

PPP method for analysis of electromagnetic fields in inhomogeneous media

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Abstract: A perturbation/iterative method (the PPP method) is proposed for the analysis of electromagnetic fields in inhomogeneous media. It consists of increasing the values of the parameters of the medium at all points in small increments, starting from their vacuum values, and computing the induced sources by means of the total field (the incident field, plus that of the induced sources as obtained in the preceding step). Thus, in all the perturbation/iterative steps the total excitation field is known, and therefore the solution of any problem using the method is based solely on the expression for the field of an elemental source in a vacuum. The method appears to be very versatile, conceptually the simplest possible and remarkably rapid. It also has extremely low memory requirements even for very large problems, provided that computing time is not critical. In addition, it can be combined very efficiently with other available methods, e.g. with the method of moments for the analysis of conducting antennas in a vacuum to analyse such antennas in the presence of arbitrary dielectric or magnetic bodies.

1 Introduction

Except in the case of waves in homogeneous media, electromagnetic structures always include boundaries, i.e. inhomogeneities. Most often the inhomogeneities are piecewise homogeneous, but not infrequently gradual variation of the electromagnetic properties of the medium is also present. The properties of the bodies constituting a system can range from those of a practically perfect dielectric of relatively low permittivity, or an imperfect dielectric with significant losses, to those of ferrites or metallic conductors.

To find the electromagnetic field in such diverse cases, many methods have been proposed and used. It is mostly assumed that the medium is linear, and we shall also restrict our attention to such cases. Analytical methods are of limited practical use. Numerical methods can be broadly classified as surface integral equation methods [1], volume integral equation methods [2, 3] and differential equation methods [4]. Directly or indirectly, all the numerical methods are aimed at establishing and solving a system of linear algebraic equations often with

a very large number of usually complex unknowns. Consequently, serious problems are frequently encountered concerning the computer memory requirements, the necessary computing time and, which is probably most important, the stability of the solution. Several techniques have been proposed, mostly of an iterative nature, to circumvent these difficulties [5, 6]. Finally, the mathematical, as well as the numerical, basis of most of these methods tends to be complicated, so that complete understanding, and thus also complete mastering, of the methods is the privilege of a relatively narrow circle of specialists.

This paper proposes a novel, conceptually and computationally simple, general perturbation/iterative numerical method for the analysis of electromagnetic fields in inhomogeneous media. In its basic form it was described in References 7 and 8. Since then, the method has been significantly improved and extended, and represents an efficient method which can be used for solving a large variety of problems as a very simple alternative to the method of moments. Although further improvements and generalisations of the method are certainly to be expected, the authors believe that even in its present form it can be used as a very powerful tool for solving many important electromagnetic field problems in a simple manner.

Basically, the method consists of allowing the values of the parameters of the (generally inhomogeneous) medium, e.g. permittivity of a dielectric body or conductivity of a conductive body, to increase at all points in small increments, from their vacuum values (e.g. unity for relative permittivity and zero for conductivity) to their actual values. In the first step, for the vacuum values of the parameters, the field at all points is computed as that due to the external sources in a vacuum. This field (the incident field) is then used to calculate the induced sources at all points of the medium with the values of the parameters increased by one small increment. In the second and subsequent steps, the induced sources are calculated on the basis of the corrected total field (the incident field plus that of the induced sources as obtained in the preceding step and situated in a vacuum). Thus, in all the perturbation/iterative steps, the total excitation field is known, and therefore the solution of any problem using the method is based solely on the expression for the field of an elemental source in a vacuum. From the initial letters of the Serbo-Croatian words *postepeno povećavanje parametara*, meaning 'gradual increase of parameters', the method is termed the PPP method.

In addition to its conceptual and computational simplicity, the method appears to be very versatile and remarkably rapid. It also offers the possibility of extremely low memory requirements for even very large

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problems, provided that computing time is not critical. In addition, it can (although need not) be combined very efficiently with other methods, e.g. with the method of moments for the analysis of conducting antennas in a vacuum to analyse such antennas in the presence of arbitrary dielectric or magnetic bodies.

2 Outline of the method

Consider an arbitrary system of sources ($\mathbf{J}_i, \mathbf{J}_{mi}$) acting in a linear, but inhomogeneous, medium of parameters $\epsilon = \epsilon_0(1 + \chi_e)$, $\mu = \mu_0(1 + \chi_m)$ and σ . The electromagnetic field in the system satisfies Maxwell's equations:

$$\begin{aligned} \text{curl } \mathbf{E} &= -\mu(\partial\mathbf{H}/\partial t) - \mathbf{J}_{mi} \\ \text{curl } \mathbf{H} &= \sigma\mathbf{E} + \epsilon(\partial\mathbf{E}/\partial t) + \mathbf{J}_i \end{aligned} \quad (1)$$

