Accurate and Efficient Curvilinear Geometrical Modeling Using Interpolation Parametric Elements in Higher Order CEM Techniques

Branislav M. Notaroš [1], Milan M. Ilić [1,2], Slobodan V. Savić [2], Nada J. Šekeljić [1], and Anđelija Ž. Ilić [3]

[1] Colorado State University, Department of Electrical and Computer Engineering, Fort Collins, CO 80523-1373 USA, notaros@colostate.edu, inadasek@engr.colostate.edu

[2] University of Belgrade, School of Electrical Engineering, 11120 Belgrade, Serbia, milanilic@etf.rs, ssavic@etf.rs

[3] Vinča Institute of Nuclear Sciences, Laboratory of Physics 010, 11001 Belgrade, Serbia, andjelijailic@ieee.org

Abstract: Accurate and efficient curvilinear geometrical modeling using Lagrange-type generalized interpolation parametric elements in higher order computational electromagnetic techniques is presented. Examples demonstrate enhanced accuracy and efficiency of the analysis when uniformly distributed Lagrange geometrical interpolation nodes on curved and large elements are combined with high-order (*p*-refined) basis functions for current modeling.

Key words: electromagnetic analysis, numerical techniques, higher order modeling, curved parametric elements, geometrical mapping, integral-equation techniques, scattering.

1. Introduction

Higher order (also known as large-domain or entire-domain) approach in computational electromagnetics (CEM) utilizes higher order basis functions for the approximation of currents and/or fields defined on large surface and/or volume geometrical elements (e.g., on the order of a wavelength in each dimension) [1]-[4]. This enables considerable reductions in the number of unknowns for a given problem, enhances the accuracy and efficiency of the CEM analysis, and results in faster (higher order) convergence of the solution, when compared to traditional low-order (also referred to small-domain or subdomain) CEM tools. In hand with higher order basis functions, novel CEM tools often employ curvilinear elements for geometrical modeling of general electromagnetic structures [5]-[11].

This paper presents accurate and efficient curvilinear geometrical modeling in higher order CEM using Lagrange-type generalized interpolation parametric quadrilaterals as basic boundary elements in the method of moments (MoM) analysis in conjunction with the surface integral equation (SIE) formulation for radiation and scattering [9] and associated parametric hexahedra for volumetric modeling based on the finite element method (FEM) [10]. In particular, the paper discusses optimal placement of interpolation nodes on high-order geometrical elements and other related issues of curvilinear geometrical modeling. It demonstrates accurate and efficient models using uniformly distributed Lagrange geometrical

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interpolation nodes on curved and large elements combined with high-order (p-refined) basis functions.

2. Curvilinear Geometrical Modeling Using Higher Order Interpolation Parametric Elements

As basic building blocks for surface geometrical modeling in MoM and hybrid techniques, we use Lagrange-type generalized curved parametric quadrilaterals of arbitrary geometrical orders K_u and K_v (K_u, $K_v \geq 1$), shown in Fig. 1 and analytically described as [9]

$$\mathbf{r}(u,v) = \sum_{k=0}^{K_u}\sum_{l=0}^{K_v} \mathbf{r}_{kl} L_k^{K_u}(u) L_l^{K_v}(v), \quad -1 \leq u,v \leq 1, \quad L_k^{K_u}(u) = \prod_{\substack{i=0 \\ i \neq k}}^{K_u} \frac{u - u_i}{u_k - u_i}, \tag{1}$$

where $\mathbf{r}_{kl} = \mathbf{r}(u_k, v_l)$ are position vectors of interpolation nodes, $L_k^{K_u}$ represent Lagrange interpolation polynomials, and u_i are the interpolation nodes along an interval $-1 \leq u \leq 1$, and similarly for $L_l^{K_v}(v)$. Electric and magnetic surface current density vectors, $\mathbf{J}_s$ and $\mathbf{M}_s$, over every generalized quadrilateral in the model are approximated by means of divergence-conforming hierarchical-type polynomial vector basis functions in parametric coordinates u and v, with arbitrary current-expansion orders N_u and N_v (N_u, $N_v \geq 1$) [9], which are entirely independent from the element geometrical orders (K_u and K_v).

