Geometrical Approach to Vector Analysis in Electromagnetics Education

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***Abstract*—A geometrical approach to teaching and learning vector calculus and analysis as applied to electromagnetic fields is proposed for junior-level undergraduate electromagnetics education. For undergraduate students, electromagnetics is typically the most challenging subject in the Electrical Engineering curriculum, and the most challenging component of the subject is the application of vector analysis to electromagnetic field theory and problem solving. According to the geometrical approach, the students are taught to "read" the figure and to "translate" this to equations at all times throughout the computation or derivation, instead of their "crunching" the formulas and numbers without even visualizing the structure. In performing vector manipulations, integrals, and derivatives, the students are taught to always deal with real geometrical entities and quantities (arrows, lengths, angles, points, lines, surfaces, volumes, etc.). They learn to "translate" the geometry and the electromagnetic physics attached to it into mathematical models (equations and symbolic or numerical values) using "first mathematical principles" instead of just "black-box" formulas as a computer would do. As opposed to the traditional formal algebraic approach to vector analysis in electromagnetics, which is very general but also very abstract and dry, the geometrical approach is problem-dependent but also much more intuitive and visual, and as such can do a great deal to increase students' understanding and appreciation of vector analysis and its application to electromagnetic theory and problem solving. This is confirmed by preliminary class testing and assessment of student learning, success, and satisfaction in the courses Electromagnetic Fields I and II at Colorado State University.**

***Index Terms*—Electromagnetics education, geometrical and visual approach, undergraduate fields courses, vector analysis.**

I. Introduction

ELECTROMAGNETIC theory or the theory of electromagnetic fields and waves is a fundamental underpinning of technical education; at the same time, it is one of the most difficult subjects for students to master. To undergraduate students, electromagnetics courses are typically the most challenging and demanding courses in the Electrical Engineering (EE) curriculum. This material is extremely abstract and mathematically rigorous and intensive, and students find it rather difficult to grasp. This is not unique to any particular school, department, country, or geographical region. It is well known and established internationally that the electromagnetic theory or fields course (or course sequence), as it is usually referred to, is always, averaged over all students in a class, the most challenging EE subject in the undergraduate curriculum.

Electromagnetics courses are taught primarily in the junior year in EE, Electrical and Computer Engineering (ECE), Physics, and similar departments and schools, typically covering some or all of the following major topics: electrostatic fields, steady electric currents, magnetostatic fields, slowly time-varying (low-frequency) electromagnetic fields, rapidly time-varying (high-frequency) electromagnetic fields, uniform plane electromagnetic waves, transmission lines, waveguides and cavity resonators, and antennas and radiation. The importance of electromagnetic theory, as a fundamental science and engineering discipline and a foundation of electrical and computer engineering as a whole, to ECE education can hardly be overstated. In addition, electromagnetics has immediate impact on a great variety of cutting-edge technologies and applications in practically all ECE areas, and a comprehensive knowledge and firm grasp of electromagnetic fundamentals is essential for students in a number of other undergraduate and graduate courses, as well as for ECE graduates as they join the workforce, now and in the future.

Perhaps the best illustration of a great struggle of educators and scholars worldwide to find an "ideal" or at least satisfactory way of teaching and learning electromagnetics, and a proof that such a way has not yet been found and established, is the fact that there are is extremely large number of quite different textbooks for undergraduate electromagnetics available and "active" (about 30 books published in North America only)—probably more than for any other discipline in science and engineering. Some initiatives to advance undergraduate electromagnetics education and surveys and experiences in teaching/learning electromagnetic fields and waves are presented in [1]–[6].

Generally, there is a great diversity in the teaching of undergraduate electromagnetics courses, in content, scope, and pedagogical philosophy. Some electromagnetics courses implement the direct or chronological order of topics, which can briefly be characterized as: first teaching static and then dynamic topics, or first teaching fields (static, quasistatic, and rapidly time-varying) and then waves (uniform plane waves, transmission lines, waveguides, and antennas)—e.g., [7] and [8]. Some courses follow the inverse (nonchronological) order of topics in teaching/learning electromagnetics: start with general Maxwell's equations and then teach everything else as applications of these equations, namely, teach all types of fields as special cases of the general high-frequency electromagnetic field—e.g., [9] and [10]. Some instructors employ the transmission-lines-first approach to teaching the course:

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start with the analysis of transmission lines, based on only pure circuit-theory concepts, with per-unit-length characteristics (distributed parameters) of the lines assumed to be known, and then continue with the chronological order of topics—e.g., [11] and [12].

Regardless of the ordering of the material, there is also a great nonuniformity in balancing the coverage and emphasis in courses in terms of general questions like: more fields versus more waves, more static versus more dynamic topics, more breadth versus more depth, more fundamentals versus more applications, etc. In addition, engineering educators are exploring various innovative ways of electromagnetics class delivery and are far from reaching consensus on pedagogical dilemmas like: traditional lecturing versus interactive in-class explorations and discussions (active teaching and learning), inclusion of teamwork and peer instruction (collaborative teaching/learning), all analysis versus considerable design component, more concepts versus more computation, etc. There are also new challenges related to a great increase in demand for distance learning, online courses, and other forms of nontraditional course delivery.

Not less importantly, about half the classes worldwide are now offered as a single mandatory fields and waves course, with the other half still being offered as a mandatory sequence of two electromagnetics courses. Another fact that must be taken into account when considering the state of the art in electromagnetics teaching is that in a very large number of programs, worldwide, fields and waves courses are taught by instructors who are not electromagnetics-trained and are teaching electromagnetics outside their personal expertise area. Finally, there is an evident decline (on average) of the mathematical and problem-solving preparedness of students taking these courses.

Two general approaches have been pursued to overcome the problems and challenges outlined above in the methodology and practice of electromagnetics teaching, with respect to the diversity and nonuniformity of curricular contents, teaching methods, pedagogical goals, instructor expertise, areas of emphasis, desired outcomes of the course or sequence of courses, the time available, and the decline in the average student's preparedness and interest and motivation for fields and waves courses. According to the first approach, the class material is very significantly reduced (many educators say almost trivialized) by simply skipping or skimming the challenging topics, concepts, examples, problems, derivations, and applications in order to attract students' attention and to remedy (bypass) any deficiencies in their mathematical and problem-solving background, as well as to save class time. Hence, the material is mostly covered only as an itemized list of final facts and selected formulas, and the examples and problems are of the pure formulaic (plug-and-chug) type. The second approach provides a rigorous and complete (as far as possible, given the available time in the course) treatment of the material and presents it to students in a consistent and pedagogically sound manner with enough detail, derivations, and explanations to be fully understandable and appreciable. The examples and problems emphasize physical conceptual reasoning, mathematical synthesis of solutions, and realistic engineering context, providing opportunities for students to develop their conceptual understanding of the material and true electromagnetic problem-solving skills. By its general philosophy and goals, this paper belongs to the second approach to electromagnetics teaching and learning.

