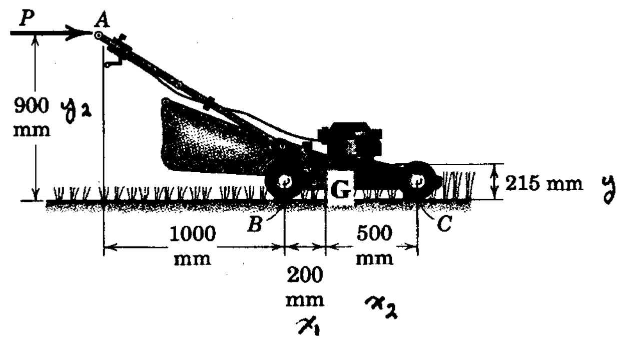
Name MASTER

CE 261 DYNAMICS Spring 1999
MIDTERM 3: CHAPTERS 6.5-6, 7 & 8
Monday, May 10, 7:00 am -9:00 am

SHOW ALL WORK   OPEN BOOK, CLOSED HOMEWORK OR SOLUTION MANUALS
ONE PAGE NOTES PERMITTED, NO CREDIT FOR ANSWER ONLY

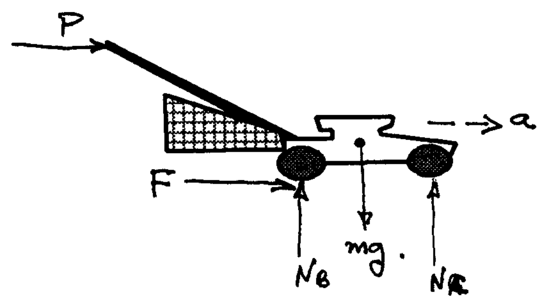
(6/2)

1. The rear-wheel drive lawn mower, when placed into gear while at rest, is observed to momentarily spin its rear tires as it accelerates. The coefficients of friction between the rear tires and the ground are $\mu_s = 0.7$ and $\mu_k = 0.5$. The operator pushes the mower with a force on the handle of P = 100 Newton. The mower wheels do not leave the ground. The mass of the mower and attached bag is 50 kg with center of mass at G.



2.

A. (5 points) Draw a free body diagram of mower.



B. (10 points) What are the upward forces N_B and N_C on the mower wheels?

$$\Sigma F_y = 0 \Rightarrow N_C + N_B - mg = 0$$

$$\Sigma M_G = 0 \Rightarrow -P(y_2 - y_1) - N_B x_1 + N_C x_2 + F y_1 = 0$$

but $F = \mu_k N_B$ ∴

$$(1) \quad N_C + N_B = 50g = 490.8$$

$$(2) \quad 0.5 N_C + \underbrace{(0.5(0.215) - 0.2)}_{-0.093} N_B = 100(.9 - .215) = 68.5$$

$$\text{so } -0.5 \times (1) + (2) = 0 \quad -0.593 N_B = -\frac{490.8 + 68.5}{2} =$$

$$N_B = \boxed{298.3 N}$$

$$N_C = 490.8 - 298.3 = \boxed{192.5 N}$$

C. (10 points) What is the forward acceleration of the mower, a.

$$\Sigma F_x = m \bar{a}_x$$

$$P + F = ma$$

$$a = \frac{P + \mu_k N_B}{m} = \frac{100 + (0.5)(298.3)}{50}$$

$$= \boxed{2.983 \ m/sec}$$

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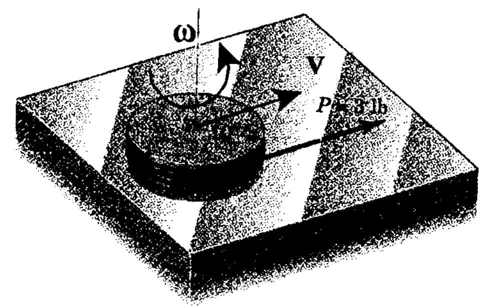
2. The 10 inch radius, 64.4 lb solid circular disk is initially at rest on the smooth horizontal surface when a 3 lb force P, constant in magnitude and direction, is applied to the cord wrapped securely around its periphery. Friction between the disk and the surface is negligible.