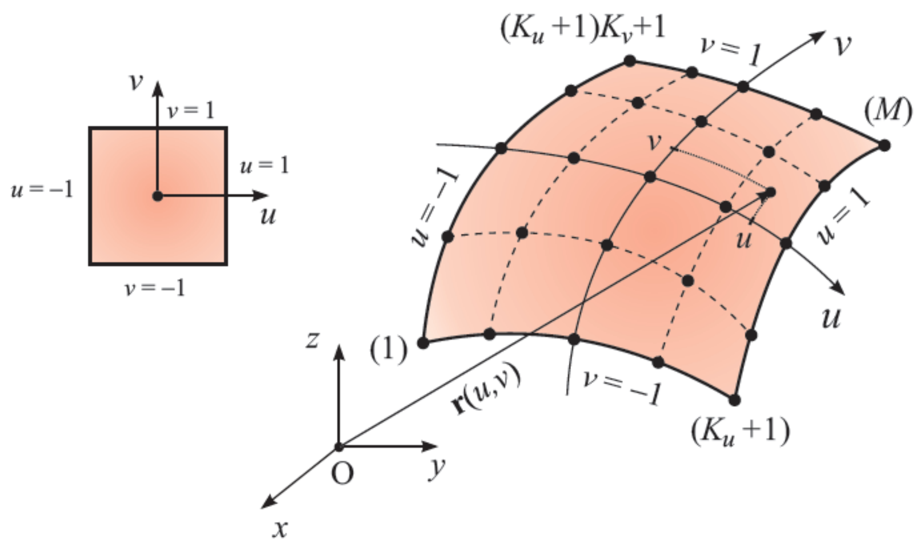

Fig. 1. Generalized curved parametric quadrilateral defined by (1); square parent domain is also shown.

In FEM and hybrid techniques, volumetric geometrical modeling of inhomogeneous materials is carried out using Lagrange-type interpolation generalized hexahedra, which are a volume (3-D) generalization of the quadrilateral patch in Fig. 1 [10], [11]. The electric field vector inside the FEM hexahedra is approximated by curl-conforming hierarchical vector expansions [10].

3. Numerical Results and Discussion

As the first example, consider a dielectric spherical scatterer of relative permittivity $\varepsilon_r = 4$ and relative permeability $\mu_r = 1$ in free space. Its surface is modeled by means of six Lagrange quadrilateral patches (Fig. 1) of the fourth geometrical order ($K_u = K_v = 4$), and with basis functions of orders $N_u = N_v = 6$ for vectors $\mathbf{J}_s$ and $\mathbf{M}_s$ on all patches. Shown in Fig. 2 is the analytically obtained (Mie's series) normalized monostatic radar cross section (RCS) of the scatterer, vs. a/λ, where a is the sphere radius and λ stands for the wavelength in the dielectric, and the computed results obtained using two models with different placements of interpolation nodes on Lagrange patches. In the first model, the interpolation nodes are obtained by mapping the uniformly distributed points of a square parent domain to one-sixth of a sphere within the cube-to-sphere mapping, that is, by projecting the square onto the spherical patch, with the common center (center of the sphere) as the projection center. Such a direct mapping actually results in a nonuniform distribution of interpolation nodes on the spherical patch, namely, it introduces significant distortion in the projected parametric space in a sense that the distances between the mapped points are quite different, whereas these distances are the same in the parent domain. The second model is created by mapping the unit parent square to one-sixth of the sphere in such a way to keep the points, that are uniformly distributed on the square, also uniformly distributed on the spherical patch, i.e., by positioning the Lagrange interpolation nodes in a uniform fashion. From the figure, we observe that the model with uniform nodes performs much better than the other model; namely, it gives accurate results in the whole range of frequencies considered, while the model with direct mapping results in noticeable errors even starting from relatively low frequencies and performs poorly at higher frequencies.