However, whatever the coverage, emphasis, and ordering of the material in a course or courses, the curricular context, level of breadth and depth, or the teaching method and pedagogical approach, the most problematic and most important component of electromagnetics teaching and learning is vector calculus and analysis as applied to electromagnetic field computation and problem solving. This is integral to all class topics and is met in practically all lectures, recitations (problem-solving sessions), homework assignments, and tests. It is a consensus of electromagnetics educators and scholars that any improvement in the pedagogy of vector analysis in electromagnetics would be welcomed by students and instructors alike.

Vector analysis in electromagnetic fields courses, if presented in a traditional manner, is extremely poorly received and not appreciated by students, primarily because of its abstract, dry, and overcomplicated pure mathematical formalisms, including multiple integrations, multivariable vector calculations, and curvilinear coordinate systems. While doing their best to solve problems, understand derivations, and perform studies, students will very often admit that with so many mathematical concepts and degrees of freedom appearing in equations, they, in fact, have little or no idea what is actually going on in their analysis or computation. Because of the lack of understanding, they soon lose confidence, then they lose motivation, and the whole learning process is sooner or later reduced to their frantically paging through the textbook in a quest for a suitable final formula or set of formulas that look applicable and that will be applied in a nearly random fashion.

This paper proposes a geometrical approach to teaching and learning vector analysis, including vector algebra, integral multivariable calculus, and differential vector calculus, as applied to electromagnetics. It is based on geometrical visualizations and emphasizes the geometry of the problem, rather than formal algebraic algorithms and brute force algebraic computation. The students are taught to "read" the figure and to "translate" it to equations, rather than to "crunch" the formulas and numbers without even visualizing the structure with which they are dealing. In the existing electromagnetics textbooks [7]–[21] and teaching/learning practices, vector algebra and calculus are used in topics on electromagnetic fields and waves in a traditional, formal, purely "algebraic" way. Briefly, the formal algebraic approach is very general, but also very abstract and dry and analogous to the way a programmer actually "instructs" a computer to do vector analysis in computer programs. The geometrical approach, on the other hand, is problem-dependent, but also much more intuitive and visual, and as such can do a great deal to increase students' understanding and appreciation of vector analysis and its application to electromagnetics.

There are hundreds of examples to illustrate this approach, a few of which are presented in this paper, but overall, the students are taught to link the equations to the picture of the real structure that is under consideration, at all times throughout the computation or derivation. Hence, in vector analysis of electromagnetic problems, including all sorts of vector manipulations, integrals, and derivatives, the students are, unlike the computer program, taught to always visualize the structure and deal with real geometrical entities and quantities (arrows, lengths, angles, points, lines, surfaces, volumes, etc.) and then to just "translate" the geometry and the electromagnetic physics attached to it into mathematical models (equations and symbolic or numerical values) using the "mathematical first principles" (and not general black-box formulas). Ultimately, in a fundamental electromagnetic course, the main objective is always to help the students really understand a theoretical statement or derivation, or a solution to a practical problem, and to develop ways of "electromagnetic thinking," rather than to offer the computationally most efficient and most generic toolboxes for different classes of electromagnetic situations and problems.

The proposed geometrical and visual approach to teaching and learning vector calculus and analysis as applied to electromagnetic field computation and problem solving has been implemented in the new undergraduate textbook *Electromagnetics* [22]. However, the book does not explicitly refer to the approach as such, never identifying it or mentioning it on the many occasions when it is applied in the book's theoretical derivations and problem solving, nor does it explain and discuss it, either from the pedagogical point of view or to draw a comparison to the traditional formal algebraic approach to vector analysis in electromagnetics. In addition, the geometrical approach to vector analysis can be used in conjunction with any other textbook on electromagnetic fields and waves. It can also be applied to teaching and learning electromagnetics at any level.

This paper is organized as follows. Section II presents and explains the geometrical and visual approach to teaching and learning vector analysis in electromagnetics. In Section III, the proposed approach is illustrated and discussed in several characteristic examples of fields class topics including vector algebra, application of Maxwell's integral equations, and spatial differential vector operators. Section IV discusses class testing and assessment of student learning, success, and satisfaction in class delivery using the proposed geometrical and visual approach in the Electromagnetic Fields I and II courses in the ECE Department at Colorado State University, Fort Collins. Section V summarizes the main conclusions of the paper and puts them in a broader perspective of current and future electromagnetics education research.

II. Geometrical and Visual Approach to Teaching and Learning Vector Analysis in Electromagnetics

In manipulations with vectors, the proposed geometrical approach to teaching and learning electromagnetic fields and waves always emphasizes that vectors are real arrows in space and not only triplets of numbers. The magnitude of a vector is primarily appreciated and used as the geometrical length of the arrow in the context of other geometrical quantities in the figure; a component of the vector is therefore just another arrow in the figure, whose length is found as the real length of the vector arrow multiplied by the cosine or sine of a real angle identified in the figure (and not found by using abstract general formulas). For example, if the components of a given vector in an adopted (e.g., cylindrical) coordinate system in a structure are needed, they are obtained geometrically as the corresponding projections of the vector in the real picture of the structure, and not formally using analytical transformations (which are not only unintuitive, but often lead to incorrect answers).

In addition, line, surface, and volume integrals in multivariable integral vector calculus are always viewed and solved, according to the proposed geometrical approach, as integrals along a real line, over a real surface, and throughout a real volume, respectively, and not formally as single, double, and triple integrals with respect to one, two, or three coordinates in an appropriate coordinate system using coordinate transformations and general algebraic-type formulas.

As an example, according to the geometrical approach, the electric flux through a Gaussian spherical surface in a problem with spherical symmetry solved applying Gauss' law (Maxwell's third equation) in integral form is understood simply as the vector magnitude E times the area of the sphere surface S, so as $ES = E4\pi r^2$, with r being the sphere radius; in contrast, the formal approach would perform a double integration with respect to the angles θ and ϕ in a spherical coordinate system, with an elemental surface area $\mathrm{d}S$ obtained using coordinate transformations. Even in cases where $\mathrm{d}S$ is needed in the integral, it is much better to obtain it geometrically, as the area of a "patch" (a surface element of the sphere) with sides equal in length to the corresponding elemental arcs in the θ and ϕ directions (lengths of arcs are computed simply as the corresponding radius times the angle and are multiplied together for the patch area) than to use formal coordinate transformations. Similarly, in a volume integral of a function $\rho(r)$ over the volume of a sphere, the elemental volume is taken to be that of a thin spherical shell of radius r and thickness $\mathrm{d}r$, so the volume element $\mathrm{d}v = 4\pi r^2 \mathrm{d}r$. This can be visualized and obtained (with no differential calculus) as the volume of a thin flat slab (a "flattened" spherical shell) of the same thickness $(\mathrm{d}r)$ and the same surface area $(S = 4\pi r^2)$, so as $\mathrm{d}v = S\mathrm{d}r$ (surface area times thickness of the slab). The integration is performed only with respect to r, rather than formally performing threefold integration. Similar geometrical visualizations, rather than algebraic algorithms and formulas, are used with spatial derivatives, including gradient, divergence, and curl, so that these important operators really come to life and their physical meaning becomes very obvious and natural.

