p-Refinement for Large-Domain Waveguide Structures Analyzed by FEM-MM Technique

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***Abstract*—Coupling of a mode matching (MM) method with a higher order large-domain finite element method (FEM) is presented in analysis of three-dimensional waveguide structures with discontinuities. The new FEM-MM technique enables very effective higher order hexahedral meshes constructed from a very small number of large curved conformal finite elements (large domains) and *p*-refined high-order field expansions. This results in solutions with minimal total numbers of unknowns.**

I. INTRODUCTION

The finite element method (FEM) provides an extremely versatile and accurate numerical solution to a large variety of electromagnetic problems. However, computational efficiency of FEM implementations remains to be a great challenge and an area of potential advancement. On one side, in order to substantially reduce the number of unknowns in FEM modeling, higher order large-domain techniques have been developed [1]-[3]. On another side, the need to solve complex and large circuit and waveguide structures has imposed the necessity for coupling of the FEM with other methods, including the mode matching (MM) technique [3], [4]. The FEM-MM hybridization enables computation of the generalized scattering matrix (GSM) of each component in the system, making the analysis much simpler and more efficient. For example, the advantage of GSM computing has been included in commercial software tools such as HFSS.

This paper presents coupling of the MM method with the higher order large-domain FEM, resulting in a very efficient and accurate FEM-MM technique, especially well suited for *p*-refinement. The technique is applied to the analysis of 3-D waveguide structures with discontinuities.

II. WAVEGUIDE SEGMENTATION AND GSM COMPUTATION

Consider a 3-D *N*-port waveguide structure with arbitrarily shaped metallic and/or dielectric discontinuities in Fig. 1. Fictitious planar surfaces ($S_{\mathrm{Port}1}$, …, $S_{\mathrm{Port}N}$) are introduced at waveguide ports to truncate the domain of computation [3]. The structure is then subdivided into waveguide subdomains introducing the same kind of fictitious surfaces between them (labeled $S_{\mathrm{Port}N+1}$, …, $S_{\mathrm{Port}P}$ in Fig. 1). Each subdomain is enclosed by PEC waveguide walls and fictitious surfaces enabling analysis by the FEM.

A standard Galerkin-type weak form of the electric field vector (**E**) wave equation yields [1], [3]:

$$\int_V \mu_r^{-1}(\nabla\times\mathbf{f})\cdot(\nabla\times\mathbf{E})\,\mathrm{d}V - k_0^2\int_V \varepsilon_r\mathbf{f}\cdot\mathbf{E}\,\mathrm{d}V = \mathrm{j}k_0Z_0\oint_S \mathbf{f}\cdot\mathbf{n}\times\mathbf{H}\,\mathrm{d}S \qquad (1)$$

where V is the volume and S is the surface of the subdomain, k_0 and Z_0 are the free-space wave number and intrinsic impedance, respectively, while **f** stands for testing/basis functions [1], [3] , **n** is the outward unit normal, and ε_r and μ_r are parameters of the dielectric inside the waveguide. The closed structure of one subdomain is tessellated using generalized Lagrange-type curved parametric hexahedra of higher geometrical orders, and the electric fields inside each of the hexahedra are expanded in terms of higher order curl-conforming hierarchical vector basis functions [1], [3].

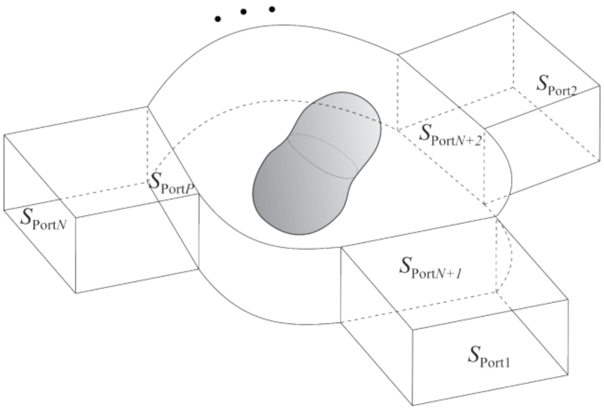

Fig. 1. *N*-port waveguide structure divided into subdomains with labeled ports.

In order to introduce the proper boundary condition at the ports of subdomains, a 3-D FEM – modal expansion method [3], [4] is invoked: the tangential electric and magnetic fields are expanded as linear combinations of the incoming and outgoing waveguide modes:

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$$\mathbf{E}_\mathrm{t}=\sum_{m=1}^{N_\mathrm{m}}(a_m+b_m)\mathbf{e}_m,\, \mathbf{H}_\mathrm{t}=\sum_{m=1}^{N_\mathrm{m}}(b_m-a_m)\mathbf{h}_m\,, \qquad (2)$$

where $\mathbf{e}_m$ and $\mathbf{h}_m$ represent the transversal electric and magnetic field components of the m-th mode on the given subdomain port, while a_m and b_m stand for the amplitudes of the incident and reflected waves, respectively. N_m represents the number of modes at the given surface. The modal forms at the subdomain ports are computed by the 2-D higher order eigenvalue FEM technique for waveguide cross-sections of arbitrary shapes [2].