(6/74)

A. (10 points) Calculate the angular velocity, ω , of the disk after the 3 lb force has been applied for 2 seconds

$\Sigma F_x = ma_x$

$\Sigma M_G = \bar{I}\alpha \Rightarrow \int_0^t M_G\,dt = I\omega_f - 0$

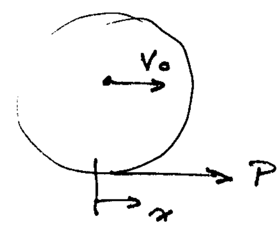
$\omega_f = \frac{\int_0^t Pr\,dt}{I} = \frac{Prt}{I} = \frac{3(\frac{10}{12})(2)}{\frac{1}{2}(\frac{64.4}{32.2})(\frac{10}{12})^2} = \frac{6(12)}{10} =$

$= \boxed{7.2 \text{ rad/sec}}$

B. (10 points) What is the ratio of the velocity speed of the cord removal to the right to the velocity of the disk to the right?

$\Sigma F_x = ma_x \Rightarrow \int_0^t P\,dt = mV_x - 0$

$V_x = \frac{Pt}{m} = \frac{3(2)}{(64.4/32.2)} = 3$



$\frac{V_{C/D}}{V_x} = \frac{r\omega}{V_x} = \frac{\frac{10}{12}(7.2)}{3} = \boxed{2}$

or $\frac{V_C}{V_D} = \frac{V_D}{V_D} + \frac{V_{C/D}}{V_D} = 1 + 2 = \boxed{3}$

C. (10 points) Calculate the linear velocity V of the center of the disk after it has moved 3 feet from rest.

$a_x = \frac{P}{m} = \frac{3}{2}$

$V^2 = 2a_x X = 2(\frac{3}{2})(3)$

$V = \boxed{3 \text{ ft/sec}}$

D. (5 points) What happens if after moving 3 feet the cord breaks? Describe the motions of the disk?

Disk continues to move but without further acceleration

$\therefore$ $V_x = 3$ ft/sec.

$\omega = \frac{V_c}{r} = \frac{2V_x}{r} = \frac{2(3)}{10/12} = 3$ ft/sec.

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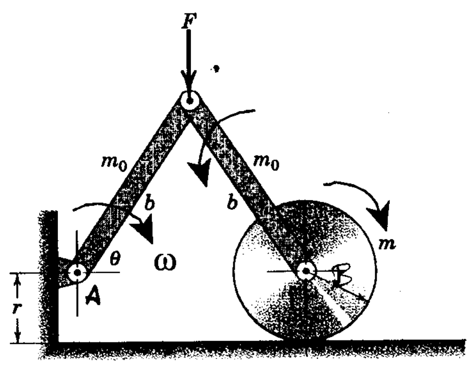
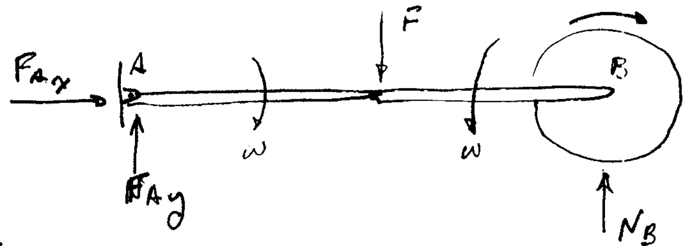
(6/120) 3. A constant force F = 100 N is applied in the vertical direction to the symmetrical linkage starting from the rest position shown, where $\theta = 45°$. Each link b = 1 m and has a mass m_o = 1 kg. The wheel is a solid circular disk of mass m = 2 kg and radius r = 0.5 m and rolls on the horizontal surface without slipping.



