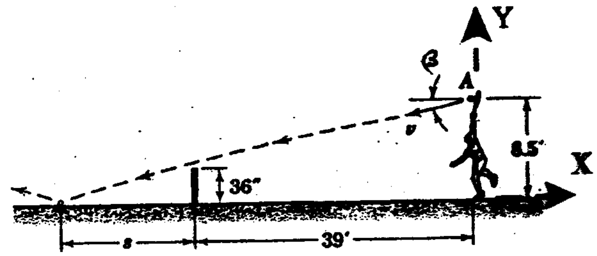
Name MASTER

CE 261 DYNAMICS
MIDTERM 1: CHAPTERS 1,2, 3 & 4
Thursday, March 4, 1999, Time: 5:00-6:30 pm

SHOW ALL WORK   OPEN BOOK, CLOSED HOMEWORK OR SOLUTION MANUALS
ONE PAGE NOTES PERMITTED, NO CREDIT FOR ANSWER ONLY

1. A tennis player serves the ball horizontally ($\beta = 0°$) at velocity v. The center of the ball clears the 36 inch high net by 6 inches. The tennis ball weighs 2 ounces and has a diameter of 3 inches.

a. (10 points) Calculate the time, t_{net}, it takes for the ball to reach the net from the racquet.



$a_x = 0: \quad x = V_{x_0} t = V t_{net}$

$39 = v\, t_{net}$

$a_y = -g: \quad y = V_{y_0} t_{net} - \frac{1}{2} g t_{net}^2$

$3.5 - 8.5 = 0 - \frac{1}{2} g\, t_{net}^2$

$t_{net} = \sqrt{\frac{2(5)}{32.2}} = \boxed{0.557 \text{ sec}}$

b. (10 points) Calculate the ball's initial velocity v.

$V = \frac{x_{net}}{t_{net}} = \frac{39}{0.557} = \boxed{70 \text{ ft/sec}}$

c. (10 points) Assuming the racquet strikes the ball for a period of 0.05 second, what average force is applied by the racquet to the ball?

$$\int_0^{\Delta t} F\,dt = mV_f - mV_i$$

$$F\Delta t = mV_f - mV_i$$

$$F = \frac{mV_f - mV_i}{\Delta t}$$

$$= \boxed{5.435\ \text{lbf}}$$

$$\frac{mV_f}{\Delta t} = \frac{(3/16)/32.2\,(\ )}{0.05}$$

d. (10 points) The tennis ball follows a curved path after being struck by the racquet. Determine the quantities $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ all relative to the x-y axes shown just after the ball is struck.

$$\vec{v} = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta = V\,\vec{e}_\theta$$

$$\boxed{\dot{\theta}}$$

$$r\dot{\theta} = V \qquad \dot{\theta} = \frac{V}{r} = \frac{70\ \text{ft/sec}}{8.5} = \boxed{8.235}\ /\text{sec}$$

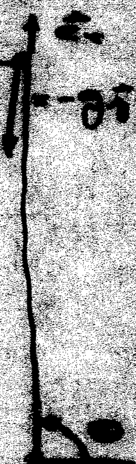
$$\vec{a} = \vec{a}_r + \vec{a}_\theta = a_r\vec{e}_r + a_\theta\vec{e}_\theta$$

$$a_r = (\ddot{r} - r\dot{\theta}^2) = -g, \quad \ddot{r} = -g + r\dot{\theta}^2$$

$$= -32.2 + 8.5(8.235)^2$$

$$= \boxed{544.3\ \text{ft/sec}^2}$$

$$a_\theta = r\ddot{\theta} + \cancel{2\dot{r}\dot{\theta}} = 0 \qquad \boxed{\ddot{\theta} = 0}$$



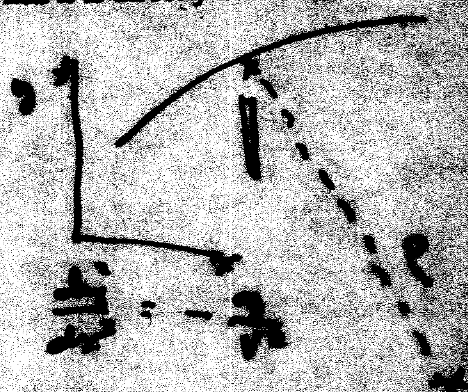
e. (10 points) What is the radius of curvature of the ball's flight when it is directly above the net?