As a part of the geometrical approach to electromagnetics education, a general strategy for solving volume and surface integrals arising in electromagnetic analysis is also employed. This strategy basically solves an integral of a function f over a volume v or a surface S by adopting as large a volume element $\mathrm{d}v$, or surface elements $\mathrm{d}S$, as possible, the only restriction being the condition that f is constant in $\mathrm{d}v$ or $\mathrm{d}S$ [22]. In other words, the larger the volume or surface element for integration, the simpler the integration; it is seldom necessary to use standard elements that are differentially small in all dimensions or along all (curvilinear) coordinates. This simple strategy is extremely useful; it is used extensively throughout the fields and waves course(s) in all sorts of volume and surface integrals that possess some kind of uniformity and/or symmetry (that is, in practically all volume and surface integrals in the courses). An example is the computation of the total charge Q of a nonuniform volume charge distribution inside a sphere with the charge density depending only on the radial distance from the sphere center, $\rho = \rho(r)$, where the largest volume element over which $\rho = \text{const}$ is a thin spherical shell of volume $\mathrm{d}v = 4\pi r^2 \mathrm{d}r$, obtained as explained in the previous paragraph. This adoption for $\mathrm{d}v$ enables the computation of Q by integrating only along r, whereas the adoption of the standard differential volume element (elementary curvilinear cuboid) in a spherical coordinate system (customarily obtained via differential calculus or coordinate transformations) would require two additional integrations, in angles θ and ϕ, respectively, in the spherical coordinate system. Moreover, the integration performed in r is visualized geometrically as volumetric integration via layering the sphere, and not as parametric integration. Another trivial but extremely frequent (in the fields courses) example is the adoption of v (S) instead of $\mathrm{d}v$ $(\mathrm{d}S)$ in cases when $f = \text{const}$ in the entire integration domain v (S), yielding $\int_v f\mathrm{d}v = fv$ (e.g., $Q = \rho v$, for $\rho = \text{const}$).

On the other hand, effective use of different types of symmetry in all sorts of electromagnetic computations and applications is promoted in the courses as being the most useful general solution technique; students are taught that this should always be considered prior to actually carrying out any computation and applied as a part of the analysis whenever possible. Ultimately, everything in nature is more or less symmetrical, and almost all industrial products possess a high level of symmetry.

To support the physical and geometrical approach to vector analysis and computation in electromagnetics, elements of vector algebra and vector calculus are presented and used gradually across the course topics. These are presented with an emphasis on physical insight and immediate links to electromagnetic field theory concepts, instead of having a purely mathematical review of vector analysis as a separate topic (traditionally the first chapter of a textbook). In general, it is pedagogically much better to have elements of vector algebra and calculus fully integrated with the development of the electromagnetic theory, where they actually belong and really come to life. Hence, the mathematical concepts of gradient, divergence, curl, and Laplacian, as well as line (circulation), surface (flux), and volume integrals, are literally derived from physics (electromagnetics), where they naturally emerge as integral parts of electromagnetic equations and laws, and where their physical meaning is almost obvious and can readily be made very visual. Furthermore, if the vector algebra and vector calculus are approached geometrically, as proposed in this paper, even students largely unfamiliar with the concepts of vector analysis will be able to acquire them directly through the electric and magnetic field topics in the course.

III. Geometrical and Visual Approach: Examples and Discussion

A. Vector Algebra

As the first example, consider a simple problem of finding the electric force on each of the three equal point charges Q placed at the three vertices of an equilateral triangle with sides a in free space. According to the traditional, formal, purely "algebraic" approach to teaching/learning vector analysis, the first step is to find the x- and y-coordinates [since this is a two-dimensional (planar) problem, the coordinate z can be set to be zero] of the charge points in an adopted Cartesian coordinate system, and the position vectors of these points, $\mathbf{r}_1 = r_{1x}\hat{\mathbf{x}} + r_{1y}\hat{\mathbf{y}}$, $\mathbf{r}_2 = r_{2x}\hat{\mathbf{x}} + r_{2y}\hat{\mathbf{y}}$, and $\mathbf{r}_3 = r_{3x}\hat{\mathbf{x}} + r_{3y}\hat{\mathbf{y}}$, with respect to the coordinate origin. The second step is to apply Coulomb's law to express the electric force $\mathbf{F}_{\mathrm{e}12}$ on charge 2 due to charge 1 [7], [8]

$$\begin{aligned}\mathbf{F}_{\mathrm{e}12} &= \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{|\mathbf{r}_2 - \mathbf{r}_1|^2}\hat{\mathbf{R}}_{12} \\ &= \frac{Q^2}{4\pi\varepsilon_0}\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3} \\ &= \frac{Q^2}{4\pi\varepsilon_0}\frac{(r_{2x} - r_{1x})\hat{\mathbf{x}} + (r_{2y} - r_{1y})\hat{\mathbf{y}}}{\left[(r_{2x} - r_{1x})^2 + (r_{2y} - r_{1y})^2\right]^{3/2}}\end{aligned} \tag{1}$$

with $\hat{\mathbf{R}}_{12}$ denoting a unit vector directed from charge 1 toward charge 2. Similar expressions can be written for forces $\mathbf{F}_{\mathrm{e}13}$ and $\mathbf{F}_{\mathrm{e}23}$. In the third step, the resultant force on charge 3, for instance, is computed using the principle of superposition, as the vector sum of partial forces due to charges 1 and 2, respectively, yielding

$$\begin{aligned}\mathbf{F}_{\mathrm{e}3} &= \mathbf{F}_{\mathrm{e}13} + \mathbf{F}_{\mathrm{e}23} \\ &= \frac{Q^2}{4\pi\varepsilon_0}\left\{\left[\frac{r_{3x} - r_{1x}}{\left[(r_{3x} - r_{1x})^2 + (r_{3y} - r_{1y})^2\right]^{3/2}}\right.\right. \\ &\qquad \left. + \frac{r_{3x} - r_{2x}}{\left[(r_{3x} - r_{2x})^2 + (r_{3y} - r_{2y})^2\right]^{3/2}}\right]\hat{\mathbf{x}} \\ &\qquad + \left[\frac{r_{3y} - r_{1y}}{\left[(r_{3x} - r_{1x})^2 + (r_{3y} - r_{1y})^2\right]^{3/2}}\right. \\ &\qquad \left.\left. + \frac{r_{3y} - r_{2y}}{\left[(r_{3x} - r_{2x})^2 + (r_{3y} - r_{2y})^2\right]^{3/2}}\right]\hat{\mathbf{y}}\right\} \\ &= F_{\mathrm{e}3x}\hat{\mathbf{x}} + F_{\mathrm{e}3y}\hat{\mathbf{y}}\end{aligned} \tag{2}$$

