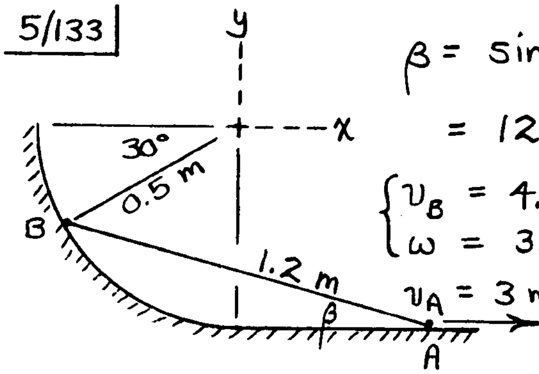
5/133



$$\beta = \sin^{-1}\frac{0.5 - 0.5\sin 30°}{1.2}$$

$$= 12.02°$$

$$\begin{cases} v_B = 4.38 \text{ m/s (Prob. 5/68)} \\ \omega = 3.23 \text{ rad/s} \end{cases}$$

$v_A = 3$ m/s = constant

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{a}_A + \underline{\alpha} \times \underline{r}_{B/A} - \omega^2 \underline{r}_{B/A}$$

$$a_{B_t}(\sin 30° \underline{i} - \cos 30° \underline{j}) + \frac{4.38^2}{0.5}(\cos 30° \underline{i} + \sin 30° \underline{j})$$

$$= \underline{0} + \alpha \underline{k} \times 1.2(-\cos 12.02° \underline{i} + \sin 12.02° \underline{j})$$

$$- 3.23^2 (1.2)(-\cos 12.02° \underline{i} + \sin 12.02° \underline{j})$$

Carry out vector algebra & equate coefficients:

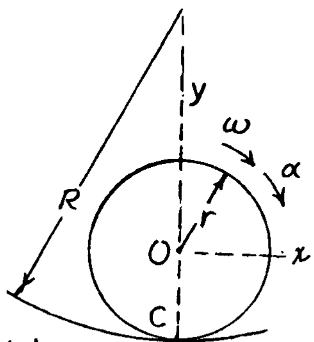
$\underline{i}$: $\frac{1}{2} a_{B_t} + 33.3 = -0.250\alpha + 12.28$

$\underline{j}$: $-\frac{\sqrt{3}}{2} a_{B_t} + 19.21 = -1.174\alpha - 2.61$

Solution: $\underline{a_{B_t} = -23.9 \text{ m/s}^2}$, $\underline{\alpha = -36.2 \text{ rad/s}^2}$

5/140

$$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$$



(a)

$$(\underline{a}_O)_n = \frac{v_O^2}{R-r}\underline{j} = \frac{r^2\omega^2}{R-r}\underline{j}$$

$$(\underline{a}_O)_t = r\alpha\,\underline{i}$$

$$(\underline{a}_{C/O})_n = r\omega^2\underline{j}$$

$$(\underline{a}_{C/O})_t = -r\alpha\,\underline{i}$$

Add & get $\underline{a}_C = \dfrac{r^2\omega^2}{R-r}\underline{j} + r\omega^2\underline{j} = \dfrac{r\omega^2}{1-r/R}\underline{j}$

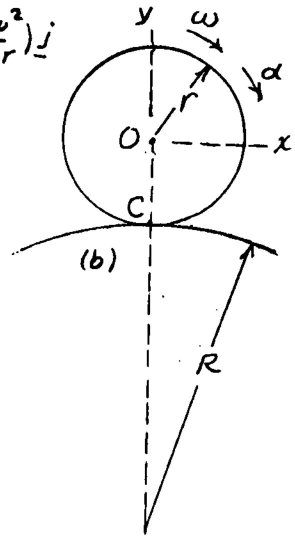
(b) $(\underline{a}_O)_n = \dfrac{v_O^2}{R+r}(-\underline{j})$; $(\underline{a}_O)_t = r\alpha\,\underline{i}$