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$y = -\tfrac{1}{2}gt^2, \quad x = vt \quad t = \frac{x}{v}$$

$$= -\tfrac{1}{2}g\left(\frac{x}{v}\right)^2 \qquad \frac{dy}{dx} = -g\frac{x}{v^2} \qquad \frac{d^2y}{dx^2} = -\frac{g}{v^2}$$

$$\rho = \frac{\left[1 + \left(-\frac{g\,(39)}{(70)^2}\right)^2\right]^{3/2}}{-g/(70)^2} = 167.4\ \text{ft}.$$



CE [illegible] Name ______________________________

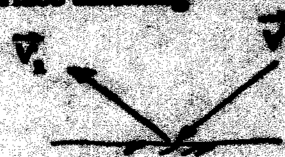
f. (10 points) Find the distance s from the net to the point where the ball hits the court surface.

$y = y_f - y_i = -\frac{1}{2} g t^2 = -\frac{1}{2} g \left(\frac{x}{v}\right)^2$

$x = \sqrt{\frac{2(8.5)}{g}} \cdot$ [illegible] $= 50.862 \text{ ft.}$

$x_{net} = 50.862 - 39 = \boxed{11.862 \text{ ft.}}$

g. (10 points) Find the velocity of the ball before and after it hits the court surface assuming the coefficient of restitution is 0.6.



$V_x = 90 \text{ ft/sec}$ $\quad t_{hit} = \frac{x_{hit}}{v} = \frac{70.862}{90}$

$v_y = -g t_{hit} = 0.727 \text{ sec}$

$= -32.2(0.727) = -23.4 \text{ ft/sec}.$

$v_{x_1} = v_{x_2} = 90 \text{ ft/sec}.$

$e = 0.6 = \frac{v_{y2} - 0}{0 - v_{y1}} = \frac{-V_{y2}}{23.4}$ $\quad V_{y2} = 0.6(23.4) = 14.04 \text{ ft/s}.$

$V_2 = \sqrt{(90)^2 + (14.04)^2} = \boxed{91.1 \text{ ft/sec}}$

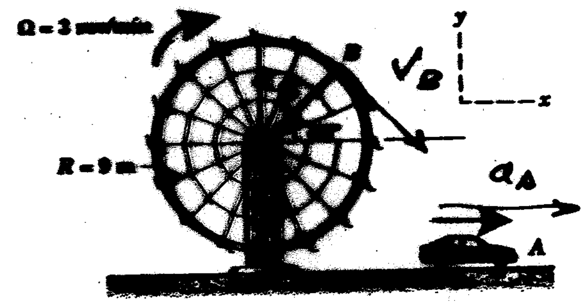
h. (10 points) What energy is lost during the ball's impact with the ground?

$\Delta T = \frac{1}{2} m \left[V_2^2 - V_1^2\right]$

$= \frac{1}{2}\left(\frac{W}{32.2}\right)\left[\left(90^2 + 14.04^2\right) - \left(90^2 + 23.4^2\right)\right]$

$= \boxed{-0.676 \text{ ft-lb}}$

2. The car A has a forward speed of 18 km/hr and is accelerating at 3 m/s². Observer B rides in a non-rotating chair on the Ferris wheel. The angular rate Ω = 3 rev/min of the Ferris wheel is constant.



a. (10 points) Determine the velocity of the car relative to observer B.