from which Cartesian components of the vector $\mathbf{F}_{\mathrm{e}3}$, $F_{\mathrm{e}3x}$ and $F_{\mathrm{e}3y}$, can be easily obtained, with analogous computations for total forces $\mathbf{F}_{\mathrm{e}1}$ and $\mathbf{F}_{\mathrm{e}2}$, on charges 1 and 2. Finally, the magnitude of $\mathbf{F}_{\mathrm{e}3}$ and the angles that it makes with the coordinate axes are found as

$$\begin{aligned}F_{\mathrm{e}3} &= \sqrt{F_{\mathrm{e}3x}^2 + F_{\mathrm{e}3y}^2} \\ \alpha_x &= \cos^{-1}\frac{F_{\mathrm{e}3x}}{F_{\mathrm{e}3}} \\ \alpha_y &= \cos^{-1}\frac{F_{\mathrm{e}3y}}{F_{\mathrm{e}3}}.\end{aligned} \tag{3}$$

This approach to evaluation of electric forces due to multiple charges is very general, as it can be implemented for any mutual position of the three (or more) equal charges in the xy-plane, but is also very formal, abstract, and dry. Although very efficient and versatile computationally, pedagogically it adds practically nothing to the understanding and mastery of the underlying physical phenomena, such as mutual interaction of charged bodies, dependence of the direction and strength of electric forces on the actual distances and mutual position of charges, individual actions of partial forces on a charge, the common action of multiple forces and their balance and

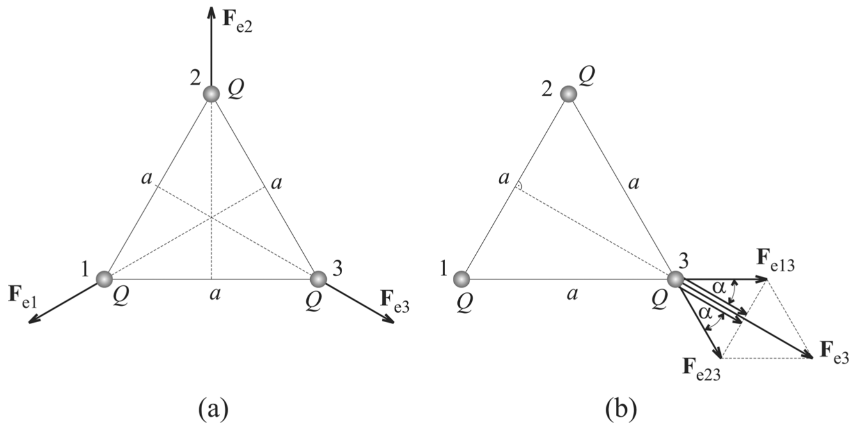

Fig. 1. Example of evaluation of electric forces due to multiple charges: (a) three equal point charges at the vertices of an equilateral triangle and (b) computation of the resultant electric force on one of the charges.

imbalance when acting together, which components of vectors add and which ones cancel, and the actual action of the resultant force on a charge in terms of the magnitude and orientation of the vector.

Essentially, the only intuitive and visual structure-dependent component of the work that the students do, based on the actual geometry and physics of the problem, is determining the Cartesian coordinates of the charge points, which are then input into the general formulas in (1)–(3). The subsequent purely mathematical "crunching" of these formulas is done in a fashion completely detached from the actual problem and from the real picture of the structure that is under consideration. Any errors that the students make in "crunching" the formulas and numbers will only effect the final result of the computation and cannot be identified and connected to a misconception or quantitative mistake in a particular component of the analysis. Errors thus cannot serve as an element of the learning process. The final result for the magnitude and the line and orientation of action of the total force on a charge emerges from a "black box," as a result of postprocessing the force components, and appears as almost a surprising and unexpected combination of numerous mutually unrelated numerical computations. Although students might use some physical intuition to check the meaningfulness of the result, this cannot be compared to their intuitively, geometrically, and visually following the entire analysis and solution process and checking all partial results as well as the final result.

This approach may be characterized as that taken by a programmer "instructing" a computer to evaluate electric forces due to multiple charges. In fact, the above steps and (1)–(3) are ideally suited for implementation in a computer program, for example, using MATLAB.

In contrast, in the proposed geometrical and visual approach to vector analysis, the students are taught to draw and then "read" the figure of the structure and to just "translate" it to equations at each stage of the computation. They are also taught to take advantage of symmetry and any specifics of the problem at hand whenever possible. In addition, the geometrical approach generally treats vectors as real arrows in space and not only triplets (or doublets in 2-D problems) of numbers. For the example with three charges at the triangle vertices, therefore, the students are taught that, even with no computation whatsoever, it can be concluded from the symmetry of the problem and the principal statement of Coulomb's law that the resultant forces on the charges, $\mathbf{F}_{\rm e1}$, $\mathbf{F}_{\rm e2}$, and $\mathbf{F}_{\rm e3}$, all have the same magnitude and are positioned in the plane of the triangle as indicated in Fig. 1(a) [22]. From Coulomb's law and Fig. 1(a), the magnitudes of the individual partial forces on charge 3 (the lower right-hand charge) are given by

$$F_{\rm e13} = F_{\rm e23} = \frac{Q^2}{4\pi\varepsilon_0 a^2} \tag{4}$$

and both forces are repulsive, as shown in Fig. 1(b). By virtue of the principle of superposition, the resultant force on this charge is obtained graphically; students do this by drawing (sketching) the force arrows in the figure, as the vector sum of vectors $\mathbf{F}_{\rm e13}$ and $\mathbf{F}_{\rm e23}$, as depicted in Fig. 1(b). The students then realize from the figure that the vector $\mathbf{F}_{\rm e3}$ is positioned along the symmetry line between charges 1 and 2, i.e., between vectors $\mathbf{F}_{\rm e13}$ and $\mathbf{F}_{\rm e23}$, and that it makes the angle $\alpha = \pi/6$ with both vectors. They simply read this from the figure (with no formulas). The magnitude of the resultant vector, $\mathbf{F}_{\rm e3}$, is therefore twice the component of any of the partial vectors along the symmetry line. In this, the magnitude of the vector $\mathbf{F}_{\rm e13}$ is considered as the geometrical length of the corresponding arrow in Fig. 1(b), and its component along (projection on) the symmetry line is just another arrow in the figure, whose length is found as the length of the $\mathbf{F}_{\rm e3}$ arrow times the cosine of the angle α, which results in

$$F_{\rm e3} = 2(F_{\rm e13}\cos\alpha) = 2F_{\rm e13}\frac{\sqrt{3}}{2} = F_{\rm e13}\sqrt{3} = \frac{\sqrt{3}Q^2}{4\pi\varepsilon_0 a^2}. \tag{5}$$

B. Application of Maxwell's Integral Equations

The next example discusses the application of Gauss' law in integral form to determine the electric field due to known charge distributions. When introducing and explaining the application of Gauss' law, in conjunction with the geometrical approach to electromagnetic field computation, students are told that while the law is always true, and can be applied to any charge distribution and any problem, only in highly symmetrical cases can it be used to analytically solve for the electric field intensity vector $\mathbf{E}$ due to a given charge distribution. Namely, the application of

Gauss' law to find $\mathbf{E}$ only gives a solution for a closed surface S (Gaussian surface) that satisfies two conditions: 1) $\mathbf{E}$ is everywhere either normal (perpendicular) or tangential to S; and 2) $E = \text{const}$ on the portion of S on which $\mathbf{E}$ is normal. In these cases, it is possible to bring the field intensity E outside the integral sign in the law equation and solve for it.

