the (α, β) plane. Similarly, if $q_1 \leq q_2$, then $B_1^{q_1}$ lies above $B_1^{q_2}$ in the (α, β) plane. Moreover, we have the following proposition. (Because of lack of space, many of the proofs here are omitted.)

Proposition 1: $B_1 \subset R_{\mathcal{U}}$.

We denote the region $R_{\mathcal{U}} \cup R_{\mathcal{L}}$ by R. We show below that R is an *invariant region* in the sense that once the system enters R, it stays there.

Proposition 2: If $(\alpha_{k_0}, \beta_{k_0}, q_{k_0}) \in R$ for some k_0 and $\{q_k\}$ decreases monotonically for $k \geq k_0$, then $(\alpha_k, \beta_k, q_k) \in R$ for all $k \geq k_0$.

From the above proposition, we can study the reduction of the total error probability when the system lies in R to determine the asymptotic decay rate.

First, we compare the step-wise reduction of the total error probability between the case where communication links fail with certain probabilities (failure case) and the case where the network has no communication link failures (non-failure case). We show that if the communication links are unreliable, then the decay of the total error probability for a single step is slower than the non-failure case.

Proposition 3: Let $L_{k+1}^{(q)} = \alpha_{k+1}^{(q)} + \beta_{k+1}^{(q)}$ be (twice) the total error probability at the next level from the current state (α_k, β_k, q). Suppose that (α_k, β_k, q_1) and $(\alpha_k, \beta_k, q_2) \in \mathcal{U}$. If $q_1 < q_2$, then

$$L_{k+1}^{(q_1)} \leq L_{k+1}^{(q_2)}$$

with equality if and only if $\alpha_k = \beta_k$.

Proof: From the recursion, we have

$$L_{k+1}^{(q)} = \frac{1-q}{1+q} L_{k+1}^{(0)} + \frac{2q}{1+q}(\alpha_k + \beta_k),$$

where $L_{k+1}^{(0)} = 2\alpha_k - \alpha_k^2 + \beta_k^2$.

It is easy to show that $2\alpha_k - \alpha_k^2 + \beta_k^2 \leq \alpha_k + \beta_k$.

$$\begin{aligned} & 2\alpha_k - \alpha_k^2 + \beta_k^2 \leq \alpha_k + \beta_k \\ \iff \quad & \alpha_k - \alpha_k^2 \leq \beta_k - \beta_k^2. \end{aligned}$$

Since $\alpha_k + \beta_k < 1$ and $\beta_k \geq \alpha_k$, we have $\beta_k - 1/2 < 1/2 - \alpha_k$. Notice that the function $x - x^2$ peaks at $x = 1/2$. Hence, $2\alpha_k - \alpha_k^2 + \beta_k^2 \leq \alpha_k + \beta_k$ with equality if and only if $\alpha_k = \beta_k$.

Notice that

$$\frac{1-q}{1+q} + \frac{2q}{1+q} = 1.$$

Therefore we can write

$$L_{k+1}^{(q_1)} = \pi_1 L_{k+1}^{(0)} + (1-\pi_1)(\alpha_k + \beta_k),$$

where $\pi_1 = (1-q_1)/(1+q_1)$. Let $\pi_2 = (1-q_2)/(1+q_2)$, it is easy to see that $\pi_1 \geq \pi_2$. Thus we have

$$\begin{aligned} L_{k+1}^{(q_1)} &= \pi_1 L_{k+1}^{(0)} + (1-\pi_1)(\alpha_k + \beta_k) \\ &\quad + (\pi_2 - \pi_1) L_{k+1}^{(0)} - (\pi_2 - \pi_1) L_{k+1}^{(0)} \\ &\leq \pi_1 L_{k+1}^{(0)} + (1-\pi_1)(\alpha_k + \beta_k) \\ &\quad + (\pi_2 - \pi_1) L_{k+1}^{(0)} - (\pi_2 - \pi_1)(\alpha_k + \beta_k) \\ &= L_{k+1}^{(q_2)}. \end{aligned}$$

■

From Proposition 3, we immediately deduce that if $q > 0$, then

$$L_{k+1}^{(0)} \leq L_{k+1}^{(q)},$$

which means that the decay of the total error probability for a single step is the fastest if the failure probability is 0 (i.e., the non-failure case). In other words, for the failure case, the step-wise shrinkage of the total error probability cannot be faster than the non-failure case, where the total error probability decays to 0 with exponent $\sqrt{N}$ [31].