In the first step, the electromagnetic field ($\mathbf{E}_0, \mathbf{H}_0$) is determined of the impressed sources acting in a vacuum. This field satisfies the equations

$$\begin{aligned} \text{curl } \mathbf{E}_0 &= -\mu_0(\partial\mathbf{H}_0/\partial t) - \mathbf{J}_{mi} \\ \text{curl } \mathbf{H}_0 &= \epsilon_0(\partial\mathbf{E}_0/\partial t) + \mathbf{J}_i \end{aligned} \quad (2)$$

As the next step, assume that χ_e, χ_m and σ are increased at all points from zero to $1/n$ of their actual values, with $n \gg 1$. We then have, approximately,

$$\begin{aligned} \text{curl } \mathbf{E}_1 &= -\mu_0(\partial\mathbf{H}_1/\partial t) \\ &\quad - [\mathbf{J}_{mi} + \mu_0(\chi_m/n)(\partial\mathbf{H}_0/\partial t)] \end{aligned} \quad (3a)$$

$$\begin{aligned} \text{curl } \mathbf{H}_1 &= \epsilon_0(\partial\mathbf{E}_1/\partial t) \\ &\quad + [\mathbf{J}_i + (\sigma/n)\mathbf{E}_0 + \epsilon_0(\chi_e/n)(\partial\mathbf{E}_0/\partial t)] \end{aligned} \quad (3b)$$

In doing this, we have approximated the effects of the 'rarefied', but inhomogeneous, medium by equivalent sources situated in a vacuum. These sources are computed on the basis of the field in the first step (iteration), neglecting the small contribution to the field due to the 'rarefied' medium.

Increasing the values of the parameters of the medium stepwise, for some $k(1 < k \leq n)$ we have

$$\begin{aligned} \text{curl } \mathbf{E}_k &= -\mu_0(\partial\mathbf{H}_k/\partial t) \\ &\quad - [\mathbf{J}_{mi} + \mu_0(k\chi_m/n)(\partial\mathbf{H}_{k-1}/\partial t)] \end{aligned} \quad (4a)$$

$$\begin{aligned} \text{curl } \mathbf{H}_k &= \epsilon_0(\partial\mathbf{E}_k/\partial t) + [\mathbf{J}_i + (k\sigma/n)\mathbf{E}_{k-1} \\ &\quad + \epsilon_0(k\chi_e/n)(\partial\mathbf{E}_{k-1}/\partial t)] \end{aligned} \quad (4b)$$

In the end, after n iterations, eqns. 4 (with $k = n$) have a form which can be rearranged as

$$\begin{aligned} \text{curl } \mathbf{E}_n &= -\mu(\partial\mathbf{H}_n/\partial t) - \mathbf{J}_{mi} \\ &\quad + [\mu_0\chi_m \partial(\mathbf{H}_n - \mathbf{H}_{n-1})/\partial t] \end{aligned} \quad (5a)$$

$$\begin{aligned} \text{curl } \mathbf{H}_n &= \sigma\mathbf{E}_n + \epsilon(\partial\mathbf{E}_n/\partial t) + \mathbf{J}_i - [\sigma(\mathbf{E}_n - \mathbf{E}_{n-1}) \\ &\quad + \epsilon_0\chi_e \partial(\mathbf{E}_n - \mathbf{E}_{n-1})/\partial t] \end{aligned} \quad (5b)$$

For $n \rightarrow \infty$, eqns. 5 will be identical with eqns. 1. Therefore, if $n \rightarrow \infty$, we have that $\mathbf{E}_n = \mathbf{E}$ and $\mathbf{H}_n = \mathbf{H}$. In addition, for any k , the exact solution corresponding to the medium with parameters $k\chi_e/n, k\chi_m/n$ and $k\sigma/n$ is obtained from eqns. 4. For a finite number of iterations n , the last conclusions are not exact, but it could be expected that, with increasing n , the solution should converge to the exact solution.

It is important to realise that in this process the sources of the field in all steps are known (these are the expressions in square brackets in eqns. 3 and 4), and that the field computed is that in a vacuum.

Some final comments about the PPP method may be useful at this point. First, increments of the values of the parameters need not be equal, but can be varied during the process. Second, the principal idea of the PPP method can be understood as an approximate mathematical model of the physical process of polarisation, namely magnetisation, of the substance under the influence of impressed fields. Third, the main problem with the method is that the iterative process can be unstable and divergent in some cases, depending on the problem considered and on the adopted model and approximations for the solution in following the general PPP procedure described by eqns. 2-4.

The general procedure outlined above will now be specialised for solving some classes of electromagnetic problems.