These basic ideas are then applied to numerous problems with spherical, cylindrical, and planar symmetry, respectively, involving both uniform and nonuniform charge distributions. For instance, for a problem with spherical symmetry, such as finding the field due to a nonuniform volume charge whose density ρ varies only in a spherically radial direction in free space, S is a spherical surface of radius r, and E on S is obtained, from Gauss' law, as simply as $E(r) = Q_S/(\varepsilon_0 S) = Q_S/(4\pi\varepsilon_0 r^2)$, with Q_S standing for the total charge enclosed by S, on the right-hand side of the law equation, which in turn is obtained applying the general integration strategy outlined in Section II. Similar discussions are carried out with regards to the application of Ampère's law in evaluating the magnetic field due to given current distributions and Faraday's law of electromagnetic induction.

Applications of Maxwell's integral equations, if presented in a traditional fashion, are extremely problematic for students, mainly because they require multifold integrations in curvilinear coordinate systems, and because so many mathematical concepts, variables, and degrees of freedom appear on both sides of the integral equation. The geometrical approach, on the other hand, teaches the students to always expand the left-hand side of Gauss' law as the field vector magnitude times the area of the appropriate Gaussian surface (or its parts), found completely geometrically (visually) from first principles. The enclosed charge on the right-hand side of the equation is determined in the simplest way possible, almost always as the corresponding charge density times the volume, surface area, or line length, depending on the nature of the charged structure. This works analogously for Ampère's and Faraday's laws. Once the students "get" this and realize that the integrals in Maxwell's equations appearing in undergraduate fields courses can practically always be reduced to the most basic geometrical manipulations, they are equipped to rapidly, smoothly, and joyfully use this understanding and knowledge on many occasions in their courses and homework, in all sorts of theoretical studies and practical applications in field computations; in analysis of capacitors, resistors, inductors, ac machines and transformers, and transmission lines; in boundary-value problems; in wave propagation; and so on.

C. Spatial Differential Vector Operators

A final example will consider how to introduce the gradient, one of the most important spatial differential operators, in an undergraduate electromagnetic fields course, first in Cartesian coordinates, and then its extension to a curvilinear, e.g., cylindrical, coordinate system. Traditionally, elements of vector algebra and vector calculus are presented as a separate set of topics in lectures (or a separate chapter in a textbook) before the actual electric and magnetic field topics (chapters). Thus, for instance, the gradient operator might be introduced by considering the spatial variation of a scalar function, for example, the temperature, T, in Cartesian (x, y, z) coordinates, with $T_1(x, y, z)$ denoting the temperature at a point $P_1(x, y, z)$ in a region of space and $T_2(x + \mathrm{d}x, y + \mathrm{d}y, z + \mathrm{d}z)$ that at a nearby point, $P_2(x + \mathrm{d}x, y + \mathrm{d}y, z + \mathrm{d}z)$, whose position vector with respect to P_1 is given by [11]

$$\mathrm{d}\mathbf{l} = \mathrm{d}x\hat{\mathbf{x}} + \mathrm{d}y\hat{\mathbf{y}} + \mathrm{d}z\hat{\mathbf{z}}. \tag{6}$$

It is pointed out to the students that, from differential calculus, the differential temperature amounts to

$$\mathrm{d}T = T_2 - T_1 = \frac{\partial T}{\partial x}\mathrm{d}x + \frac{\partial T}{\partial y}\,\mathrm{d}y + \frac{\partial T}{\partial z}\,\mathrm{d}z \tag{7}$$

and, since $\mathrm{d}x = \hat{\mathbf{x}} \cdot \mathrm{d}\mathbf{l}$, $\mathrm{d}y = \hat{\mathbf{y}} \cdot \mathrm{d}\mathbf{l}$, and $\mathrm{d}z = \hat{\mathbf{z}} \cdot \mathrm{d}\mathbf{l}$, (7) can be recast as

$$\begin{aligned}\mathrm{d}T &= \frac{\partial T}{\partial x}\,\hat{\mathbf{x}} \cdot \mathrm{d}\mathbf{l} + \frac{\partial T}{\partial y}\,\hat{\mathbf{y}} \cdot \mathrm{d}\mathbf{l} + \frac{\partial T}{\partial z}\hat{\mathbf{z}} \cdot \mathrm{d}\mathbf{l} \\ &= \left(\frac{\partial T}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\,\hat{\mathbf{z}}\right) \cdot \mathrm{d}\mathbf{l} = \nabla T \cdot \mathrm{d}\mathbf{l}.\end{aligned} \tag{8}$$

The gradient of the scalar function T is then introduced as the above expression in the parentheses, sometimes written as $\text{grad}\, T$, but much more frequently using the del or nabla operator (∇), so as ∇T,

$$\text{grad}\, T = \nabla T = \frac{\partial T}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\,\hat{\mathbf{z}} \tag{9}$$

and is characterized as defining the change in temperature $\mathrm{d}T$ corresponding to a vector change in position $\mathrm{d}\mathbf{l}$ in (8).

