THE ESTIMATION OF TURBULENT DISPERSION FROM VELOCITY STATISTICS. PART I: THE LAGRANGIAN AUTOCORRELATION FUNCTION

Lionel V. Baldwin*, Gearold R. Johnson**,
Wen-Whai Li, and Robert N. Meroney***

Colorado State University
Civil Engineering Department
Fort Collins, Colorado 80523

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* : President, National Technological University, Fort Collins, Colorado 80523

** : Abell Professor, College of Engineering, Colorado State University

***: Professor-in-charge, Fluid Mechanics and Wind Engineering Program, Colorado State University

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Abstract

Lagrangian autocorrelations and time scales are estimated from Eulerian time measurements of the axial turbulent velocity at two points aligned with the mean velocity. Three Eulerian statistical measures are used to specifiy the steady turbulent flow: the general space-time correlation evaluated in the mean flow frame of reference; the integral scale of the space correlation; and, the variance of the turbulent velocity fluctuation. The computational model involves a spatially-weighted, time-varying average of the Eulerian space-time correlation which, when evaluated along the mean flow axis, is a first approximation to the Lagrangian autocorrelation. The proposed estimation method is examined when the requirements of both Eulerian isotropy and Lagrangian isotropy are imposed in a hypothetical homogeneous, stationary turbulence of incompressible fluid.

1. Introduction

The basic approaches to statistical turbulence are either Lagrangian or Eulerian. Whereas the Lagrangian approach follows the motion of a single fluid particle, and is difficult to measure, the Eulerian approach concentrates on the balance of particle fluxes through a fixed point in the flow field and is normally easier to determine. Recent research on the turbulent dispersion phenomenon suggests that the concentration field in a wide variety of situations can be generated if the Lagrangian statistics/properties of the flow field are known. Attempts have been made to deduce the Lagrangian autocorrelation from the Eulerian turbulent velocity at fixed point in space. Theoretical and empirical approaches to the Lagrangian-Eulerian relationship are quite diverse. Nevertheless, most of the attempts have been based on the assumption that the Lagrangian autocorrelation and the Eulerian autocorrelation, or Eulerian space-time cross correlation, are of similar shape but different scales. Whereas the importance of the shapes of those two autocorrelation functions is still debatable except for short range dispersion, the importance of the integral scales in turbulent diffusion has been confirmed. The purpose of this research is to demonstrate how a systematic scheme based on a probability method can estimate Lagrangian statistics by a few anemometers located in the fixed Eulerian frame of reference.

Diffusion of a fluid particle in a uniform mean velocity, stationary, homogeneous turbulent flow was first described by Taylor (1921). The mean square particle displacement was predicted to depend on the Lagrangian velocity variance and the Lagrangian autocorrelation,

$$[X_i^2(t)] = 2[v_i^2] \int_o^t \int_o^{t_1} {}_L R_{ii}(\tau)d\tau dt_1 \ , \qquad (1.1)$$

where the square bracket indicates an ensemble average of N fluid particles, v_i is the Lagrangian velocity fluctuation in the ith direction, $[v_i^2]$ is an abbreviation for $[v_i^2(t)]$ during stationary turbulence, and ${}_LR_{ij}(\tau)$ is a normalized Lagrangian autocorrelation function,

$$ {}_LR_{ij}(\tau) = \frac{[v_i(t)v_j(t+\tau)]}{[v_i{}^2]^{\frac{1}{2}}\ [v_j{}^2]^{\frac{1}{2}}} \quad . \tag{1.2} $$

In a homogenous turbulent flow with uniform shear Γ and mean velocity U, $U = \Gamma\, x_3$, Corrsin (1953) derived expressions for the components of the dispersion tensor $[X_i(t)X_j(t)]$:

$$ [X_1{}^2(t)] = \underbrace{\Gamma^2[v_3{}^2]\{\tfrac{2}{3}t^3\int_0^t {}_LR_{33}(\tau)d\tau - t^2\int_0^t \tau\,{}_LR_{33}(\tau)d\tau + \tfrac{1}{3}\int_0^t \tau^3\,{}_LR_{33}(\tau)d\tau\}}_{(I)} $$

$$ + \underbrace{2[v_1{}^2]\int_0^t (t-\tau)\,{}_LR_{11}(\tau)d\tau}_{(II)} \tag{1.3} $$