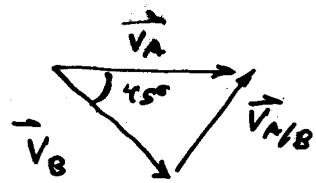
$\vec V_A = \vec V_B + \vec V_{A/B}$

$V_A = \frac{18(1000)}{3600} = 5\vec i$

$|V_B| = R\Omega = 9\left(\frac{3(2\pi)}{60}\right) = 2.83$

$\vec V_B = 2.83\cos 45\,\vec i - 2.83\sin 45\,\vec j$

$\vec V_{A/B} = 3\vec i + 2\vec j$ m/sec. $= \sqrt{13}$



$= \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\Theta}$

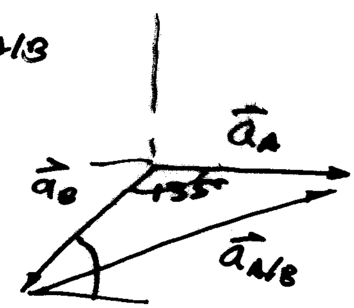
$= \sqrt{25 + 8 + 2(5)\sqrt{8}\cos 45} = \boxed{3.606 \text{ m/sec}}$

b. (10 points) Determine the acceleration of the car relative to observer B.

$\vec a_A = \vec a_B + \vec a_{A/B}$

$3\vec i = \frac{V_B^2}{R}(-\cos 45°\,\vec i - \sin 45°\,\vec j) + \vec a_{A/B}$

$\boxed{\vec a_{A/B} = 3.63\,\vec i + 0.63\,\vec j \quad m/s^2}$



$a_{A/B} = \sqrt{9 + 0.789 - 2(3)(0.88)\cos 135°}$

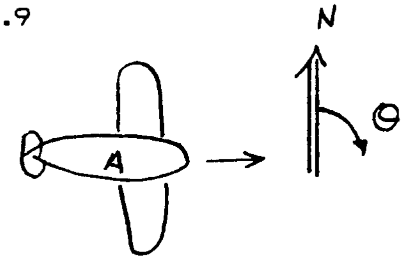
$= \boxed{3.683 \ m/s^2}$

CE261 HWK
NOTEBOOK

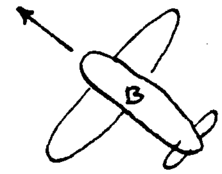
Name *MASTER.*

CE261 - DNMAMICS
Midterm I: Chapters 1.1-7, 2.1-2.9
24 February 1988

SHOW ALL WORK   CLOSED BOOK
CLOSED HOMEWORK OR SOLUTION MANUALS
NO CREDIT FOR ANSWER ONLY

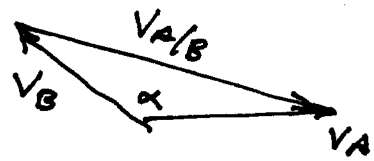


1. Aircraft A is flying east with a velocity of V_A = 600 mph, while aircraft B is flyin northwest with a velocity of 400 mph.



a.) (10 points) Determine the mag tude, $V_{A/B}$ (mph), which A appears to have tc passenger in B.

$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$$



Cos law

$$V_{A/B}^2 = V_A^2 + V_B^2 - 2V_A V_B \cos\alpha$$

$$\alpha = 135^\circ$$

$$V_{AB}^2 = (600)^2 + (400)^2 - 2(600)(400)\cos 135$$

$$V_{A/B} = \boxed{927 \text{ mph}} \times \frac{5280}{3600} \times \frac{0.3048}{1} = 414 \text{ m/sec}$$