2.1 Analysis of electrostatic problems by means of the PPP method

Let us consider a dielectric body situated in a vacuum in external electrostatic field \mathbf{E}_0 . Let the relative permittivity of the body be $\epsilon_r(\mathbf{r}) = 1 + \chi_e(\mathbf{r})$, where \mathbf{r} is the position vector of the field point, and let the step in susceptibility be $\Delta\chi_e(\mathbf{r}) = \chi_e(\mathbf{r})/n$. The induced sources in this case are polarisation charges, which can be expressed in terms of the polarisation vector, \mathbf{P} . The equations for the PPP method are the following (we omit henceforth the explicit dependence on \mathbf{r} ; \mathbf{n} is the unit vector normal to the body surface, directed outward, \mathbf{r}' is the position vector of the source point, and $\mathbf{R} = \mathbf{r} - \mathbf{r}'$):

$$\mathbf{P}^{(1)} = \epsilon_0 \Delta\chi_e \mathbf{E}_0 \quad (6)$$

$$\mathbf{P}^{(k)} = \epsilon_0 k \Delta\chi_e (\mathbf{E}_0 + \mathbf{E}_d^{(k-1)}) \quad k = 2, 3, \dots, n \quad (7)$$

The field due to the polarised dielectric in all steps is obtained as the field of a known distribution of surface and volume charges in a vacuum,

$$\begin{aligned} \mathbf{E}_d^{(k-1)} &= \frac{1}{4\pi\epsilon_0} \left(\int_S \frac{\sigma^{(k-1)} dS}{R^2} \mathbf{r}_0 + \int_v \frac{\rho^{(k-1)} dv}{R^2} \mathbf{r}_0 \right) \\ &\quad \mathbf{r}_0 = \mathbf{R}/R \end{aligned} \quad (8)$$

$$\sigma^{(k-1)} = \mathbf{P}^{(k-1)} \cdot \mathbf{n} \quad (9)$$

$$\rho^{(k-1)} = -\text{div } \mathbf{P}^{(k-1)} \quad (10)$$

The simplest (but not the only possible) way of solving the problem by means of these equations is as follows. We approximate the volume of the body by N small homogeneous cubes, and assume that the polarisation vector in every cube is constant. We thus obtain the following iteration scheme:

$$\mathbf{P}_j^{(1)} = \epsilon_0 \Delta\chi_e \mathbf{E}_0 \quad j = 1, 2, \dots, N \quad (11)$$

$$\begin{aligned} \mathbf{P}_j^{(k)} &= \frac{\epsilon_0 k \Delta\chi_e}{1 - \epsilon_0 k \Delta\chi_e a_{self}} \left[\mathbf{E}_0 + \sum_{\substack{i=1 \\ i \neq j}}^N (\mathbf{E}_d)_{ji}^{(k-1)} \right] \\ &\quad j = 1, 2, \dots, N; \quad k = 2, 3, \dots, n \end{aligned} \quad (12)$$

where \mathbf{P}_j is the polarisation vector at the center of the j th cube and $(\mathbf{E}_d)_{ji}$ the field at the centre of the j th cube due to the surface charges on the i th cube. In this iteration scheme, the value of the self field $a_{self} \mathbf{P}_j$, as the dominant term, is computed on the basis of the polarisation from that particular iteration $\mathbf{P}_j^{(k)}$, instead of the polarisation $\mathbf{P}_j^{(k-1)}$ from the preceding one. Without this improvement, the iteration process appeared to be unstable and to diverge for relative permittivities greater than about five. The iteration scheme in eqns. 11 and 12 was used in

all the examples to be presented in Section 3, yielding a fast convergence with increasing the number n of the steps in susceptibility.

All the coefficients in eqns. 12 are evaluated analytically.

For a body made of a homogeneous dielectric, or of piecewise-homogeneous dielectric, a surface model is preferred, in which only boundary surfaces are considered; they are approximated by small flat surface elements. The bound charges on these surface elements are approximated by point charges.

In two-dimensional problems, volume elements become surface elements, and surface elements become line elements, in a cross-section of the system.

2.2 Analysis of scatterers by means of the PPP method

Consider now a body made of imperfect inhomogeneous dielectric situated in a vacuum in an incident time-harmonic electromagnetic field of angular frequency ω . Suppose that the complex incident electric field \mathbf{E}_0 is known at all points, that the relative permittivity ϵ_r and conductivity σ of the body are known functions of the co-ordinates, and that permeability at all points is μ_0 . The body can be replaced by equivalent polarisation and free charges, and the corresponding currents throughout its volume v and over its surface S situated in a vacuum. If we introduce the complex permittivity, $\epsilon_c = \epsilon - j\sigma/\omega$, the procedure to be used according to the PPP method is described by eqns. 6 and 7, in which $\Delta\chi_e$ should be replaced by $\Delta\chi_e = (\epsilon_c/\epsilon_0 - 1)/n$. The field of the polarised dielectric is computed as the field of a known charge and current distribution in a vacuum,

$$\begin{aligned} \mathbf{E}_d^{(k-1)} = & \frac{1}{\epsilon_0} \left(\int_v \rho^{(k-1)} G(\mathbf{R}) \mathbf{r}_0 dv + \int_S \sigma^{(k-1)} G(\mathbf{R}) \mathbf{r}_0 dS \right) \\ & - j\omega\mu_0 \int_v \mathbf{J}^{(k-1)} g(\mathbf{R}) dv \\ \mathbf{r}_0 = & \mathbf{R}/R; \quad k = 2, 3, \dots, n \end{aligned} \quad (13)$$

where

$$\begin{aligned} g(\mathbf{R}) &= \frac{e^{-j\beta R}}{4\pi R} \\ G(\mathbf{R}) &= -\frac{dg(\mathbf{R})}{dR} = \frac{1 + j\beta R}{R} g(\mathbf{R}) \end{aligned} \quad (14)$$