$$ + \underbrace{\Gamma[v_1v_3]\int_0^t (t-\tau)\,{}_LR_{31}(\tau)d\tau + \Gamma[v_1v_3]\int_0^t (t^2-\tau^2)\,{}_LR_{13}(\tau)d\tau}_{(III)} \ , $$

$$ [X_1(t)X_3(t)] = \underbrace{\Gamma[v_3{}^2]\int_0^t \tau(t-\tau)\,{}_LR_{33}(\tau)d\tau}_{(I)} + \tag{1.4} $$

$$[v_1^2]^{\frac{1}{2}}[v_3^2]^{\frac{1}{2}} \int_o^t (t-\tau)\{_L R_{13}(\tau)+_L R_{31}(\tau)\}d\tau \qquad (1.5)$$

,

(II)

$$[X_2^2(t)] = 2[v_2^2] \int_o^t (t-\tau)_L R_{22}(\tau)d\tau \qquad \text{and}$$

$$[X_3^2(t)] = 2[v_3^2] \int_o^t (t-\tau)_L R_{33}(\tau)d\tau \ . \qquad (1.6)$$

Notice that the shear-enhanced term (I) in Equation 1.3 and Equation 1.4 dominate the long term dispersion ($t > {}_L T_{11}$). The turbulent shear correlation contribution terms, (III) in Equation 1.3 and (II) in Equation 1.4 are often neglected in the absence of data for ${}_L R_{ij}$, $i \neq j$. Term (II) in Equations 1.3, 1.4, and 1.5 are the Taylor's diffusion terms.

2. Estimation of Lagrangian Autocorrelation Function

The Lagrangian autocorrelation function, Equation 1.2, must be formed from an ensemble average of the time lagged product of single fluid point velocities which have a common origin. A review of the difficulties encountered in attempts to relate the Lagrangian function to more readily measured Eulerian correlation functions is given by Hinze (1975). Corrsin (1953, 1963a) proposed that the Lagrangian function might be estimated from the general Eulerian space-time correlation function by properly weighting the latter to account for fluid point distribution in space and time. His concept is sometimes called the Independence Hypothesis. The fluid point velocity is, of course, equal to the local instantaneous fluid velocity at the position occupied by the point, so this hypothesis has strong physical appeal. The hypothesis may be expressed in terms of the Lagrangian kinematics of a fluid particle (Corrsin, 1963a Weinstock, 1976) as follows:

$${}_L R_{ij}(\tau) = \iiint_{-\infty}^{\infty} {}_E R_{ij}(x_i,x_j,x_k;\tau)P(x;\tau)dx_i dx_j dx_k \qquad (2.1)$$

Note that the coordinate system has its new origin in the mean convective frame, as shown in Figure 1.

The weighting function, $P(x,\tau)$, is the probability of finding a fluid point injected at the origin in the space (x_i', x_j', x_b') at any time τ.

Consider a hypothetical joint normal distribution for P such that

$$P(x';\tau) = (2\pi)^{\frac{3}{2}} \ \Sigma^{\frac{1}{2}} \exp\{-\tfrac{1}{2} x' \ \Sigma^{-1} \ x'^{T}\} ,$$

where $\quad x' = (x_1', x_2', x_3')$ and

$$\Sigma = \begin{bmatrix} X_1^2(\tau) & X_1(\tau)X_2(\tau) & X_1(\tau)X_3(\tau) \\ X_2(\tau)X_1(\tau) & X_2^2(\tau) & X_2(\tau)X_3(\tau) \\ X_3(\tau)X_1(\tau) & X_3(\tau)X_2(\tau) & X_3^2(\tau) \end{bmatrix} . \tag{2.2}$$

Equation 2.2 for $P(\chi;\tau)$ introduces a great deal of mathematical complexity to the formulation of the Lagrangian autocorrelation. Further simplifications of the problem are necessary!