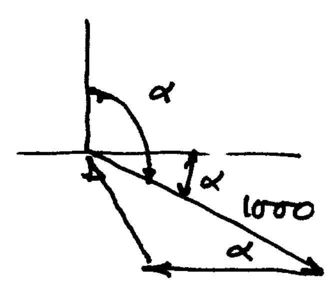
or $$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B = +600\,\hat{i} - \left[-400\cos 45\,\hat{i} + 400\,\hat{j}\cos 45\right]$$

$$= 882.8\,\hat{i} - 282.8\,\hat{j}$$

$$= \boxed{927 \text{ mph}} = 414 \text{ m/sec}.$$

b.) (10 points) Assuming $V_{A/B}$ = 1000 mph, find direction of $V_{A/B}$ with respect to North in degrees.

$$\alpha = \tan^{-1}\frac{(V_{A/B})_y}{(V_{A/B})_x} = \tan^{-1}\frac{282.8}{882.8} = 17.76^\circ$$



$$\Theta = 17.76 + 90 = \boxed{107.76^\circ}$$

or

$$\alpha = \sin^{-1}\left(\frac{400}{1000}\sin 45\right) = 16.43$$

$$\Theta = 90 + 16.43 = \boxed{106.43}$$

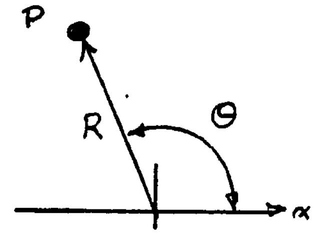
or $$\frac{400}{\sin\alpha} = \frac{1000}{\sin 45}$$

$$\sin\alpha = \frac{400\sin 45}{1000}$$

$$\alpha = \sin^{-1}\left(\frac{400\sin 45}{1000}\right)$$

CE261, MIDTERM I   Name________________

2. Given a particle P which varies position as $R = K\Theta$, and where initial position is $\Theta_o = \pi/4$ at $t = 0$ and the angular acceleration is constant, $\ddot{\Theta} = B$. $\dot{\Theta}_0 = 0$



a. (10 points) Find the time, t, when the particle P is at $\Theta = 3\pi/4$.

$$\omega = \dot{\Theta} = \int_0^t \alpha\, dt + \dot{\Theta}_0 = Bt$$

$$\Theta = \Theta_0 + \int_0^t \omega\, dt = \frac{\pi}{4} + B\int_0^t t\, dt = \frac{\pi}{4} + \frac{Bt^2}{2}$$

$$\frac{3\pi}{4} = \frac{\pi}{4} + \frac{Bt^2}{2} \Rightarrow t^2 = \frac{\pi}{B}$$

$$\boxed{t = \sqrt{\frac{\pi}{B}}}$$

b. (10 points) At time, $t = 2\,[\pi/B]^{1/2}$, find the radial acceleration of P, a_r.

$$r = k\Theta = k\left[\frac{\pi}{4} + \frac{B}{2}4\frac{\pi}{B}\right] = \frac{9k\pi}{4}$$

$$\dot{r} = k\dot{\Theta} = kBt = kB2\left(\frac{\pi}{B}\right)^{1/2} = 2\sqrt{\pi B}\,k$$

$$\ddot{r} = k\ddot{\Theta} = kB$$

$$\Theta = \frac{9\pi}{4}, \quad \dot{\Theta} = 2\sqrt{\pi B}, \quad \ddot{\Theta} = B$$

$$a_r = \ddot{r} - r(\dot{\Theta})^2 = kB - \frac{9k}{4}[4\pi B]\pi = kB[1 - 9\pi^2]$$

$$\boxed{a_r = kB[1 - 9\pi^2]}$$

c. (10 points) At time, $t = 2\,[\pi/B]^{1/2}$, find the angular acceleration of P, a_Θ.

$$a_\Theta = 2\dot{r}\dot{\Theta} + r\ddot{\Theta}$$

$$= 2\left(2\sqrt{\pi B}\,k\right)\left(2\sqrt{\pi B}\right) + \frac{9k\pi}{4}B$$

$$= \pi kB\left[8 + \frac{9}{4}\right] = \boxed{\frac{41}{4}\pi kB}$$