β being the free-space phase coefficient, $\beta = \omega\sqrt{\epsilon_0\mu_0} = 2\pi/\lambda$.

The volume and surface total charge densities $\rho^{(k-1)}$ and $\sigma^{(k-1)}$ are computed by means of eqns. 10 and 9, respectively, and the total current density is given by $\mathbf{J}^{(k-1)} = j\omega\mathbf{P}^{(k-1)}$.

The simplest (but not the only possible) way of performing the described PPP procedure is to approximate the domain of the body by means of N small homogeneous cubes, and to assume the complex polarisation vector to be constant in every cube. (In this model, the charges are represented by surface charges over the sides of the small cubes.)

The electric field due to the equivalent sources is evaluated as follows. The cubes being electrically small, the self field is the same as the surface-charge static field, $(\mathbf{E}_s)_j^{(k-1)} = -\mathbf{P}_j^{(k-1)}/3\epsilon_0$. If the centre of the j th cube ($j \neq i$) is closer to the centre of the i th cube than a certain distance r_g ($r_{ji} \leq r_g$), $e^{-j\beta R}$ is expanded in Maclaurin series and the first four terms are retained, and the elec-

tric field due to the surface charges is calculated analytically. (In the examples to be presented in Section 3, it was adopted that $r_g = \lambda/(10\sqrt{3})$.) For $r_{ji} > r_g$, the surface charges on cube sides are approximated by point charges at side centres. The field of the cube currents in all cases ($j \neq i$) is computed as that due to a current element at the cube centre. (This approximation for the electric field of the current suffices, because the electric field due to the charges is dominant for close cubes.) For $r_{ji} \gg r_g$, the field can be taken to be due to a Hertzian dipole at the cube centre.

In all cases analysed using the described procedure, the convergence was found to be remarkably rapid, i.e. $n \ll 3N$ resulted in accurate solution. Therefore, the present method is considerably faster than a direct solution of the moment method system of equations. (The necessary computing time is especially small if a structure needs to be analysed in a wide range of values of the medium parameters.) In addition, the PPP method can be formulated as a version with almost no memory requirements, in which the coefficients of the iteration matrix are recomputed in every iteration. Although this increases the computing time greatly, it enables small computers to solve very large problems, which possibly could not be solved using much larger computers and the usual method of moments.

2.3 PPP method in combination with existing methods for analysis of electromagnetic systems in homogeneous media

The PPP method can be combined with existing methods for the analysis of electromagnetic systems in homogeneous media to obtain an efficient procedure for the analysis of such systems in the presence of (imperfect) dielectric and/or magnetic bodies.

Consider an electromagnetic system situated in a homogeneous medium and assume that a method is available for its analysis in an arbitrary impressed field $(\mathbf{E}_i, \mathbf{H}_i)$. Suppose that a body of arbitrary electromagnetic properties is introduced in the near field of the system. This will result in secondary sources in the body, which will change the original current distribution in the system. However, if in the first step the body is assumed to have the values of its electromagnetic parameters, equal to $1/n$ ($n \gg 1$) of their actual values, the secondary sources in the body can be considered to be due to the original near field only, and not to the field of the body itself. Therefore, in this first step, we can determine the secondary sources directly.

We have next to recompute the system currents, as the field of the induced sources has to be added to the original impressed field. This, however, changes only the right-hand side matrix, and so the original system matrix is used for determining this new distribution of the system currents. In addition, this new system (and all subsequent ones in later iterations) can be solved by back substitution only, i.e. it requires very little computing time compared with the first solution of the system. We then need to compute the new near field of the system currents (considered in a vacuum) and the new values of the induced sources (taking into account their own field), and the process continues in a way similar to the basic PPP scheme.

A notable example of this general combined method is the analysis of wire antennas in the presence of dielectric and/or magnetic bodies. Several powerful computer programs exist for the analysis of wire antennas assembled from arbitrarily interconnected straight-wire segments

and situated in a vacuum. One such program has been used by one of the authors and many antenna designers very successfully for a number of years. Its possibilities are described in Reference 9. The program was adapted and changed as required by the general combined method described above to solve the problem of wire antennas in the presence of dielectric or magnetic bodies, and was found to be very rapid. It is much faster than the usual subdomain-type solution obtained by the method of moments. In addition, it can be formulated with memory requirements practically equal to those for the wire antenna in a vacuum, for arbitrary bodies in the antenna vicinity (at the expense of a large increase in computing time), so that large systems can be analysed with even personal computers.