Experimental evidence (Frenkiel, 1953) suggests that the probability density function of finding a fluid particle in a spherical cloud should preserve a Gaussian form. Asymptotically, the probability density function is not only expected to be joint-normally distributed, but it should be independent in each direction such that

$$\begin{aligned} P(x';\tau) &= \frac{1}{\sqrt{2\pi}^3 [X_1^2(\tau)]^{\frac{1}{2}} [X_2^2(\tau)]^{\frac{1}{2}} [X_3^2(\tau)]^{\frac{1}{2}}} \cdot \\ &\quad \exp-\left\{ \frac{x_1'^2}{2[X_1^2(\tau)]} + \frac{x_2'^2}{2[X_2^2(\tau)]} + \frac{x_3'^2}{2[X_3^2(\tau)]} \right\} . \end{aligned} \tag{2.3}$$

In accordance with the expressions for the dispersion tensor $[X_i(\tau)X_j(\tau)]$ developed by Riley and Corrsin (1974), the turbulent flow field is further constrained in a homogenous uniform shear flow. The homogeneity requires invariance conditions with respect to the x_1x_3 plane such that in a uniform shear flow ${}_LR_{12}(\tau) = {}_LR_{21}(\tau) = [X_1(\tau)X_2(\tau)] = [X_2(\tau)X_3(\tau)] = 0$.

However, $[X_1(\tau)X_3(\tau)]$ cannot be ignored in a uniformly sheared flow, because it increases with time significantly faster than the variance of transverse displacements. This implies that an elliptic cloud evolves with two mutually correlated displacements along the x_1 and x_3 axes.

Based on Equation 2.2 and Riley and Corrsin's, expressions the Lagrangian autocorrelation function may be estimated from four integral equations iterated simultaneously with Taylor's Equation. The system of equations is

$${}_LR_{ij}(\tau) = \iiint \frac{{}_ER_{ij}(x_1,x_2,x_3;\tau)}{\sqrt{2\pi}\ \Sigma} \exp-\{x\Sigma x^T\}dx_1'dx_2'dx_3' \ ,$$

where

$$\Sigma = \begin{matrix} [X_1^2(\tau)] & 0 & [X_1(\tau)X_3(\tau)] \\ 0 & [X_2^2(\tau)] & 0 \\ [X_1(\tau)X_3(\tau)] & 0 & [X_3^2(\tau)] \end{matrix} \ , \qquad (2.4)$$

$$x = (x_1' - \Gamma t x_3', x_2', x_3') \ , \text{ and}$$

${}_LR_{ij}(\tau) = 0$ for $i \neq j$ except at $i=1$, $j=3$; and $[X_1^2(\tau)]$, $[X_1(\tau)X_3(\tau)]$, $[X_2^2(\tau)]$ and $[X_3^2(\tau)]$ retain their earlier definitions in Equations 1.3, 1.4, 1.5, and 1.6, respectively.

The present analysis requires that the general Eulerian space-time correlation be known. It is proposed in this study that the general Eulerian space-time correlation may be expressed as the product of time correlation and space correlation in at convective moving frame,

$${}_ER_{ij}(x_1,x_2,x_3;\tau) = {}_ER_{ij}(U\tau,0,0,\tau)\ {}_ER_{ij}(x_1',x_2',x_3') \qquad (2.5)$$

where x_i is the Eulerian fixed point coordinate and x_1' is the Eulerian moving frame coordinate. Predictions of the general Eulerian space-time correlation are rare and empirical, yet the convective Eulerian space-time correlations are well documented (Li and Meroney, 1984). Equation 2.5 represents an appropriate approximation which physically takes into account both the eddy lifetime and the eddy size effect.

For convenience ${}_ER_{ij}(\upsilon\tau, o, o;\tau)$ is hereafter represented by $F_1(\tau/{}_ST_{ij})$ where ${}_sT_{ij} = \int_0^\infty {}_ER_{ij}(\upsilon\tau, o, o; \tau)d\tau$.

2.1. Isotropic Homogeneous Turbulence with uniform flow

If Lagrangian isotropy exists in a stationary isotropic turbulence, the mean square displacement tensors will be identical for all diagonal terms and vanish for all off-diagonal terms, i.e.

$$[X_1^2(\tau)] = [X_2^2(\tau)] = [X_3^2(\tau)] \ ,$$
$$[X_i(\tau)X_j(\tau)] = 0 \ , \ \text{if } i \neq j \ ,$$

and (2.6)

$$_LR_{11}(\tau) = {}_LR_{22}(\tau) = {}_LR_{33}(\tau) \ ,$$
$$_LR_{ij}(\tau) = 0 \ , \ \text{if } i \neq j \ .$$

A consequence of Lagrangian isotropy is a spherical symmetric probability density function of finding a fluid particle in the turbulence field; thus,

$$P(r,\tau) = (2\pi[X_1^2(\tau)]^{-\frac{3}{2}} \exp- \left\{ \frac{r^2}{2[X_1^2(\tau)]} \right\} \ . \qquad (2.7)$$

where $r = (x_1'^2 + x_2'^2 + x_3'^2)^{1/2}$.