In order to compute $\{a\}$ and $\{b\}$ coefficients for one subdomain, the 3-D electric-field expansion in [3] and 2-D magnetic-field expansion from (2) are substituted in (1) and the continuity of tangential electric fields over the subdomain port surfaces is constrained, which yields the following two matrix systems of equations, respectively:

$$\begin{aligned}&([A]-k_0^2[B])\{\alpha\}=[P]\{a\}-[P]\{b\}\\&[C]\{\alpha\}=[D]\{a\}+[D]\{b\}\end{aligned}\,, \qquad (3)$$

where α represent the unknown coefficient in the FEM electric field expansion, while $[A]$, $[B]$, $[C]$, $[D]$, and $[P]$ matrices are given in [4]. Matrices needed for the above analysis are computed independently of frequency and stored, to be recalled afterwards, and $[GSM]$ ($\{b\}=[GSM]\{a\}$) is found for multiple frequencies after eliminating α from (3).

Once $[GSM]$ matrices for all subdomains are computed, the $[GSM]$ matrix of the whole structure can be obtained by equating appropriate coefficients of the incoming and outgoing waves at each of the fictitious ports for the two subdomains that share the port.

III. NUMERICAL RESULTS AND DISCUSSION

One of the very useful properties of the FEM-MM method is that the waveguide ports can be positioned very close to the waveguide discontinuity region, lowering down the number of FEM unknowns. Using modal expansion at the ports, all necessary modes can be taken into account, which yields an accurate numerical solution. The results shown here are obtained for a WR90 ($a=22.86\,\mathrm{mm}$, $b=10.16\,\mathrm{mm}$) waveguide of length $l=4\,\mathrm{mm}$ loaded with a cylindrical metallic post of radius $r=1\,\mathrm{mm}$ positioned at the center of the structure (Fig. 2). The waveguide ports are indicated in the figure. The waveguide is excited by the dominant mode, and the S_{21} is computed for the GSM matrix including different TE modes. An error is computed as the absolute difference between $|S_{21}|$ results in dB obtained by the FEM-MM technique and the HFSS reference solution. HFSS is applied in the dominant-mode analysis of the longer model, $l=20\,\mathrm{mm}$. Fig. 3 shows the average error over the frequency range 8–12.4 GHz as a function of the number of unknowns N along long edges in the model as shown in Fig. 2. An excellent error behavior with p-refinement (increasing N) can be observed.

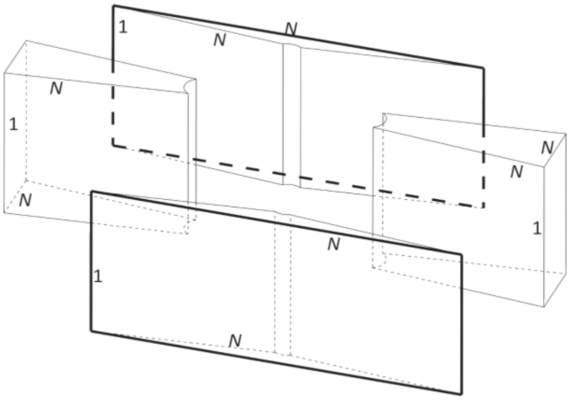

Fig. 2. Higher order large-domain FEM-MM model of a WR90 waveguide with a centrally positioned cylindrical metallic post, constructed from only four second-geometrical order finite elements (field-approximation orders in different directions are also shown).

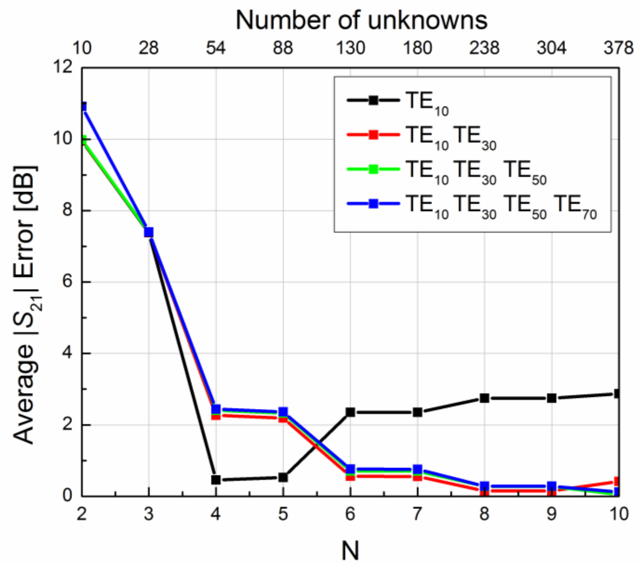

Fig. 3. S_{21} error for the model in Fig. 2 with p-refinement.

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