To convert (9) to a cylindrical coordinate system, shown in Fig. 2(a), where an arbitrary point is represented by coordinates (r, ϕ, z), the student can use relations (transformations) between Cartesian and cylindrical coordinates [11]

$$r = \sqrt{x^2 + y^2} \qquad \tan\phi = \frac{y}{x} \tag{10}$$

and then differential calculus

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial T}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial T}{\partial z}\frac{\partial z}{\partial x} \tag{11}$$

where $\partial z/\partial x = 0$ because z is orthogonal to x. From the coordinate relations in (10), the students obtain

$$\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \cos\phi \qquad \frac{\partial \phi}{\partial x} = -\frac{1}{r}\,\sin\phi \tag{12}$$

and hence

$$\frac{\partial T}{\partial x} = \cos\phi\,\frac{\partial T}{\partial r} - \frac{\sin\phi}{r}\frac{\partial T}{\partial \phi}. \tag{13}$$

They can use this expression to replace the x-component of the vector ∇T in (9), and a similar procedure can be carried out to obtain an expression for $\partial T/\partial y$ in terms of r and ϕ. Finally, the students employ another set of cylindrical-to-Cartesian coordinate transformations

$$\hat{\mathbf{x}} = \cos\phi\hat{\mathbf{r}} - \sin\phi\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{y}} = \sin\phi\hat{\mathbf{r}} + \cos\phi\hat{\boldsymbol{\phi}} \tag{14}$$

which completes the conversion of (9) into the expression for the gradient in cylindrical coordinates

$$\text{grad}\, T = \nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\,\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}. \tag{15}$$

As a part of the proposed physical and geometrical approach to vector analysis in electromagnetics, on the other side,

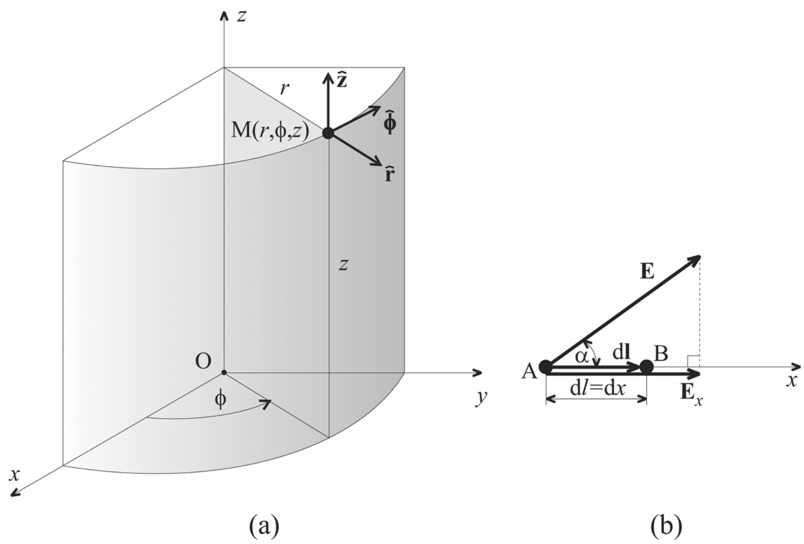

Fig. 2. (a) Point $\mathrm{M}(r,\phi,z)$ and coordinate unit vectors in a cylindrical coordinate system. (b) Derivation of the differential relationship in (19) between the electric field intensity vector $\mathbf{E}$ and the electric potential V in electrostatics.

elements of vector algebra and vector calculus are introduced fully integrated with electromagnetic field theory concepts, where they naturally belong and really come to life. Thus, the gradient is introduced through a differential relationship between the electric field intensity vector $\mathbf{E}$ and the electric potential V in electrostatics. This relationship, in turn, is derived from first principles and geometrical manipulations. The students consider a movement (displacement) from a point A in an electrostatic field at which the potential is V_{A} to a point B for an elementary distance $\mathrm{d}l = \mathrm{d}x$ along an x-axis, as shown in Fig. 2(b) [22], with the resulting change in the potential amounting to the potential at B (new potential) minus the potential at A (starting potential), that is

$$\mathrm{d}V = V_{\mathrm{B}} - V_{\mathrm{A}}. \tag{16}$$

On the other hand, potential difference (voltage) between points A and B, which equals the line integral of $\mathbf{E}$ from A to B, where the integral sign is actually not needed because the path is differentially small, can be written as

$$\begin{aligned} V_{\mathrm{A}} - V_{\mathrm{B}} &= \int_{\mathrm{A}}^{\mathrm{B}} \mathbf{E}\cdot \mathrm{d}\mathbf{l} \\ &= \mathbf{E}\cdot\mathrm{d}\mathbf{l} = E\mathrm{d}l\cos\alpha = E\cos\alpha\mathrm{d}x = E_x\mathrm{d}x \end{aligned} \tag{17}$$

with E_x standing for the x-component of $\mathbf{E}$, which equals the projection of $\mathbf{E}$ on the x-axis, $E\cos\alpha$, in Fig. 2(b). Combining (16) and (17), the students realize that

$$E_x = -\frac{\mathrm{d}V}{\mathrm{d}x} \tag{18}$$

and, similarly, the projections of the vector $\mathbf{E}$ on the other two axes of the Cartesian coordinate system are obtained as $E_y = -\mathrm{d}V/\mathrm{d}y$ and $E_z = -\mathrm{d}V/\mathrm{d}z$, so the complete vector expression for the electric field $\mathbf{E}$ is given by

$$\begin{aligned} \mathbf{E} &= E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} + E_z\hat{\mathbf{z}} \\ &= -\left(\frac{\partial V}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\,\hat{\mathbf{z}}\right) = -\nabla V \end{aligned} \tag{19}$$

where the expression in the parentheses is the gradient in Cartesian coordinates of the electric potential, $V = V(x,y,z)$, denoted as ∇V, as in (9).

To find the expression for the gradient in a cylindrical coordinate system, in Fig. 2(a), it is explained to the students that since ϕ is not a length coordinate but an angular one, an incremental distance $\mathrm{d}l$ corresponding to an elementary increment in the coordinate, $\mathrm{d}\phi$, equals $\mathrm{d}l = r\mathrm{d}\phi$ (the length of an arc of radius r defined by the angle $\mathrm{d}\phi$). They are further told that this is exactly the displacement $\mathrm{d}l$ in Fig. 2(b) in computing the change in potential $\mathrm{d}V$ in (16)–(18), now in the ϕ direction. Therefore, the ϕ-component of the electric field vector at the point M in Fig. 2(a) equals [22]

$$E_\phi = -\frac{\mathrm{d}V}{\mathrm{d}l} = -\frac{\mathrm{d}V}{r\mathrm{d}\phi} \tag{20}$$

and not just $-\mathrm{d}V/\mathrm{d}\phi$. The other two cylindrical coordinates, r and z, are length coordinates, so no modification is needed, $E_r = -\mathrm{d}V/\mathrm{d}r$ and $E_z = -\mathrm{d}V/\mathrm{d}z$. Consequently, the electric field vector can be computed as the gradient of $V = V(r,\phi,z)$ in cylindrical coordinates, in place of (19), as follows:

$$\begin{aligned} \mathbf{E} &= E_r\hat{\mathbf{r}} + E_\phi\hat{\boldsymbol{\phi}} + E_z\hat{\mathbf{z}} \\ &= -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}\right) \end{aligned} \tag{21}$$

which is equivalent to the result in (15).