Based on a study of the kinematics of isotropic turbulence, Karman and Howarth (1938) derived an equation which relates the general double velocity correlation tensor to the space correlation of the axial velocity taken along the mean flow axis, f(r), without additional approximation.

$$ {}_E R_{ij}(x_1', x_2', x_3') = -\frac{1}{2\gamma}\frac{\partial f(r)}{\partial r} x_i' x_j' + \{f(r) + \frac{\gamma}{2}\frac{\partial f(r)}{\partial \gamma}\}\, \delta_{ij} \quad (2.8) $$

δij is the Kronecker delta function.

Experimental measurements of f(r) are satisfactorily approximated by a simple exponential function. (Hinze, 1975; Comte-Bellot and Corrsin, 1971). The exponential fit does not satisfy the requirement of eveness as $r \to o$, but over the entire range of positive correlation the exponential function rather closely follows the best fit of the measurements. It also satisfies the inertial subrange theory as noted by Tennekes (1979). Therefore, f(r)=exp(-r/L) is adopted in the present analysis. L is the Eulerian integral space scale.

Equation 2.1 can be solved in conjunction with Equation 1.1, 2.7 and 2.8,

$$ {}_L R_{11}(t_*) = F_1(t_*)\ \{e^{\alpha^2 I(t_*)}[1-\mathrm{erf}\sqrt{\alpha^2 I(t_*)}][1+4\alpha^2 I(t_*) + \frac{4}{3}\alpha^4 I^2(t_*)] $$

$$ - \frac{2}{3}\left(\frac{\alpha^2 I(t_*)}{\pi}\right)^{\frac{1}{2}}[5 + 2\alpha^2 I(t_*)]\} , $$

$$ I(t_*) = \int_o^{t_*}\int_o^{t_{1*}} {}_L R_{11}(t_{2*})dt_{2*}dt_{1*} , \quad (2.9) $$

where $t_* = \dfrac{\tau}{{}_S T_{11}}$, $\alpha = \dfrac{[u_1{}^2]^{\frac{1}{2}}\ {}_S T_{11}}{L}$ and

erf represents an error function of its own argument.

To satisfy the requirement of Lagrangian isotropy, the convective space-time correlation in the lateral direction must be

$$F_2(t_*) = \frac{\iiint {}_ER_{11}(r,\theta)P(r,\tau)dr_1 d\theta_2 d\phi_3}{\iiint {}_ER_{22}(r,\theta)P(r,\tau)drd\theta d\phi} \; F_1(t_*) \;, \qquad (2.10)$$

where r, θ, ϕ are spherical coordinates.

After some manipulation, it is found that

$$F_2(t_*) = \frac{H(\alpha,t_*) - \frac{1}{3}\,K(\alpha,t_*)}{H(\alpha,t_*) - \frac{1}{3}\,K(\alpha,t_*)} \; F_1(t_*) \;,$$

where

$$H(\alpha,t_*) = \alpha^2 I(t_*)\{-4\alpha^2 I(t_*)+\sqrt{4\pi\alpha^2 I(t_*)}(2\alpha^2 I(t_*)+1) \;.$$

$$(1-\mathrm{erf}\sqrt{\alpha^2 I(t_*)})\cdot\exp(\alpha^2 I(t_*))\}$$

and

$$K(\alpha,t_*) = \alpha^2 I(t_*)\{8\alpha^2 I(t_*)(1+\alpha^2 I(t_*))-\sqrt{4\pi\alpha^2 I(t_*)}(4\alpha^4 I^2(t_*)+6\alpha^2 I(t_*))\cdot$$

$$(1-\mathrm{erf}\sqrt{\alpha^2 I(t_*)})\cdot\exp(\alpha^2 I(t_*))\} \;.$$

Important points to note are (1) that the estimation of Lagrangian autocorrelation for fluid tracer requires the specification of a single dimensionless parameter, α , formed from readily available Eulerian measures, (2) the time dependent portion of the Eulerian space-time correlation, $F(t_*)$.

Equation 2.9 was solved numerically by an iterative procedure using a digital computer for various values of the Eulerian diffusivity parameter α. The algorithm makes direct use of the fact that as $\alpha \to o$, and the bracketed term { } on the right hand side of Equation 2.9 approaches unity.

