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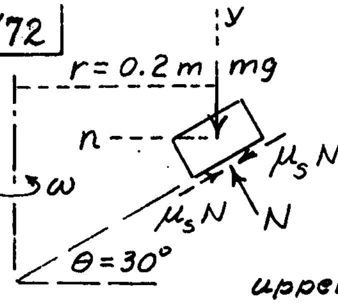

$\mu_s N$ is down for ω_{max}

" " up " ω_{min}

$\Sigma F_y = 0$; $N\cos\theta \mp \mu_s N \sin\theta = mg$

$\Sigma F_n = ma_n$; $N\sin\theta \pm \mu_s N\cos\theta = mr\omega^2$

upper sign for ω_{max}

lower sign for ω_{min}

Combine & get $\dfrac{\sin\theta \pm \mu_s \cos\theta}{\cos\theta \mp \mu_s \sin\theta} = \dfrac{r\omega^2}{g}$

$$\omega = \sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s \sin\theta}} = \sqrt{\frac{9.81}{0.2}}\sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

Upper sign $\omega_{max} = \underline{7.21 \text{ rad/s}}$

Lower sign $\omega_{min} = \underline{3.41 \text{ rad/s}}$

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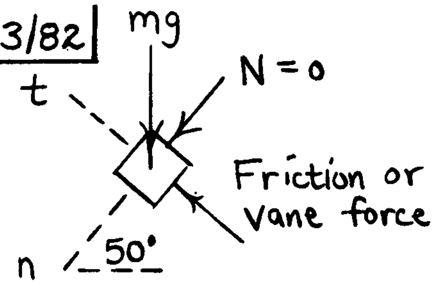

$\Sigma F_n = ma_n: \quad mg\sin 50° = mr\Omega^2$

$$\Omega = \sqrt{\frac{g\sin 50°}{r}} = \sqrt{\frac{9.81\sin 50°}{0.330}} = \underline{4.77 \text{ rad/s}}$$

(45.6 rev/min)

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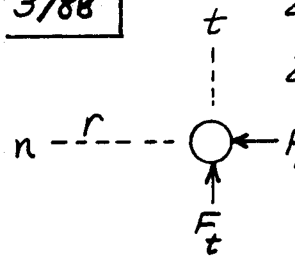

$\Sigma F_t = m a_t;\quad F_t = m r \alpha$

$\Sigma F_n = m a_n;\quad F_n = m r \omega^2$

$F_{max} = \mu_s N = \mu_s m g$ so

$$m r \sqrt{\alpha^2 + \omega^4} = \mu_s m g$$

& $\omega^2 = \frac{1}{r}\sqrt{\mu_s^2 g^2 - r^2\alpha^2}$ But for constant α,
$\omega^2 = 2\alpha\theta = 2\alpha(2\pi N)$

Thus no. of rev. $N = \frac{\omega^2}{4\pi\alpha} = \underline{\frac{1}{4\pi}\sqrt{\left(\frac{\mu_s g}{r\alpha}\right)^2 - 1}}$

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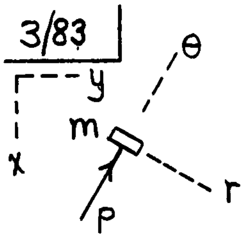


(Note: mg & static normal $\perp$ to paper)

$$\Sigma F_r = ma_r: \quad 0 = m\left(\ddot r - r\Omega^2\right) \quad (1)$$

$$\Sigma F_\theta = ma_\theta: \quad P = m\left(\underbrace{r\ddot\theta}_{0} + 2\dot r\Omega\right) \quad (2)$$

$$(1): \quad \ddot r = \dot r \frac{d\dot r}{dr} = r\Omega^2$$

$$\int_{\dot r_0}^{\dot r} \dot r \, d\dot r = \int_{r_0}^{r} \Omega^2 r \, dr \Rightarrow \dot r^2 = \dot r_0^2 + \Omega^2\left(r^2 - r_0^2\right)$$

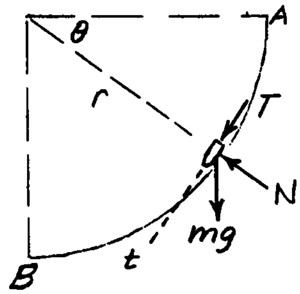
Numbers: (at end of tube) $\dot r = \left[60^2 + 7^2\left(3^2 - \left(\frac{6}{12}\right)^2\right)\right]^{1/2} = 63.5 \ \frac{ft}{sec}$

$$(2): \quad P = m\left(2\dot r\Omega\right) = \frac{5/16}{32.2}(2)(63.5)(7)$$

$$= \underline{8.62 \text{ lb}}$$

3/86 $\Sigma F_t = ma_t$; $T + mg\cos\theta = ma_t$

$a_t = \frac{T}{m} + g\cos\theta$



$v\,dv = a_t(r\,d\theta)$

$$\int_0^v v\,dv = \int_0^{\pi/2}\left(\frac{T}{m} + g\cos\theta\right) r\,d\theta$$

$$\frac{v^2}{2} = \frac{Tr}{m}\frac{\pi}{2} + gr\sin\theta\Big]_0^{\pi/2}$$

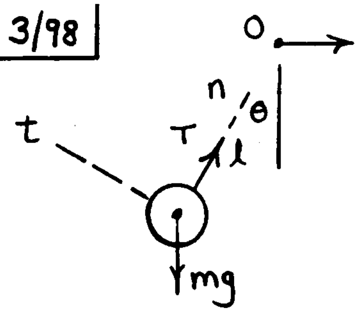
$$= \frac{Tr\pi}{2m} + gr$$

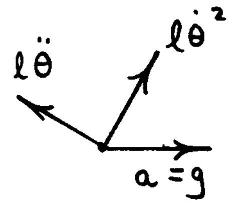
$v^2 = r\left(\frac{\pi T}{m} + 2g\right)$ $\quad \underline{v = \sqrt{r\left(\frac{\pi T}{m} + 2g\right)}}$

At B, $\Sigma F_n = ma_n$; $N - mg = m\frac{v^2}{r}$

$N = mg + T\pi + 2mg$, $\quad \underline{N = T\pi + 3mg}$

▶ 3/98





$\underline{a}_m = \underline{a}_o + \underline{a}_{m/o}$

$\Sigma F_t = ma_t: \quad -mg\sin\theta = m(\ell\ddot{\theta} - g\cos\theta)$

$$\ddot{\theta} = \frac{g}{\ell}(\cos\theta - \sin\theta) = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$$

$$\int_0^\theta \frac{g}{\ell}(\cos\theta - \sin\theta)\,d\theta = \int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta}$$

$$\frac{g}{\ell}\left[\sin\theta + \cos\theta - 1\right] = \dot{\theta}^2/2 \qquad *$$

$\theta = \theta_{max}$ when $\dot{\theta} = 0$: $\quad \sin\theta + \cos\theta = 1$

$$\Rightarrow \theta_{min} = 0, \quad \underline{\theta_{max} = \frac{\pi}{2}}$$

$\Sigma F_n = ma_n: \quad T - mg\cos\theta = m(\ell\dot{\theta}^2 + g\sin\theta)$

$T = m(\ell\dot{\theta}^2 + g\sin\theta + g\cos\theta)$

With $*$: $\quad \underline{T = mg(3\sin\theta + 3\cos\theta - 2)}$