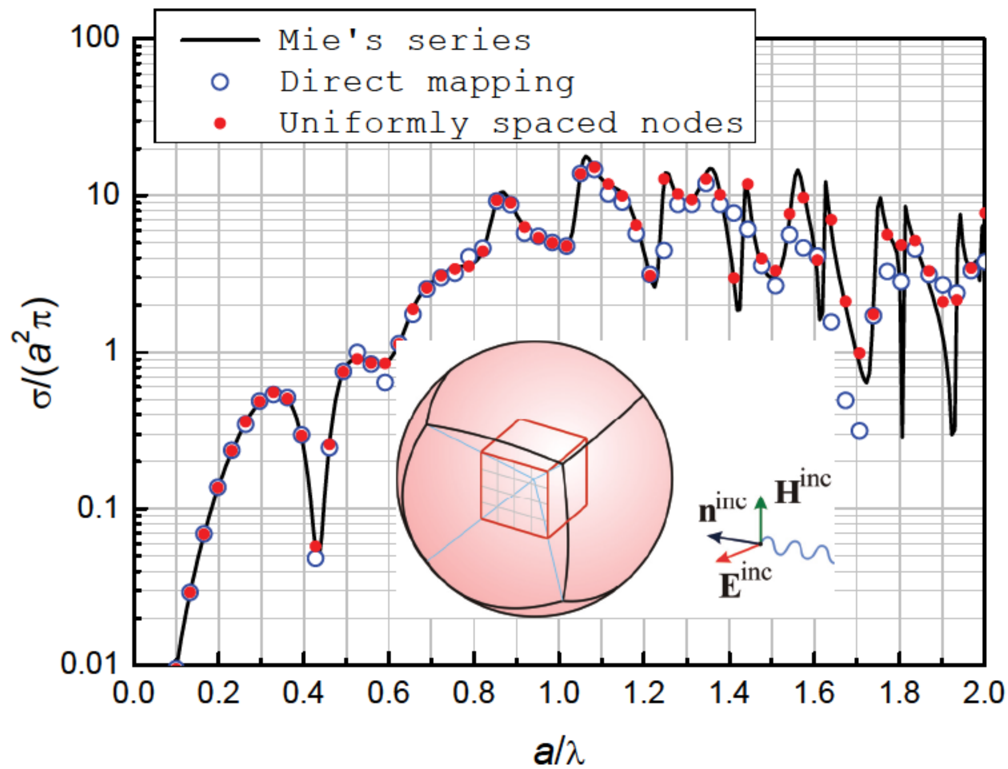

Fig. 2. Normalized monostatic RCS of a spherical dielectric ($\varepsilon_r = 4$) scatterer (λ is the wavelength in the dielectric) computed by the higher order MoM-SIE technique using two geometrical models with different placements of interpolation nodes on Lagrange patches and by Mie's series (exact solution); mesh obtained by the cube-to-sphere mapping with uniformly spaced nodes is shown in the figure inset.

Accuracy improvement in the model with uniform nodes in the previous example is partially due to (i) more accurate geometrical representation of the spherical surface and (ii) better parametrization. The sole influence of the parametrization on the solution accuracy will generally vary with offset of the control points, frequency, adopted polynomial current approximation orders, integration accuracy, and on the particular model being analyzed (e.g., if the edge effects are more or less pronounced), and hence it requires a thorough investigation. However, as an initial attempt to identify the sole influence of the parametrization on the solution accuracy, we consider here, as the second example, a simple metallic square plate scatterer of side length $a = 1\,\mathrm{m}$ illuminated by a plane wave with $E = 1\,\mathrm{V/m}$ parallel to a square edge (v-axis in the parent domain and x-axis in the child domain). In Fig. 3, we compare the RCS solutions and the surface current distributions (dominant, v-components) for the model with uniform nodes (simple bilinear quadrilateral, with $K_u = K_v = 1$) and a model with a fourth-order quadrilateral ($K_u = K_v = 4$) for which we offset the interpolation nodes in planes $v = \pm 0.5$ by $\pm 0.1\,\mathrm{m}$, as shown in the bottom inset in Fig. 3. A significant difference in the current distributions can be observed, as well as a significantly more accurate RCS solution of the model with uniform nodes (when compared to a highly accurate *hp*-refined reference solution obtained by WIPL-D). (Note also the difference in the scales in the current distribution plots, which is by an order of magnitude, arising primarily due to pronounced edge effects in this example.)