That is, the α=o case is equivalent to assuming the weighting factor P(x, τ) in Equation 2.7 is a Dirac delta function. This limiting case has been called Burger's hypothesis in the literature (Baldwin and Walsh, 1961).

2.2. Homogeneous uniform shear flow

In a uniform shear flow, a plume evolves as an ellipsoid whose size, eccentricity, and direction are time dependent due to continuity (Elrick, 1962). For lack of information about ${}_LR_{13}(\tau)$ and ${}_ER_{13}(x_1, x_2, x_3;\tau)$, the probability density function of finding a fluid particle in a moving frame traveling with a mean velocity Γx_3 may be reasonably approximated by a three dimensional Gaussian distribution such as

$$P(x_1',x_2',x_3';\tau) = (2\pi)^{-3/2}\ ([X^2(\tau)][X_2^{\ 2}(\tau)][X_3^{\ 2}(\tau)])^{-1/2}\ .$$
$$\exp-\{\frac{(x_1'-\Gamma\tau x_3)^2}{2[X_1^{\ 2}(\tau)]} + \frac{x_2'^{\,2}}{2[X_2^{\ 2}(\tau)]} + \frac{x_3'^{\,2}}{2[X_3^{\ 2}(\tau)]}\} \tag{2.11}$$

where $[X_i^2(\tau)]$ is defined in Equation 1.3, 1.5, and 1.6 for i=1,2,3, respectively.

The definition of the Eulerian parameter is generalized to account for the anisotropy, i.e., $\alpha_i = \frac{[u_i^2]^{1/2}\ {}_ST_{ii}}{L}$. Since the general space correlation in a non-isotropic flow is still unknown, the Karman-Howarth relationship is retained and the Lagrangian autocorrelation functions are assumed to be the same in all three directions. Such assumptions require that,

$$[X_1^{\ 2}(t_*)] = L^2[\Gamma^2\,{}_ST_{11}^{\ 2}\alpha_3^{\ 2}II(t_*)+2\alpha_1^{\ 2}I(t_*)]\ ,$$
$$[X_2^{\ 2}(t_*)] = 2\alpha_2^{\ 2}L^2I(t_*)\ ,$$
$$[X_3^{\ 2}(t_*)] = 2\alpha_3^{\ 2}L^2I(t_*)\ ,\ \text{and}$$

$$II(t_*) = \frac{2}{3}\,t_*^{\ 2}\int_o^{t_*}{}_LR_{33}(\tau_*)d\tau\ -t_*^{\ 2}\int_o^{t_*}\tau_*\ {}_LR_{33}(\tau_*)d\tau\ + \frac{1}{3}\int_o^{t_*}\tau_*^{\ 3}\,{}_LR_{33}(\tau_*)d\tau_*,$$

The Lagrangian autocorrelation function in a homogeneous uniform shear flow becomes

$$
{}_{L}R_{11}(t_*) = \frac{F_1(t_*)}{2^{\frac{5}{2}}\pi^{\frac{3}{2}}\alpha_2\alpha_3 I(t_*)A^{\frac{1}{2}}(t_*)} \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} [\{(\frac{\sqrt{\pi}}{4B^{\frac{3}{2}}(t_*)} + \frac{\sqrt{\pi}}{8B^{\frac{5}{2}}(t_*)})e^{\frac{1}{4B(t_*)}}
$$

$$
(1-\mathrm{erf}(\frac{1}{2B^{\frac{1}{2}}(t_*)})) - \frac{1}{4B^2(t_*)}\} + \frac{\sin^2\theta}{2}\{(\frac{\sqrt{\pi}}{16B^{\frac{7}{2}}(t_*)} + \frac{3\sqrt{\pi}}{8B^{\frac{5}{2}}(t_*)})e^{\frac{1}{4B(t_*)}}
$$

$$
(1-\mathrm{erf}(\frac{1}{2B^{\frac{1}{2}}(t_*)})) - (\frac{1}{8B^3(t_*)} + \frac{1}{2B^2(t_*)})\}]\ \sin\theta d\phi d\theta \ , \tag{2.12}
$$

where

$$
A(t_*) = \Gamma^2{}_S T_{11}{}^2\alpha_3{}^2 II(t_*) + 2\alpha_1{}^2 I(t_*) \quad \text{and}
$$

$$
B(t_*) = \frac{(\cos\theta - \Gamma_S T_{11} t_* \sin\theta\cos\phi)^2}{2A(t_*)} + \frac{\sin^2\theta\sin^2\phi}{4\alpha_2{}^2 I(t_*)} + \frac{\sin^2\theta\cos^2\phi}{4\alpha_3{}^2 I(t_*)} .
$$

Equation 2.9 may be shown to be a special case of Equation 2.12 if one assumes that $\alpha_1 = \alpha_2 = \alpha_3$ and $\Gamma = 0$ and carries out the integration.