Next we assume that the communication failure probabilities are identical at all levels, that is, $\ell_k = C$ for all k, where $C \in (0,1)$. Denote $L_k = \alpha_k + \beta_k$ to be (twice) the total error probability for nodes at level k. Let $P_N = L_{\log N}$, which is (twice) the total error probability at the fusion center. We provide an upper bound for $\log P_N^{-1}$.

Theorem 1: Suppose that $(\alpha_0, \beta_0, q_0) \in R$ and $\ell_k = C$ for all k, where $C \in (0,1)$. Then,

$$\log P_N^{-1} \leq \sqrt{N}\left(\log L_0^{-1} + 1\right).$$

In this paper, $\log$ stands for binary logarithm. Theorem 1 provides an upper bound for $\log P_N^{-1}$. Moreover, we can show that in the asymptotic regime,

$$\log P_N^{-1} = o(\sqrt{N}).$$

This implies that the convergence rate is strictly slower than $\sqrt{N}$ (note that the convergence rate for the non-failure case is exactly $\sqrt{N}$).

Finally, we assume that the failure probabilities decay quadratically to 0, that is, $\ell_{k+1} = \ell_k^2$, where $k = 0, 1, \ldots, \log N - 1$. In consequence, if α_0, β_0, and q_0 are fixed, then we have $q_k \leq \alpha_k$ and $q_k \leq \beta_k$ for sufficiently large k. With these, we derive upper and lower bounds for $\log P_N^{-1}$.

Proposition 4: Suppose that $(\alpha_k, \beta_k, q_k) \in R$, $\alpha_k \geq q_k$, and $\beta_k \geq q_k$. Then,

$$\frac{1}{2} \leq \frac{L_{k+2}}{L_k^2} \leq 4.$$

Proposition 4 gives rise to bounds on the change in the total error probability every two steps: $L_{k+2} \leq 4L_k^2$ and $L_{k+2} \geq L_k^2/2$. From these, we can derive bounds for $\log P_N^{-1}$ for even-height trees, i.e., $k = \log N$ is even.

Theorem 2: Suppose that $(\alpha_0, \beta_0, q_0) \in R$ and $\ell_{k+1} = \ell_k^2$, where $k = 0, 1, \ldots, \log N - 1$. If $\log N$ is even, then

$$\sqrt{N}\left(\log L_0^{-1} - 2\right) \leq \log P_N^{-1} \leq \sqrt{N}\left(\log L_0^{-1} + 1\right).$$

Proof: If $(\alpha_0, \beta_0, q_0) \in R$, then we have $(\alpha_k, \beta_k, q_k) \in R$ for $k = 0, 1, \ldots, \log N - 2$. From Proposition 4, we have

$$L_{k+2} = a_k L_k^2$$

for $k = 0, 1, \ldots, \log N - 2$ and some $a_k \in [1/2, 4]$. Therefore, for $k = 2, 4, \ldots, \log N$, we have

$$L_k = a_{(k-2)/2} \cdot a_{(k-4)/2}^2 \cdots a_0^{2^{(k-2)/2}} L_0^{2^{k/2}},$$

where $a_i \in [1/2, 4]$. Substituting $k = \log N$, we have

$$\begin{aligned}\log P_N^{-1} = &-\log a_{(k-2)/2} - 2\log a_{(k-4)/2} - \ldots \\ &- 2^{(k-2)/2}\log a_0 + \sqrt{N}\log L_0^{-1}.\end{aligned}$$