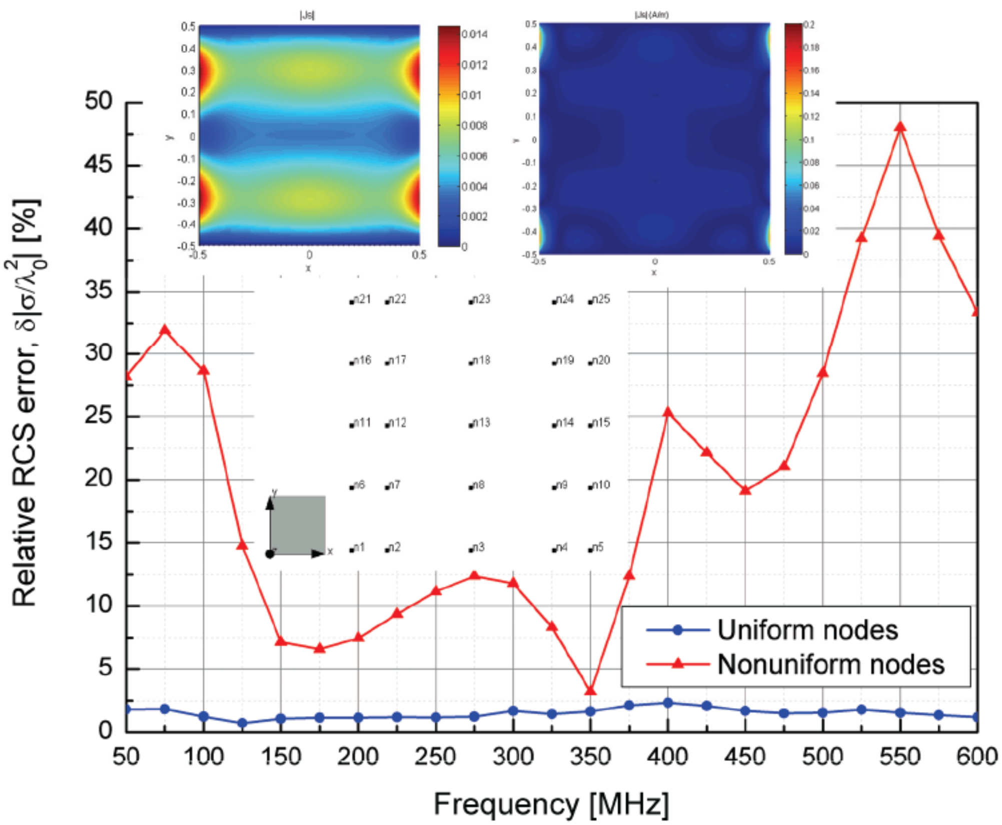

Fig. 3. Comparisons of the relative RCS errors and surface current distributions for a metallic square plate scatterer obtained by higher order MoM-SIE models with uniform and nonuniform distributions of geometrical interpolation nodes.

As the last example, we perform a higher order MoM-SIE RCS analysis of the NASA almond, a benchmark target established by the Electromagnetic Code Consortium (EMCC), at a frequency of $f = 7$ GHz [12]. The model of the almond, shown in Fig. 4, is built (based on geometrical equations from [12]) using 56 quadrilateral curved elements with $K_u = K_v = 2$ and ensuring a nearly uniform distribution of interpolation nodes (there are a total of 226 nodes). We observe in Fig. 5 an excellent convergence of higher order MoM-SIE results with p-refinement, namely, with increasing the orders of basis functions from $N_u = N_v = 2$ to $N_u = N_v = 7$, while keeping the orders of Gauss-Legendre integration formulas in computations of generalized Galerkin impedances and potential and field integrals to be $NGL_u = NGL_v = N_u + 2 = N_v + 2$ in all cases, as well as an excellent agreement of results for N_u , $N_v \geq 4$ with measurements.

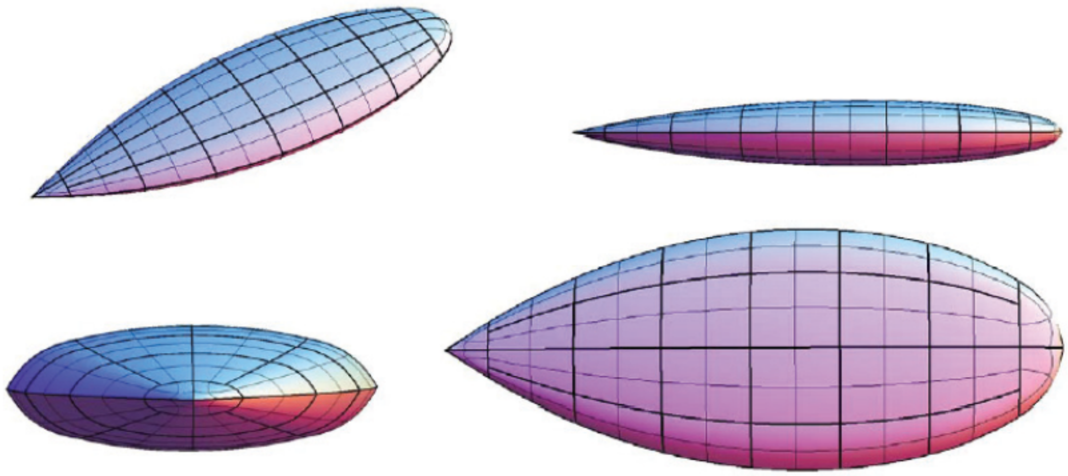

Fig. 4. Higher order geometrical model of the NASA metallic almond [12] using 56 curved quadrilateral elements with a nearly uniform distribution of interpolation nodes.