A. (5 points) What is the velocity of the center of the disk and its angular velocity when $\theta = 0°$? (Hint sketch the system in this position)

$$\boxed{V_B = 0}$$

$$\omega_{disk} = \frac{V_B}{r} = \boxed{0}$$



B. (5 points) What is the horizontal force of the pin at the wall on the left link when $\theta = 0°$?

$A_x \neq 0$

See attached sheet

C. (10 points) Determine the angular velocity ω which the links acquire as the reach the position $\theta = 0°$.

$$Work + T_1 + V_{1g} = T_2 + V_{2g}.$$

$$F(b \sin\theta) + 2m_o g\left(\frac{b}{2}\sin\theta\right) = 2\left(\frac{1}{2} I_o \omega^2\right]$$

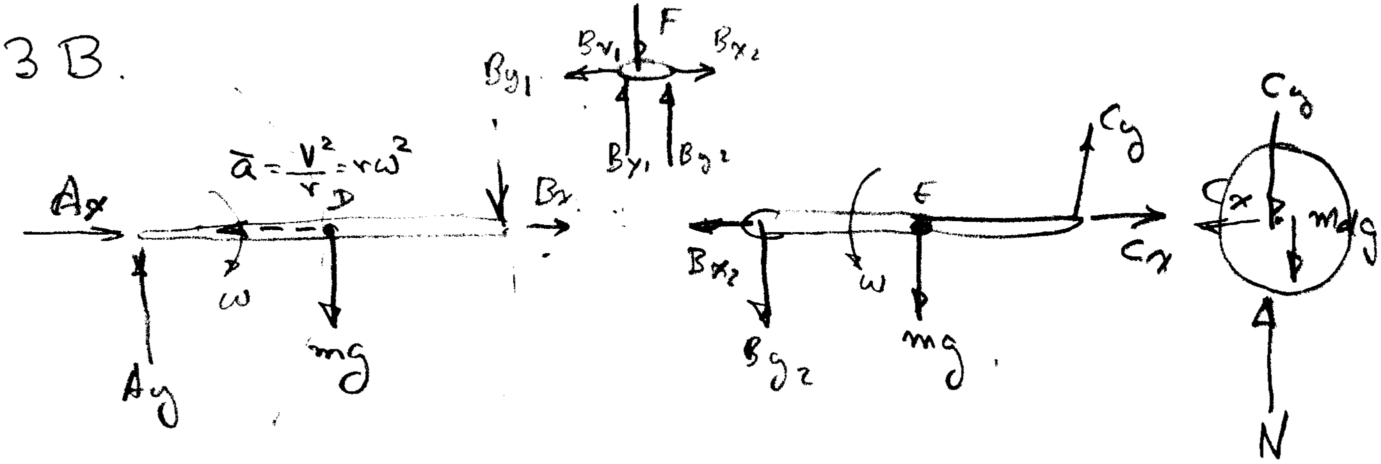
$$I_o = \frac{1}{3} m_o b^2$$

$$\omega = \sqrt{\frac{\left(F b + \frac{2}{2} m_o g b\right)\sin}{\frac{1}{3} m_o b^2}} = \sqrt{\frac{3\left(F + \frac{2}{2} m_o g\right)\sin\theta}{m_o b}}$$

$$= \sqrt{\frac{3\left(100 + \frac{2}{2}(1)(g)\right)\sin 45°}{(1)(1)}}$$

$$= \boxed{15.96 \text{ rad/sec.}}$$

PROB 3B.



$V_A = 0$

$a_A = 0$

$\vec{V}_D = \cancel{\vec{V}_A}^{0} + \vec{\omega}_1 \times \vec{r}_{AD} = -\omega \frac{l}{2} \vec{j}$

$\vec{V}_B = \vec{V}_A^{0} + \vec{\omega}_1 \times \vec{r}_{AB} = -\omega l \vec{j}$

$\vec{V}_E = \vec{V}_B + \vec{\omega}_2 \times \vec{r}_{BE} = -\omega l j + \omega \frac{l}{2} j = -\omega \frac{l}{2} \vec{j}$

$\vec{V}_C = \vec{V}_B + \vec{\omega}_2 \times \vec{r}_{BC} = -\omega l j + \omega l j = 0$

$B_{x_1} = B_{x_2}$

$\bar{a}_{Dx} = \cancel{a_{Ax}}^{0} + \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_{AD} = -\omega^2 \frac{l}{2} \vec{i}$

$\bar{a}_{Bx} = a_{Ax}^{0} + \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_{AB} = -\omega^2 l \vec{i}$

$a_{Ex} = a_{Bx} + \vec{\omega}_2 \times \vec{\omega}_2 \times \vec{r}_{BE} = -\omega^2 l \vec{i} - \omega^2 \frac{l}{2} = -\frac{3}{2}\omega^2 l \vec{i}$