Notice that $\log L_0^{-1} > 0$ and for each i, $-1 \leq \log a_i \leq 2$. Thus,

$$\begin{aligned}\log P_N^{-1} &\leq \sqrt{N}\log L_0^{-1} + \sqrt{N} \\ &= \sqrt{N}\left(\log L_0^{-1} + 1\right).\end{aligned}$$

Finally,

$$\begin{aligned}\log P_N^{-1} &\geq -2\sqrt{N} + \sqrt{N}\log L_0^{-1} \\ &= \sqrt{N}\left(\log L_0^{-1} - 2\right).\end{aligned}$$

■

For odd-height trees, we need to calculate the decrease in the total error probability in a single step. For this, we have the following proposition.

Proposition 5: If $(\alpha_k, \beta_k, q_k) \in \mathcal{U}$, then we have

$$\frac{L_{k+1}}{L_k^2} \geq 1$$

and

$$\frac{L_{k+1}}{L_k} \leq 1.$$

From Propositions 4 and 5, we give bounds for the total error probability at the fusion center for trees with odd height.

Theorem 3: Suppose that $(\alpha_0, \beta_0, q_0) \in R$ and $\ell_{k+1} = \ell_k^2$, where $k = 0, 1, \ldots, \log N - 1$. If $\log N$ is odd, then

$$\sqrt{\frac{N}{2}}\left(\log L_0^{-1} - 2\right) \leq \log P_N \leq \sqrt{2N}\left(\log L_0^{-1} + 1\right).$$

We have derived error probability bounds for balanced binary relay trees with unreliable communication links. In the next section, we will use these bounds to study the asymptotic rate of convergence.

IV. Asymptotic Rates

Notice that when N is very large and $\{q_k\}$ decreases monotonically, the sequence $\{(\alpha_k, \beta_k, q_k)\}$ enters the invariant region R at some level and stays inside afterward. In consequence, the decay rate in the invariant region determines the asymptotic rate. Since the error probability bounds for odd-height trees differ from those of even-height trees simply in a constant term, without loss of generality, we will consider trees with even height to calculate the decay rate.

Proposition 6: Suppose that $L_0 = \alpha_0 + \beta_0$ is fixed and $\{q_k\}$ decreases monotonically. If $q_k \leq \alpha_k$ and $q_k \leq \beta_k$ for sufficiently large k, then

$$\log P_N^{-1} = \Theta(\sqrt{N}).$$

This implies that the convergence of the total error probability is sub-exponential with exponent $\sqrt{N}$. Compared to the exponent for the non-failure case, the scaling law of the asymptotic rate does not change when we have unreliable communications, provided the probabilities of communication failure probabilities decay to 0 sufficiently fast, even though the step-wise shrinkage for the failure case is worse.

Given $L_0 \in (0,1)$ and $\varepsilon \in (0,1)$, suppose that we wish to determine how many sensors we need to have so that $P_N \leq$

ε. The solution is simply to find an N (e.g., the smallest) satisfying the inequality

$$\sqrt{N}\left(\log L_0^{-1} - 2\right) \geq -\log \varepsilon.$$

The smallest N grows like $\Theta((\log \varepsilon)^2)$ (cf., [31], in which the growth rate is the same, and [30], where a looser bound was derived).

V. Conclusion

We have studied the detection performance of balanced binary relay trees with communication link failures. We have shown that, under the assumption of identical communication link failure probability in the tree, the exponent of the total error probability is $o(\sqrt{N})$. We have also developed total error probability bounds at the fusion center as functions of N for both even-height trees and odd-height trees in the case where the communication link failure probabilities decay to 0 quadratically. These bounds imply that the total error probability converges to 0 sub-exponentially with exponent $\sqrt{N}$. Compared to balanced binary relay trees with no communication failures, the step-wise shrinkage of the total error probability in the failure case is slower, but the asymptotic decay rate follows the same scaling law.

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