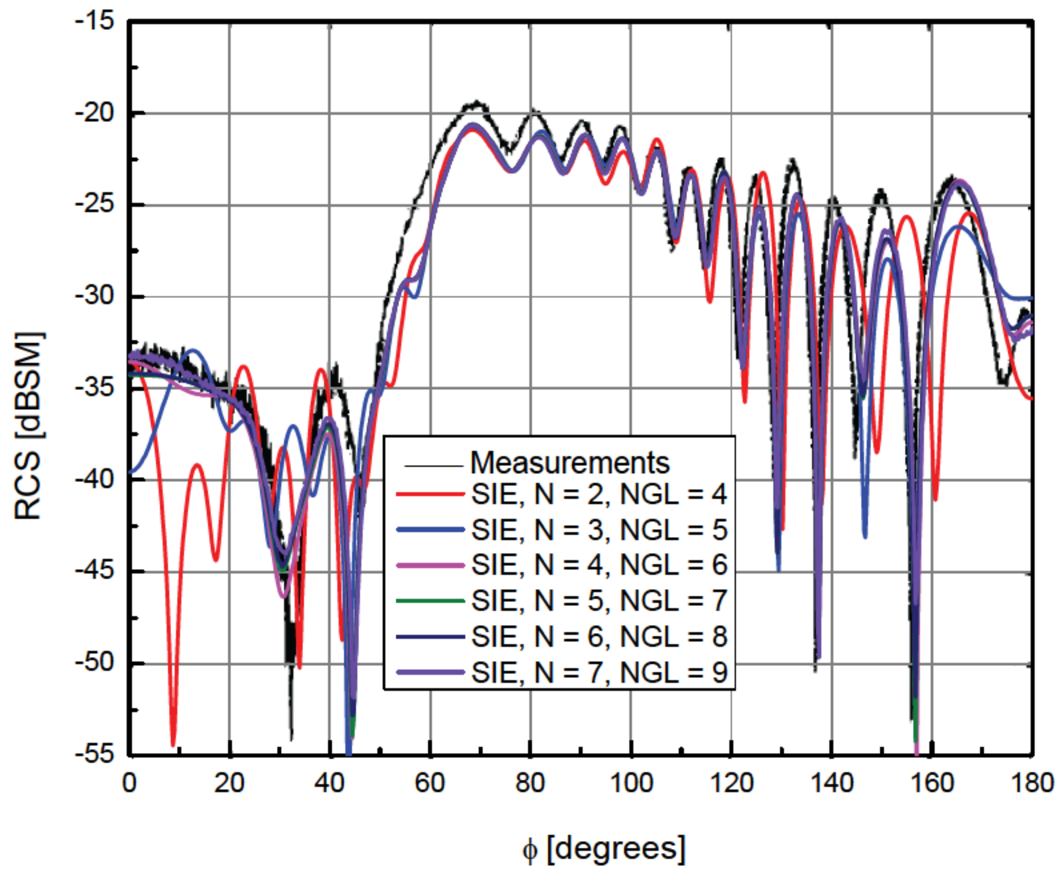

Fig. 5. Higher order MoM-SIE RCS analysis of the NASA almond at $f = 7$ GHz for the HH polarization using the geometrical model in Fig. 4: *p*-refinement and comparison with experimental results [12].

5. Conclusions

This paper has presented accurate and efficient geometrical models for higher order MoM-SIE analysis using uniformly distributed Lagrange geometrical interpolation nodes on curved and large elements combined with high-order (*p*-refined) basis functions for the approximation of electric and magnetic surface currents. Examples have included RCS analysis of a spherical dielectric scatterer using two geometrical models with different placements of interpolation nodes on Lagrange patches, evaluation of the RCS and current distributions for a metallic square plate scatterer obtained by higher order models with uniform and nonunform distributions of geometrical interpolation nodes, and RCS analysis of a higher order model of the NASA metallic almond using curved quadrilateral elements with a nearly uniform distribution of geometrical interpolation nodes. Examples at the conference will also include higher order FEM and hybrid models.

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