3 Numerical results

In this Section, some of the numerical results obtained using the PPP method are presented, to illustrate the generality, simplicity, accuracy and rapid convergence of the method.

3.1 Homogeneous dielectric circular cylinder and dielectric cube in an electrostatic field

Let us compute first the dipole moment \mathbf{p} of two simple dielectric bodies situated in a uniform electric field \mathbf{E}_0 .

Fig. 1 shows the normalised electric dipole moment, $\mathbf{p}_{norm} = \mathbf{p}/(\epsilon_0 |\mathbf{E}_0| v)$, of an infinitely long circular dielectric

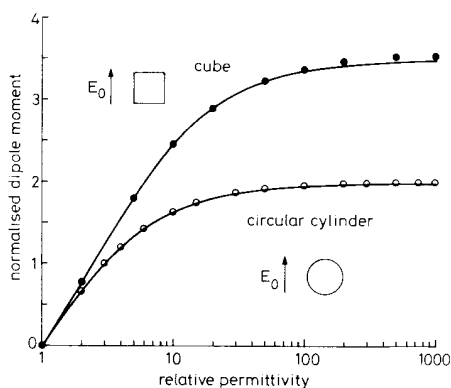


Fig. 1 Normalised electric dipole moment of a homogeneous dielectric circular cylinder and a cube, in a uniform electrostatic field, against relative permittivity

— PPP method
○ Analytical solution
● Method of moments [10]

cylinder and a dielectric cube, against the dielectric relative permittivity. (v is the volume of the body, and for the cylinder the dipole moment per unit length is considered.) The surface model was first adopted. The cylinder and the cube surfaces were approximated by $N = 100$ strips and $N = 600$ equal squares, respectively. Almost identical results for the cube were obtained by means of the volume model, with $N = 343$ small cubes (resulting in a total of 1029 unknowns). The number of iterations was from five to 20 (depending on ϵ_r). The PPP results for the circular cylinder were compared with the analytical results, and those for the cube with the moment-method results presented in Reference 10. In both cases, agreement of the two sets of results is good.

Convergence of the PPP method is illustrated in Table 1 for a cube of $\epsilon_r = 100$. The faster PPP procedure is almost an order of magnitude faster than the classical method of moments if an error not exceeding 1% is

Table 1: Normalised cube dipole moment ($\epsilon_r = 100$) and required processor time, T_{cpu1} , for the faster version of the PPP method (the system matrix stored) and T_{cpu2} for the slower one (with extremely low memory requirements) against the number n of the steps in susceptibility

n	ρ_{norm}	T_{cpu1} (s)	T_{cpu2} (s)	Classical method of moments*
3	8.996 850	50.20	90.90	$\rho_{norm} = 3.339203$ $T_{cpu3} = 554.39$ s
5	4.080 631	55.26	181.75	
7	3.547 200	60.31	272.59	
9	3.383 388	65.31	374.93	
11	3.344 493	70.36	454.34	
15	3.339 332	80.47	636.09	
25	3.339 237	104.91	1090.43	

The surface model ($N = 600$) was adopted.

* Including the Gauss elimination procedure.

accepted, in which case even the slower PPP procedure is faster than the classical method. The results were obtained on a PC-386 computer, 25 MHz, with a coprocessor 80387 and 4 Mbyte RAM.

3.2 Microstrip line: combined quasistatic analysis

Consider a microstrip transmission line of the cross-section shown in Fig. 2. We assume that the conducting strips are infinitely thin.

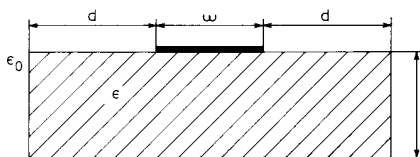


Fig. 2 Cross-section of microstrip transmission line

It is a relatively simple matter to analyse the microstrip line without dielectric substrate. We then proceed with the PPP method to analyse the line with the substrate. Following the general reasoning explained in Section 2, we assume the field in the first iteration to be that due to the line without substrate and that it polarises the dielectric of susceptibility χ_e/n . In the next and further iterations, the polarisation of the dielectric is assumed to be due to the distribution of charges over the conductors changed by the field of the polarised dielectric, and to the polarised dielectric itself, as calculated in the preceding iteration. Fig. 3 shows the effective relative permittivity (defined by $\epsilon_{ef} = C'/C'_0$, where C' is the capacitance per unit length of the actual line, and C'_0 of the same line without dielectric) against the relative permittivity of the dielectric ϵ_r . The adopted number of divisions of the line contours was about 350 (depending on h). The necessary number of iterations ranged from 10 to 20, depending on ϵ_r , i.e. it was much smaller than the number of unknowns. The results were compared with those obtained by means of the approximate formula, $\epsilon_{ef} = 0.5[(\epsilon_r + 1) + (\epsilon_r - 1)(1 + 12h/w)^{-1/2}]$, proposed in Reference 11. Excellent agreement of the two sets of results was observed.