A numerical iterative procedure was developed to calculate the Lagrangian autocorrelation function as stated previously provided that the convective Eulerian space-time correlation is specified. The computer program accounts for experimental values $F_1(t_*)$ and for the presence of uniform shear. During the computation at each time step, the temporary Lagrangian autocorrelation function is assumed to have an exponential form:

$$
{}_{L}R_{11}(t_*) = \exp-\{A\alpha_1 t_*\} \tag{2.13}
$$

where A is a function of the Eulerian parameter and time.

After initialization at $t_*=0$, A is perturbed by a small magnitude to compute the mean square particle displacement according to the Taylor's integral relation. The Lagrangian autocorrelation function at each successive time step is evaluated from Equation 2.12 and compared for convergence to Equation 2.13. The new value of A is determined by using a Newton-Raphson's technique. The relative convergence criterion for A is set to 10^{-5} which provides a relative error less than 10^{-4} for the estimate of the Lagrangian autocorrelation.

The procedure may be run for various values of α_i and the turbulent shear parameter $\Gamma_S T_{11}$. The double integral of Equation 2.12 is computed by a Gauss-Legendre quadrature integration scheme. Such a scheme is maintained self-convergent during the iteration by using up to 1024 weighting points. Detailed description of the Gauss-Legendre quadrature method may be found in Carnahan et al. (1969).

3. Eulerian Space-Time Correlation

The formulation of Equation 2.5 assumes that the general Eulerian space-time correlation can be represented by multiplying the space function by the empirical function obtained at the origin of the mean convective frame, ${}_ER_{11}(o,o,o;\tau)$. By taking the correlation of two axial velocity signals at a time lag τ, which is equal to the transit time associated with the fixed probe separation $U\tau$, the data become equivalent to the autocorrelation which an anemometer at the origin of the mean convective frame would measure. Figure 2 is a summary of such correlation data reported by several investigators. The dashed curve fitting the data for grid turbulence is an empirical fit proposed by Comte-Bellot and Corrsin (1971). The solid curve represents filtered correlation data in a simulated boundary layer reported by Li and Meroney (1984). To evaluate the adequacy of the dashed curve (or the solid curve) for other turbulence measurements, Figure 3 was prepared. Figure 3 is a summary plot of the time lagged, peak correlations obtained from two probes separated at various distance along the axis of a pipe. The solid points (Baldwin and Walsh, 1961; Baldwin and Mickelsen, 1963) were obtained in an 8-inch pipe with room air as the fluid and the open points were measured by Atesman (1971) in a 6-inch pipe of flowing water. The water flow data extend over a sufficient range to indicate that the empirical curve, $F_1(t_*)$, is indeed an adequate approximation for pipe core flows as well as grid turbulence.

A quantitative test of Equation 2.5 can be made based on data reported by Champagne et al. (1970) and Comte-Bellot and Corrsin (1971). These data were obtained from the time lagged correlation of two probes separated at a fixed axial separation but at several different transverse separations. Table 1 summarizes the Eulerian space-time correlations for calculated and measured results, where the calculated results were obtained by taking $x_1'=0$ in the Karman Howarth relationship. The calculated values become systematically lower than the measured values at larger separations, but the comparison is considered rather good in view of the simplicity of the formation.

4. Comparison of Present Predictions with Previous Analysis

A sample of the Lagrangian autocorrelation functions predicted using Equation 2.9 are plotted in Figure 4(Model I:$F_1(t_*)=\exp(-t_*)$; Model II:$F_1(t_*)=\exp(-\frac{\pi t_*^2}{4})$; Model III:Comte-Bellot and Corrsin; Model IV:present measurement). As noted before, the $\alpha=0$ curve is the time dependent portion $F_1(t_*)$, of the general Eulerian space-time correlation. Despite the complexity of Equation 2.9, the predicted Lagrangian autocorrelations show a clear resemblance in functional form to the input Eulerian function ($\alpha=0$). Figure 5 illustrates that when the predicted Lagrangian autocorrelations, Figure 4, are normalized by each Lagrangian integral time scale, ${}_LT_{11}$, the functional form of the autocorrelations is remarkably similar, except for a Gaussian form for $F_1(t_*)$.