$a_{Cx} = a_{Bx} + \vec{\omega}_2 \times \omega_B \times \vec{r}_{BC} = -\omega^2 l \vec{i} - \omega^2 l \vec{i} = -2\omega^2 l \vec{i}$

link$_{AB}$ $\underline{\Sigma F_x = m\bar{a}_x}$ $\quad A_x + B_x = -m\omega^2 \frac{l}{2}$

link BC $\underline{\Sigma F_x = m\bar{a}_x}$ $\quad -B_x + C_x = -m\frac{3}{2}\omega^2 l$

disk $\quad -C_x = -m_d(2\omega^2 l)$

$\therefore \quad B_x = m_d(2\omega^2 l) + m\frac{3}{2}\omega^2 l = (2m_d + \frac{3}{2}m)\omega^2 l$

$A_x = -m\omega^2 \frac{l}{2} - B_x = -m\omega^2 \frac{l}{2} - (2m_d + \frac{3}{2}m)\omega^2 l$

$= -(2m_d + \frac{5}{2}m)\omega^2 l$

$$\boxed{A_x = -\left(2m_d + \frac{5}{2}m\right)\omega^2 l}$$

$A_x \neq 0$

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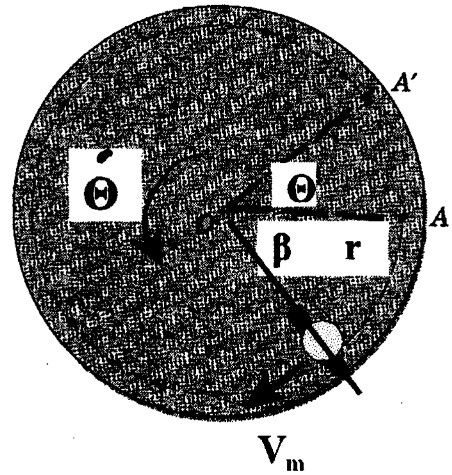
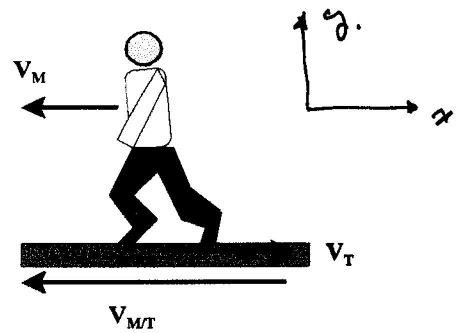
(6/190) 4. A man of mass m = 75 kg stands at point A on the horizontal turntable, initially at rest. The turntable is free to rotate about O with negligible friction and has a moment of inertia $I_o = 1000$ kg-m^2 about the center O. The man starts to walk clockwise in a circle of radius r = 4 m.



A. (5 points) What is relation between absolute velocity of man, V_m, absolute velocity of turntable under man, V_T, and velocity of man relative to turn table, $V_{M/T}$?

$$\boxed{\vec{V}_m = \vec{V}_T + \vec{V}_{M/T}}$$

B. (5 points) Express the relation above in terms of the angular changes of position (angular velocities) of man and turntable or $\dot{\Theta}$ and $\dot{\beta}$, respectively? (Be careful with signs)



$$V_m = -r\dot{\beta}$$
$$V_T = r\dot{\Theta}$$
$$-r\dot{\beta} = r\dot{\Theta} + V_{m/T}$$
$$\boxed{V_{M/T} = -r(\dot{\Theta} + \dot{\beta})}$$

C. (10 points) If the man walks with a speed of $V_{m/T} = -1$ m/s relative to the turntable, what is the corresponding angular velocity Θ of the turntable?