Pedagogically, the approach in (16)–(21) has multiple advantages over that in (6)–(15), and similar conclusions hold for derivations and computations involving other spatial differential operators, e.g., divergence and curl. The gradient operator in Cartesian coordinates is derived in (16)–(19) from the integral relationship between the electric field intensity vector and the potential in electrostatics, namely, from the fact that the voltage between the two close points equals the component of the electric field vector along the path between the points times the path length, in Fig. 2(b). This fact is very familiar to students by the point in the course when the gradient is introduced. Simple geometrical manipulations to find the field component from the figure, by considering the lengths of the $\mathbf{E}$ and $\mathbf{E}_x$ arrows in order to find the magnitudes of the corresponding vectors, are used as well. In addition, the gradient is literally derived from physics (electrostatic field theory), giving directly the differential relationship between $\mathbf{E}$ and V. It also makes very obvious and visual the otherwise very abstract physical meaning of field components as being equal to the negative of the rate of change of the potential in that direction. With this approach, furthermore, the extension of the gradient concept to cylindrical (or spherical) coordinates comes out extremely smoothly and naturally. The entire derivation of the expression in (21) essentially consists of geometrically and visually identifying the displacement (path) for finding the potential change (voltage) corresponding to an angular increment as the length of the associated arc in the figure (that is, the radius times the angle), with the incremental distance $\mathrm{d}l$ in (20) being treated not through differential calculus but as a real length (of an arc).

The purely mathematical and formal approach to introducing the gradient in Cartesian coordinates in (6)–(9) lacks all the above pedagogical components. In addition, the differential representation for $\mathrm{d}T$ in (7), taken from differential calculus, is completely artificial, unintuitive, and nonvisual to the students, so the introduction of $\mathrm{grad}\, T$ in (8) appears as a purely formal mathematical manipulation. The conversion of the expression for $\mathrm{grad}\ T$ to cylindrical (or spherical) coordinates in (10)–(15) is yet another striking example of the complete dryness and unnecessary pure mathematical complexity of the traditional formal algebraic approach to vector analysis in electromagnetics. It is based on Cartesian-to-cylindrical transformations of coordinates, in (10), and cylindrical-to-Cartesian transformations of coordinate unit vectors, in (14), along with another artificially invoked formula from differential calculus, in (11). This sort of approach—when presented in a classroom or read from a book—will not engage the students at all. It contains too many formulas, which are completely detached from physics; moreover, coordinate transformations in (10) and (14), if taken from a general table (sheet) of formulas, break a continuous flow of thought and understandable derivation. Also, formal use of generic analytical transformations by the students in similar situations (not explicitly worked out in a book or a lecture/recitation), without the actual picture of the geometry under consideration, often leads to incorrect answers and errors. On the other hand, if the relationships in (10) and (14) were derived and justified from the geometry, the already large and pedagogically overwhelming number of equations, (10)–(15), would be even larger, in comparison to just one simple equation, (20), involved in the geometrical approach leading to the gradient expression in (21).

IV. Class Testing and Assessment

The geometrical approach to teaching and learning vector analysis as applied to electromagnetics was class tested in the ECE 341 Electromagnetic Fields I and ECE 342 Electromagnetic Fields II courses in the Department of Electrical and Computer Engineering at Colorado State University (CSU) during the academic years 2008–2009, 2009–2010, and 2010–2011. The content of ECE 341 includes electrostatic fields in free space and in dielectrics, capacitance, electric energy, steady electric currents, magnetostatic fields in free space and in material media, electromagnetic induction, inductance, magnetic energy, slowly time-varying (quasistatic) electromagnetic fields, and general Maxwell's equations. ECE 342 covers rapidly time-varying electromagnetic fields, propagation of uniform plane electromagnetic waves in free space and in various media, wave reflection, transmission, and refraction, transmission-line theory using frequency- and time-domain analysis, rectangular metallic waveguides, and fundamentals of radiation and antennas. The student learning, success, and satisfaction in these courses have been dramatically improved when compared to the class delivery not using the geometrical approach.

Student attendance in all classes was practically 100% throughout the semester (versus less than 50% with the conventional class delivery), with an extremely high official retention in classes. The passing rate of students in courses was improved by about 50%, and the average final course grade by about one grade point.

Evaluations of the course and instructor by students in classes taught using the geometrical and visual approach were consistently extremely high, despite students being expected to perform at a very high level on all assignments. For instance, student evaluation numerical ratings averaged over multiple questions on student course surveys were about 9.7 (on the scale from 0 to 10), about a 70% increase from the previous average score. Also, the percentage of students who "Strongly Agree" or "Agree" that "Overall, I would rate this course as good" was 100% in every course and every year (which is extremely unusual for fields courses), and the same percentage of students (100%) gave the same combination of responses to the question "Overall, I would rate this teacher as good." In addition, all written comments by students in their evaluation forms were extremely positive.

In a recent anonymous unofficial survey of students in the ECE 341 class, 31 students, out of 32 surveyed, answered positively to the question "Did the geometrical approach to teaching and learning vector analysis as applied to electromagnetic fields help your understanding of electromagnetic field concepts and solving electromagnetic problems?" (with 19 students stating "Yes, a lot"), and only one student answered negatively (with no students with "No opinion"). Note that students in this class are very well aware of what is meant by the "geometrical approach" in the course. Almost all of the students surveyed provided comments emphasizing the great difference this approach had made in their understanding of concepts and their problem-solving skills, with most of them comparing this approach to that used in their Electromagnetism Physics class (PH 142), which was their only other college experience in the similar topics.

The Electromagnetics Concept Inventory (EMCI) [23] is an assessment tool for measuring students' understanding of fundamental concepts in electromagnetics that was developed within the NSF Foundation Coalition project [24]. Whether used independently or with EMCI questions incorporated in class midterm and final exams, this also showed great improvements. Unfortunately, as yet there are no nationally and internationally calibrated, accepted, and established standard assessment tools for junior-level electromagnetic field courses, with established and validated performance norms and statistics over many institutions and many years of testing. Therefore, there were no opportunities to evaluate the performance of students taught electromagnetic fields using the geometrical approach to vector analysis against national and international performance norms on standardized assessment tools. Note that an example of such an established assessment tool is the Force Concept Inventory (FCI) [25], designed to measure conceptual understanding of Newtonian Mechanics; the EMCI (like other Foundation Coalition concept inventories) is inspired by the FCI and its impact on physics education.

With the geometrical approach to teaching/learning electromagnetic fields in ECE 341 and 342, student interest in follow-up elective courses in the electromagnetics, antennas, and microwaves area has dramatically increased. For example, the class enrollment in ECE 444 Antennas and Radiation at CSU rose from six and four in Fall 2007 and Fall 2008 to 13, 13, and 14 in Fall 2009, Fall 2010, and Fall 2011, respectively. Students' interest in research opportunities for undergraduates and in senior design projects in the electromagnetics area has also dramatically risen. For example, the total number of senior design students in this area grew from four per year in both academic years 2007–2008 and 2008–2009 to eight per year in both 2011–2012 and 2012–2013.