3.3 Homogeneous perfect-dielectric scatterer

As an example of the application of the PPP method to the analysis of dynamic fields, consider a dielectric scat-

terer in the form of a parallelepiped in the field of a linearly polarised plane wave of electric field intensity E_0 (Fig. 4).

Fig. 5 shows the scattered field for a cube ($a = b = c = \lambda/5$, λ being the free-space wavelength) made of lossless dielectric ($\epsilon_r = 9$, $\sigma_d = 0$). The dielectric cube was represented as $N = 6 \times 6 \times 6 = 216$ small cubes (a total of 648 scalar complex unknowns), and the

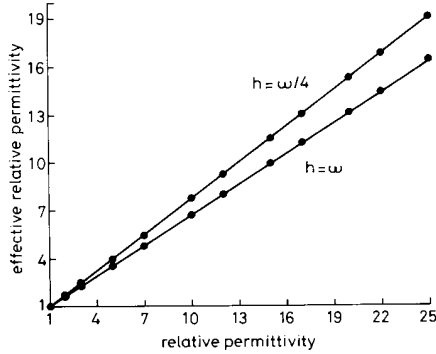


Fig. 3 Effective relative permittivity, ϵ_{ef} , of the microstrip transmission line sketched in Fig. 2 ($w = 1$ mm, $d = 10$ mm) against relative permittivity ϵ_r of the dielectric substrate

— PPP method
● Approximate formula proposed in Reference 11

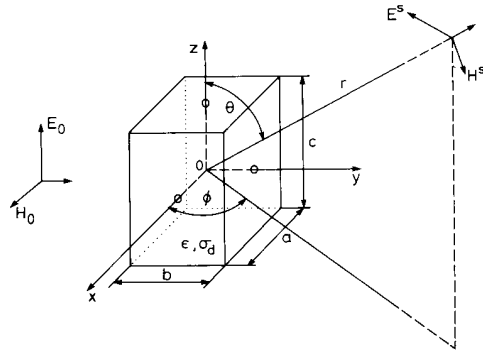


Fig. 4 Dielectric scatterer in the form of a parallelepiped in the field of a linearly polarised plane wave

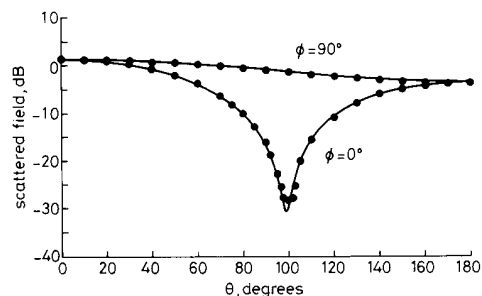


Fig. 5 Scattered field, $20 \log |E^S|$, for a homogeneous perfect-dielectric cube of $\epsilon_r = 9$ (see Fig. 4, $a = b = c = 20$ cm), for various θ in two characteristic planes ($r = 30$ m)

$E_0 = 377e^{-j\beta z}i_x$ (V/m), $f = 300$ MHz
— PPP method
● Method of moments [3] (volume formulation)

number of iterations adopted was 20. The PPP results were compared with those obtained by the method of moments [3]. Very good agreement of the two sets of results was observed.

3.4 Inhomogeneous imperfect-dielectric scatterer

As an example of an inhomogeneous lossy dielectric scatterer, consider a rod-like scatterer of parameters $\epsilon_r = 2\xi(z)$, $\sigma_d = 0.6$ S/m $\xi(z)$, where $\xi(z) = 5.5 + 9z/c$, $-c/2 \leq z \leq c/2$ (see Fig. 4). Fig. 6 shows the normalised

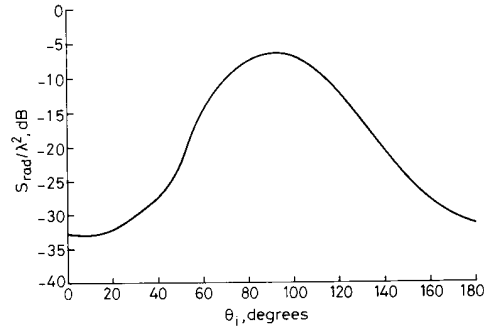


Fig. 6 Normalised radar cross-section of an inhomogeneous lossy-dielectric rod-like scatterer (see Fig. 4, $a = b = 2.8$ mm, $c = 22.4$ mm) of parameters $\epsilon_r = 2\xi(z)$, $\sigma_d = 0.6$ S/m $\xi(z)$, where $\xi(z) = 5.5 + 9z/c$, $-c/2 \leq z \leq c/2$, against the incident angle θ_i

$\phi_i = -90^\circ$, E_0 is in the plane $x = 0$, $f = 9.5$ GHz

radar (backscattering) cross-section S_{rad}/λ^2 against the incident angle θ_i ($\phi_i = -90^\circ$, E_0 is in the plane $x = 0$). The rod was divided into $3 \times 3 \times 24 = 216$ small cubes (648 complex unknowns) and a total of 25 iterations was adopted. Asymmetry of the results with respect to $\theta_i = 90^\circ$ was observed, which is a consequence of the inhomogeneity of the dielectric along the rod axis.