$(\underline{a}_{C/O})_n = r\omega^2\underline{j}$; $(\underline{a}_{C/O})_t = -r\alpha\,\underline{i}$

Add & get

$$\underline{a}_C = \left(r\omega^2 - \frac{r^2\omega^2}{R+r}\right)\underline{j}$$

$$\underline{a}_C = \frac{r\omega^2}{1+\frac{r}{R}}\underline{j}$$

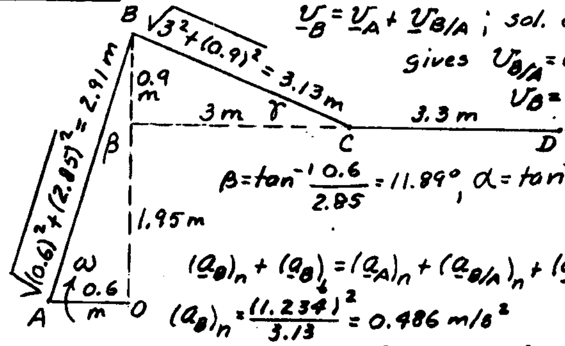


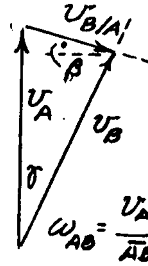
(b)

▶5/149 $\omega = \frac{2\pi}{3} = 2.09$ rad/s , $v_A = 0.6(2.09) = 1.257$ m/s

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$; sol. of polygon gives $v_{B/A} = 0.362$ m/s

$v_B = 1.234$ m/s





$\beta = \tan^{-1}\frac{0.6}{2.85} = 11.89°$, $\alpha = \tan^{-1}\frac{0.9}{3} = 16.70°$

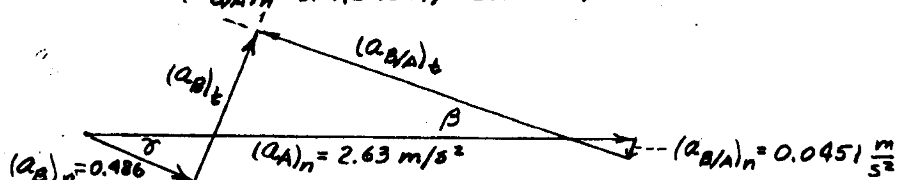
$\omega_{AB} = \frac{v_{A/B}}{\overline{AB}} = \frac{0.362}{2.91} = 0.1244$ rad/s

$(\underline{a}_B)_n + (\underline{a}_B)_t = (\underline{a}_A)_n + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(a_B)_n = \frac{(1.234)^2}{3.13} = 0.486$ m/s²

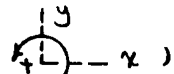
$(a_A)_n = 0.6(2.09)^2 = 2.63$ m/s²

$(a_{B/A})_n = 2.91(0.1244)^2 = 0.0451$ m/s²



Sol. by vector algebra or vector geometry gives

$(a_B)_t = 0.539$ m/s², $a_D = \overline{CD}\alpha_{CD} = \overline{CD}\alpha_{BC} = 3.3\,\frac{0.539}{3.13} = \underline{0.568 \text{ m/s}^2}$

5/153 For the coordinates 

, the no-slip constraints are $v_0 = -r\omega$ & $a_0 = -r\alpha$. So

$$\omega = -\frac{v_0}{r} = -\frac{-3}{0.30} = 10 \text{ rad/s}$$

$$\alpha = -\frac{a_0}{r} = -\frac{5}{0.30} = -16.67 \text{ rad/s}^2$$

Use the frame Oxy as disk-fixed.