V. Conclusion

This paper has proposed a geometrical approach to teaching and learning vector calculus and analysis as applied to electromagnetic field theory and problem solving, and a general "visual" approach to understanding the physics and using mathematical models in electromagnetics, for junior-level electromagnetics courses in the undergraduate Electrical Engineering curriculum. As opposed to the traditional formal algebraic approach, used in the existing electromagnetics textbooks and teaching/learning practices, the proposed approach is based on geometrical visualizations and emphasizes the geometry of the problem, rather than formal algebraic algorithms and brute-force algebraic computation. It has been illustrated and discussed via several characteristic examples of vector algebra, application of Maxwell's integral equations, and spatial differential vector operators. It can also be applied to electromagnetics education at any level.

The ultimate goal of this present work and of the continued future work in this area is to significantly improve students' understanding of electromagnetics and their attitude toward it. Such an improvement has been achieved and confirmed by preliminary class testing and assessment of student learning, success, and satisfaction in the Electromagnetic Fields I and II courses in the ECE Department at Colorado State University. Of course, for the proposed approach to become an established way of teaching a very major component of junior-level fields courses in EE, ECE, and physics curricula, and of electromagnetics education in general, it needs to be tried by a large number of instructors in diverse institutions and programs, with diverse teaching styles and curricular and course objectives. Thus, the main purpose of this paper is to present this approach to electromagnetics educators and education researchers and to explain its perceived advantages and benefits. It is also meant to open a discussion on this important and timely topic. In general, unlike in some other areas of science and engineering, there seems to be a lack of such discussions in electromagnetics education research (within EE and related disciplines). In addition, the paper is intended to alert mathematics instructors teaching vector analysis courses as to the real needs of students being prepared for electromagnetics courses and to possibly initiate or enhance a conversation between engineering and mathematics instructors and educators on this issue. The disconnect between electromagnetics and vector mathematics courses is precisely due to the fact that electromagnetics really needs the geometrical approach, whereas students are trained to use a more mathematical methodology invoking coordinate transformations and the like. The geometrical approach may indeed be useful not only to electromagnetics instructors, but also (and perhaps even more so) to mathematics instructors of (vector) calculus as well, as a complementary means to expand its appeal to engineering and physics students. Finally, the goal is to possibly initiate and motivate the development and dissemination of other ideas, strategies, and implementations in electromagnetic fields instruction and pedagogy, toward an "ideal" way of teaching and learning electromagnetics. This is especially important given both the growing relevance of electromagnetics in emerging technologies and applications in practically all ECE areas and the evident decline (on average) of the mathematical and problem-solving preparedness of students taking fields courses.

References

[1] K. F. Warnick, R. H. Selfridge, and D. V. Arnold, "Teaching electromagnetic field theory using differential forms," *IEEE Trans. Educ.*, vol. 40, no. 1, pp. 53–68, Feb. 1997.

[2] B. Beker, D. W. Bailey, and G. J. Cokkinides, "An application-enhanced approach to introductory electromagnetics," *IEEE Trans. Educ.*, vol. 41, no. 1, pp. 31–36, Feb. 1998.

[3] M. Popović and D. D. Giannacopoulos, "Assessment-based use of CAD tools in electromagnetic field courses," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1824–1827, May 2005.

[4] M. F. Iskander, "Technology-based electromagnetic education," *IEEE Trans. Microw. Theory Tech.*, vol. 50, no. 3, pp. 1015–1020, Mar. 2002.

[5] S. Rengarajan, D. Kelley, C. Furse, and L. Shafai, "Electromagnetics education in North America," in *Proc. URSI Gen. Assembly*, New Delhi, India, Oct. 2005.

[6] J. Mur, A. Usón, J. Letosa, M. Samplón, and S. J. Artal, "Teaching electricity and magnetism in electrical engineering curriculum: Applied methods and trends," in *Proc. Int. Conf. Eng. Educ.*, Gainesville, FL, Oct. 16–21, 2004.

[7] W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, 6th ed. New York: McGraw-Hill, 2001.

[8] M. N. O. Sadiku, *Elements of Electromagnetics*, 3rd ed. New York: Oxford Univ. Press, 2001.

[9] N. N. Rao, *Elements of Engineering Electromagnetics*, 6th ed. Upper Saddle River, NJ: Prentice-Hall, 2004.

[10] M. F. Iskander, *Electromagnetic Fields and Waves*. Prospect Hills, IL: Waveland Press, 2000.

[11] F. T. Ulaby, *Fundamentals of Applied Electromagnetics*. Upper Saddle River, NJ: Prentice-Hall, 2001.

[12] U. S. Inan and A. S. Inan, *Engineering Electromagnetics*. Menlo Park, CA: Addison-Wesley-Longman, 1999.

[13] D. K. Cheng, *Field and Wave Electromagnetics*, 2nd ed. Reading, MA: Addison-Wesley, 1989.

[14] J. D. Kraus and D. A. Fleisch, *Electromagnetics With Applications*, 5th ed. New York: McGraw-Hill, 1999.

[15] K. R. Demarest, *Engineering Electromagnetics*. Upper Saddle River, NJ: Prentice-Hall, 1998.

[16] S. M. Wentworth, *Fundamentals of Electromagnetics With Engineering Applications*. New York: Wiley, 2005.

[17] C. R. Paul, *Electromagnetics for Engineers With Applications*. New York: Wiley, 2004.

[18] K. E. Lonngren, S. V. Savov, and R. J. Jost, *Fundamentals of Electromagnetics With MATLAB*, 2nd ed. Raleigh, NC: SciTech, 2007.
[19] C. T. A. Johnk, *Engineering Electromagnetic Fields and Waves*, 2nd ed. New York: Wiley, 1988.
[20] N. Ida, *Engineering Electromagnetics*, 2nd ed. New York: Springer, 2004.
[21] D. J. Griffiths, *Introduction to Electrodynamics*, 3rd ed. Upper Saddle River, NJ: Pearson-Addison-Wesley, 1999.
[22] B. M. Notaroš, *Electromagnetics*. Upper Saddle River, NJ: Pearson Prentice-Hall, 2010.
[23] B. M. Notaros, "Concept inventory assessment instruments for electromagnetics education," in *IEEE Antennas Propag. Soc. Int. Symp. Dig.*, San Antonio, TX, June 16–21, 2002, vol. 1, pp. 684–687.
[24] D. L. Evans, D. Gray, S. Krause, J. Martin, C. Midkiff, B. M. Notaros, M. Pavelich, D. Rancour, T. Reed-Rhoads, P. Steif, R. Streveler, and K. Wage, "Progress on concept inventory assessment tools," in *Proc. 33rd ASEE/IEEE FIE*, Boulder, CO, Nov. 5–8, 2003, pp. T4G.1–T4G.8.
[25] D. Hestenes and I. Halloun, "Interpreting the force concept inventory," *Phys. Teacher*, vol. 33, no. 8, pp. 502–506, 1995.

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