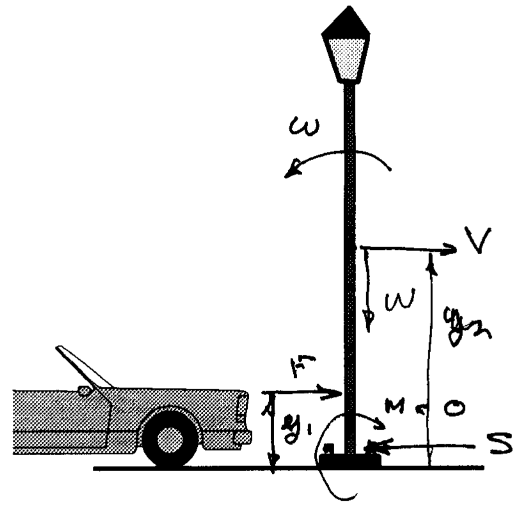
$$H_1 = H_2 \quad ; \quad H_1 = 0 \quad , \quad H_2 = I_o\dot{\Theta} + mrV_m$$
$$0 = I_o\dot{\Theta} + mrV_m = I_o\dot{\Theta} - mr(r\dot{\Theta} + V_{m/T})$$
$$\dot{\Theta} = \frac{-mrV_{m/T}}{I_o + mr^2} = \frac{-75(4)(-1)}{1000 + 75(4)^2} =$$
$$= \boxed{0.136 \text{ rad/sec.}}$$

D. (Xtra credit 10 points) Determine the angle Θ through which the turntable rotates until the man reaches his starting point in the new position A'. (Hint use results of A-C above and fact that $\Theta + \beta = 2\pi$.)

$$\dot{\Theta}(I_o + \cancel{mr^2}) = -mr(-r(\dot{\Theta} + \dot{\beta})) = \cancel{mr^2\dot{\Theta}} + mr^2\dot{\beta}$$
$$I_o\dot{\Theta} = mr^2\dot{\beta} \Rightarrow I_o\Theta = mr^2\beta$$
$$\beta = \frac{I_o}{mr^2}\Theta$$
but $$\Theta + \beta = \Theta + \frac{I_o}{mr^2}\Theta = 2\pi$$
$$\Theta = \frac{2\pi}{1 + I_o/mr^2} = \frac{2\pi}{1 + \frac{1000}{75(4)^2}} = \boxed{\begin{array}{c}3.427 \text{ rad} \\ \text{or} \quad 196^\circ\end{array}}$$

Name $\underline{\text{MASTER}}$

5. Engineers desire tp design a streetlight pole to shear off at ground level when struck by a vehicle. They desire to estimate the force S required to shear off the ground support bolts. From videotapes of a test impact they know the angular velocity of the pole is ω = 0.74 rad/sec and the horizontal velocity of its centerpoint is V = 22 ft/sec after impact, whose duration is Δt = 0.01 sec. The pole can be modeled as a 20-ft, 140 lb pole. The car impacts the pole 2 ft above the ground, the couple exerted on the pole can be neglected.



A. (5 points) Draw a free body diagram on the sketch to right of the pole including impact force, shear force, weight, and indicate directions of motion, V and ω.

A. (5 points) From the principle of linear impulse and momentum write an equation including the shear force S.

$$\int \Sigma F dt = mV_2 - mV_1$$

$$(F-S)\Delta t = mV$$

$$S = F - \frac{mV}{\Delta t} = F - \frac{(140)(22)}{(32.2)(.01)} = F - 9656$$

B. (5 points) From the principle of angular impulse and angular momentum write an equation for angular impulse about the pole base. (Be careful with your negative and positive sense of rotation of the angular rotations due to center velocity and pole rotation about its center

$$\int M_o dt = H_2 - H_1 = \left[\bar{I}\omega + (\bar{r} \times m\bar{V})\cdot\hat{k}\right]_1^2$$

$$-y_1 F\Delta t = \bar{I}\omega - y_2 mV$$

$$F = \frac{-\bar{I}\omega + y_2 mV}{y_1 \Delta t} = \frac{-\frac{1}{12}ml^2\omega + y_2 mV}{y_1 \Delta t} = \frac{-\frac{1}{12}\left(\frac{140}{32.2}\right)(20)^2(.74) + 10\left(\frac{140}{32.2}\right)(22)}{2(.01)} = 42{,}466$$

C. (5 points) Solve these equations for the average shear force S required.

$$S = \frac{y_2 mV - \bar{I}\omega}{y_1 \Delta t} - \frac{mV}{\Delta t}\left(\frac{y_1}{y_1}\right)$$

$$= \frac{(y_2 - y_1)mV - \bar{I}\omega}{y_1 \Delta t}$$

or

$$S = F - 9565 = 42{,}466 - 9565 = \boxed{32{,}901 \text{ lbf}}$$