(5/12): $\underline{v}_A = \underline{v}_0 + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

(5/14): $\underline{a}_A = \underline{a}_0 + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Ingredients:
$$\begin{cases} \underline{v}_0 = -3\underline{i} \text{ m/s} \\ \underline{a}_0 = 5\underline{i} \text{ m/s}^2 \\ \underline{\omega} = 10\underline{k} \text{ rad/s} \\ \underline{\alpha} = -16.67\underline{k} \text{ rad/s}^2 \end{cases}$$

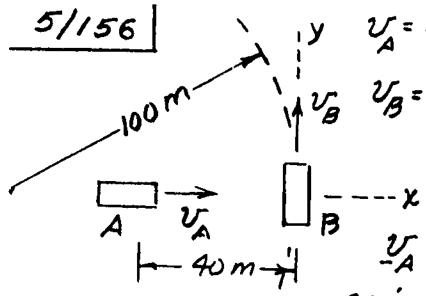
$$\underline{r} = 0.24\underline{j} \text{ m}$$
$$\underline{v}_{rel} = 2\underline{i} \text{ m/s}$$
$$\underline{a}_{rel} = -7\underline{i} - \frac{2^2}{0.24}\underline{j} = -7\underline{i} - 16.67\underline{j} \text{ m/s}^2$$

Substitute into (5/12) & (5/14) & simplify:

$$\underline{v}_A = -3.4\underline{i} \text{ m/s}$$

$$\underline{a}_A = 2\underline{i} - 0.667\underline{j} \text{ m/s}^2$$

5/156



$v_A = \frac{72 \times 1000}{3600} = 20$ m/s

$v_B = \frac{54 \times 1000}{3600} = 15$ m/s

$\omega = \omega_B = \frac{15}{100} = 0.15$ rad/s

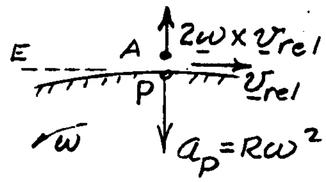
$$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$$

$$20\underline{i} = 15\underline{j} + 0.15\underline{k} \times (-40\underline{i}) + \underline{v}_{rel}$$

$\underline{v}_{rel} = 20\underline{i} - 9\underline{j}$ m/s

$(v_{rel})_{\text{rotating axes}}$ differs from $(v_{rel})_{\text{translating axes}}$ by $\underline{\omega} \times \underline{r}$

5/160 Let P be a point on the road coincident with A. $\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



For zero vertical acceleration,

$|2\underline{\omega} \times \underline{v}_{rel}| = R\omega^2$

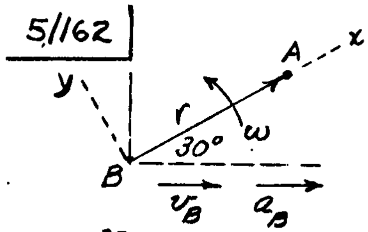
$v_{rel} = \underline{\frac{1}{2}R\omega}$

For $R = 6378$ km, $\omega = 0.7292(10^{-4})$ rad/s,

$v_{rel} = \frac{1}{2}(6378)(10^3)(0.7292 \times 10^{-4}) = 233$ m/s

or $v_{rel} = 233(3.6) = \underline{837 \text{ km/h}}$

5/162



$\underline{r} = (20+b)\underline{i} = 25\underline{i}$ ft

$\underline{v}_{rel} = \dot{r}\underline{i} = 2\underline{i}$ ft/sec

$\underline{a}_{rel} = \ddot{r}\underline{i} = -1\underline{i}$ ft/sec^2

$\underline{\omega} = \frac{10}{180}\pi\underline{k} \doteq 0.1745\underline{k}$ rad/sec

$\dot{\underline{\omega}} = \underline{0}$

$v_B = \frac{35}{30}44 = 51.3$ ft/sec , $\underline{v}_B = 51.3(\underline{i}\cos 30° - \underline{j}\sin 30°)$

$a_B = -10$ ft/sec^2 $\quad \underline{a}_B = -10(\underline{i}\cos 30° - \underline{j}\sin 30°)$