A note on the convergence of the procedure is in order here. In all the described examples, rapid convergence was obtained with increasing number of iterations. However, it was found that the larger ϵ_r and σ_d and the larger the number of unknowns $3N$, the slower the convergence, i.e. the larger n must be to provide a stable solution. It is a consequence of the system of equations becoming progressively more ill-conditioned, i.e. the condition number becoming progressively larger than one, with increasing ϵ_r , σ_d and N . This results in progressively slower convergence of iterative solutions of the system of equations, i.e. possible instability and inaccurate results in the case of direct solutions. In our case, this problem can probably be made less pronounced by more accurate evaluation of all the coefficients of the PPP method iteration matrix.

3.5 Wire dipole antenna in the presence of an imperfectly conducting body

As an example of the combined method for the analysis of wire antennas in the presence of dielectric bodies, consider a symmetrical centre-fed dipole antenna parallel to a dielectric parallelepiped, as sketched in Fig. 7.

Fig. 8 shows the difference of the impedance of a half-wave dipole in the presence of a lossy-dielectric parallelepiped [$\epsilon_r \approx (71 - j132)\epsilon_0$] and the impedance of the dipole when isolated, $Z_d = Z_a - Z_0$, which is a measure of the influence of the dielectric body on the dipole, against the spacing y_0 between the dipole and the body.

The body was approximated by $1 \times 4 \times 16 = 64$ small cubes (resulting in 192 complex unknowns), and a total of 20 iterations was adopted. The results obtained by the PPP method were compared with those obtained by the

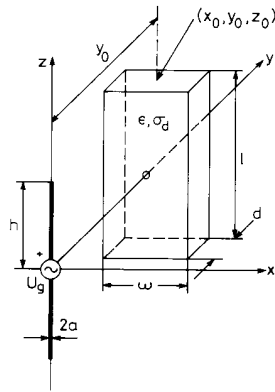


Fig. 7 Wire dipole antenna in the presence of an imperfectly conducting body
Location of body relative to antenna is specified by the reference point (x_0, y_0, z_0)

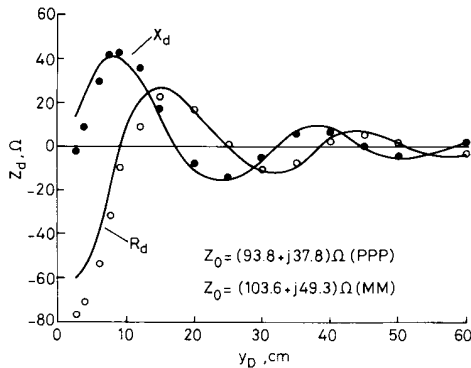


Fig. 8 Difference of the impedance of a half-wave dipole in the presence of a lossy dielectric parallelepiped, shown in Fig. 7, and the impedance of the dipole when isolated, $Z_d = Z_a - Z_0$, against the spacing y_0 between the dipole and the body

$f = 600$ MHz, $h = 12.5$ cm $= \lambda/4$, $a = 0.3125$ cm, $w = 6.25$ cm, $d = 1.5625$ cm, $l = 25$ cm, $\epsilon_r = 71$, $\sigma_d = 4.4$ S/m, $x_0 = 0$, $z_0 = 12.5$ cm
○ PPP method
● R_d , method of moments [12]
● X_d , method of moments [12]

method of moments [12]. Satisfactory agreement of the two sets of results is observed. Therefore it is reasonable to state that the combined PPP method is accurate and efficient in such and similar cases.

Assume now that the generator frequency f is 50 MHz and that the parallelepiped sketched in Fig. 7 is made of a conductor of permittivity ϵ_0 and conductivity $\sigma = 2.777$ S/m ($\epsilon_r = (1 - j1000)\epsilon_0$). Fig. 9 shows the half-wave dipole impedance Z_a against the number of iterations n . Stable results are obtained if n is as small as about 15.

3.6 Square-loop wire antenna in the presence of a dielectric body

As another example of the application of the combined PPP method, consider a square loop antenna encircling a homogeneous perfect-dielectric parallelepiped, as sketch-

ed in Fig. 10. Fig. 11 shows the loop-antenna impedance when a dielectric parallelepiped is present, Z_a , against ϵ_r . All the results were obtained in a single iterative process, i.e. the electric susceptibility was increased from zero to

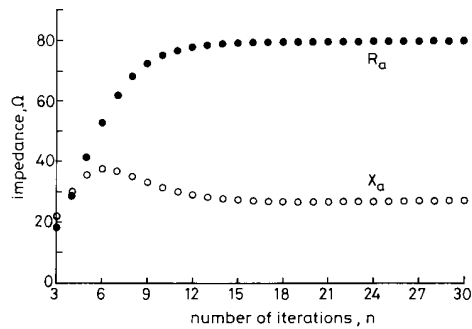


Fig. 9 Impedance of a dipole antenna in the presence of a conducting cylinder, shown in Fig. 7 ($f = 50$ MHz, $h = 150$ cm $= \lambda/4$, $a = h/100$, $w = d = 40$ cm, $l = 140$ cm, $\epsilon_{re} = 1 - j1000$, $x_0 = 0$, $y_0 = 40$ cm, $z_0 = 70$ cm) against the number of iterations n

$N = 2 \times 2 \times 7 = 28$ small cubes
○ X_a
● R_a

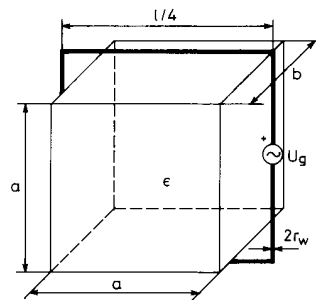


Fig. 10 Square-loop wire antenna encircling a dielectric parallelepiped
The centre of the parallelepiped coincides with the centre of the loop

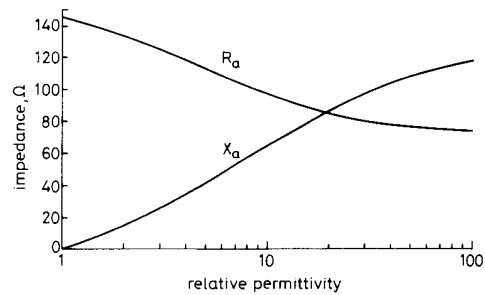


Fig. 11 Impedance of the loop antenna shown in Fig. 10 ($f = 300$ MHz ($\lambda = 1$ m), $l = 116.32$ cm, $r_w = 0.5$ cm, $a = 20$ cm, $b = 10$ cm) against the parallelepiped relative permittivity ϵ_r ($\sigma_d = 0$ S/m)
 $N = 6 \times 6 \times 3 = 108$ small cubes (324 complex unknowns); the loop, if isolated ($\epsilon_r = 1$), is resonant at 300 MHz

99, in the beginning with small, and then in progressively larger, increments. At certain characteristic points, few iterations were performed without increasing the susceptibility, to increase the accuracy of the solution, and the faster PPP procedure was used. In this way, the complete result was obtained much faster than by means of the corresponding form of the method of moments (in which

case the complete antenna-body system of equations would have to be solved many times).

4 Conclusions

The paper presents the method of gradual increase of the values of medium parameters, the PPP method, for the analysis of electromagnetic fields in inhomogeneous media. The method is of the iterative/perturbation type.

In summary, the following are advantages and deficiencies of the PPP method compared with other available methods for solving electromagnetic field problems in inhomogeneous media:

1. The PPP method is a general procedure for solving electromagnetic field problems. It enables, in principle, any problem to be solved. It could be regarded as an alternative to the method of moments in many areas of numerical electromagnetics, and is of remarkable conceptual and computational simplicity.

2. The PPP method can be interpreted as an approximate mathematical simulation of physical processes. This type of approach is not characteristic of any other method as far as we know. It enables solutions to be reduced to the application of first principles. The PPP method performs a construction of the solution, whereas other methods reconstruct the solution.

3. The PPP method is conceptually simpler than any available method of comparable generality, enabling very simple programming. The algorithm consists of a sequence of computations of the fields of known sources in a homogeneous medium.

4. The PPP method can be formulated as a faster and a slower version. The faster version has memory storage requirements comparable to those for the standard moment method solution, but is on average at least an order of magnitude more rapid than the latter. The slower version has almost no memory storage requirements, but the computing time is increased greatly, although in many cases it is still less than that required by the standard method of moments. (The possibility of obtaining the solution with very small memory requirements is typical for most iterative methods.)

5. The PPP method is particularly efficient if the analysis of a structure is required for a wide range of the values of medium parameters. The authors are aware of only one method having this property [13], relating to scattering of transverse magnetic waves on a homogeneous dielectric cylinder.

6. Due to its perturbation/iterative character, the PPP method can be easily and efficiently combined with any of the existing methods for the analysis of electromagnetic structures in homogeneous media. It is believed that this is also a unique feature of the method.

7. The basic deficiency of the PPP method is the sensitivity of the process in the case of a large number of unknowns and large values of the medium parameters. This, however, is typical to a greater or lesser extent, not only for all available iterative methods, but also for all known direct methods.

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