Eq. 5/14, $\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

so $\underline{a}_A = -10(0.866\underline{i} - 0.5\underline{j}) + \underline{0} + (0.1745)^2\underline{k}\times(\underline{k}\times 25\underline{i})$

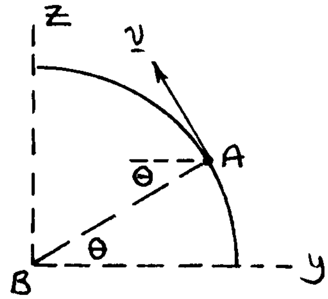
$\quad + 2(0.1745\underline{k})\times 2\underline{i} - 1\underline{i}$

(b) $\underline{a}_A = -10.42\underline{i} + 5.70\underline{j}$ ft/sec^2 with respect to ground

(a) $\underline{a}_A - \underline{a}_B = -10.42\underline{i} + 5.70\underline{j} - (-10)(0.866\underline{i} - 0.5\underline{j})$

$= -1.76\underline{i} + 0.70\underline{j}$ ft/sec^2 with respect to truck

5/167



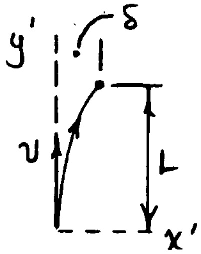
$$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$$

$$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}$$

$$= 2\Omega\underline{k} \times v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$$

$$= 2\Omega v \sin\theta\,\underline{i} \quad \text{(west)}$$

With no westward force mechanism available, the ball will drift to the east (relative to the ground) with an acceleration of magnitude a_{cor}.



$$a_{x'} = 2\Omega v \sin\theta$$

$$\delta = \frac{1}{2} a_{x'} t^2 = \frac{1}{2}(2\Omega v \sin\theta)\left(\frac{L}{v}\right)^2$$

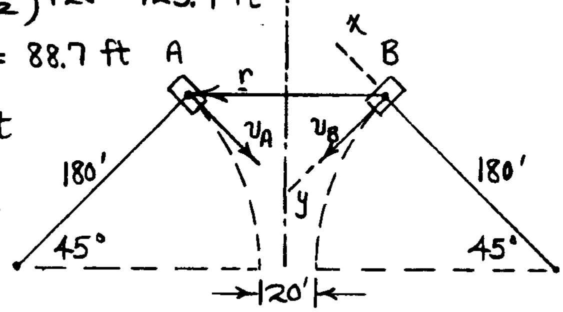
$$= \underline{\frac{\Omega L^2}{v} \sin\theta} \quad (\text{assumes } \delta \ll L)$$

With $\Omega = 7.292(10^{-5})$ rad/sec, $v = 15$ ft/sec, $L = 60$ ft, & $\theta = 40°$: $\delta = 0.01125$ ft

$\underline{(0.1350 \text{ in.})}$

5/170 $r = 2\left(180 - \frac{180}{\sqrt{2}}\right) + 20 = 125.4 \text{ ft}$

$r_x = r_y = 125.4\,\frac{\sqrt{2}}{2} = 88.7 \text{ ft}$



$\underline{r} = 88.7(\underline{i} + \underline{j}) \text{ ft}$

$\underline{v}_A = -44\underline{i} \text{ ft/sec}$

$\underline{v}_B = 44\underline{j} \text{ ft/sec}$

$\underline{\omega} = \frac{44}{180}\underline{k} = 0.244\underline{k} \text{ rad/s}$

(5/12): $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

Substitute & obtain $\underline{v}_{rel} = -22.3\underline{i} - 65.7\underline{j} \text{ ft/sec}$

(5/14): $\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \frac{44^2}{180}\underline{j} = 10.76\underline{j} \text{ ft/sec}^2$; $\underline{a}_B = -10.76\underline{i} \text{ ft/sec}^2$

$\dot{\underline{\omega}} = \underline{0}$

Substitute & obtain $\underline{a}_{rel} = -16.06\underline{i} + 27.0\underline{j} \text{ ft/sec}^2$