Estimates of the Lagrangian integral time scale are more readily available than the autocorrelation function itself. Figure 6 shows that the predicted Lagrangian autocorrelations are a function of α and approach zero monotonically in a well-behaved fashion at large time lags. Figure 6 also presents results reported by other researchers. Discussion of those data are deferred to a later section.

The fixed-point Eulerian autocorrelation can be obtained as a result of Equation 2.8 by substituting $x_1' = -U\tau$ and $x_2'=x_3'=0$. Therefore, ${}_ET_{11}$ may be determined as

$${}_ET_{11} = {}_ST_{11} \int_o^\infty e^{-\left|\frac{\alpha}{i}\right| t_*} F_1(t_*)dt_*$$

The ratio ${}_ST_{11}/{}_ET_{11}$ calculated from the above relationship is shown in Figure 7. Another ratio of time scale, $\beta = {}_LT_{11}/{}_ET_{11}$ (Pasquill's Beta), is readily evaluated by multiplying ${}_LT_{11}/{}_ST_{11}$ by ${}_ST_{11}/{}_ET_{11}$. Figure 8a is produced from Figures 6 and 7 for selected α. Based on atmospheric observations, Hay and Pasquill (1959) recommended a value of 4.0 for α, which is independent of wind speed and stability. Wandel and Kofoed-Hansen (1962) examined the Lagrangian and Eulerian energy spectra for a fully developed isotropic homogeneous turbulence. They established a more complicated relation between the Eulerian and Lagrangian correlation based on the statistical theory of "shot" noise and the Helmholts theorem. A rough approximation in the case of smooth energy spectra indicates that $\beta \simeq \frac{\sqrt{\pi}/4}{i}$ where i is the turbulence intensity $\frac{[u_1^2]^{1/2}}{U}$.

Corrsin (1963b) compared the shape of Lagrangian and Eulerian energy spectra over the inertial subrange. By assuming that the total turbulent energy was identical in the Lagrangian and Eulerian system, he arrived at a theoretical prediction for β of

$$\beta = {}^c/_i$$

where c is a constant.

By monitoring the trajectories of tetroons and a tethered balloon system at height of 750 m, Angell (1964) observed an average value for α near 3.3 and recommended a relation equal to 0.4/i for β. Angell et al. (1971) made further observations in the atmosphere near Las Vegas, Nevada, by releasing tetroons past tall towers. β was again found to have average values near 3 and varied inversely with the turbulence intensity.

Snyder and Lumley (1971) performed direct measurements of Lagrangian velocity in a grid-generated turbulence field in a laboratory. The fluid particle was simulated by releasing single spherical beads with different weights and sizes. Since light particles have only small inertia and cross-trajectory effect, light-particle correlations were utilized to estimate Lagrangian fluid properties. They concluded that the Lagrangian autocorrelation function has similar shape to the Eulerian spatial correlation and the β is roughly equal to 3 when approximated by 1/i.

Turbulence measurements reported by Hanna (1981) were conducted in the daytime mixing layer near Boulder, Colorado. The average Lagrangian time scale detected was about 70 seconds for a sampling time of 15 minutes. The ratio β was found to be 1.7 and inversely proportional to turbulence intensity, $\beta = 0.7/i$.

These results are presented together with some atmospheric observations in Figure 8b for comparison.

If the atmospheric turbulence intensity is expected to be in the vicinity of 0.1, the present analysis gives a reasonable estimation of $3<\beta<7$ which agrees with the atmospheric observations.

Figure 9 presents ${}_LR_{11}(t_*)$ for various values of α and $\Gamma{}_ST_{11}$. The existence of mean shear does not change the Lagrangian autocorrelation function at small t_* ($t_* < 0.25$) but results in a faster decay at larger t_*. $\Gamma{}_LT_{11}/{}_ST_{11}$ is plotted in terms of $\Gamma{}_ST_{11}$ for different α in Figure 10 to emphasize the shear effect. ${}_{l}T_{11}/{}_{S}T_{11}$ approach 5.0 for large α. Figure 10 may be used together with Figure 7 to predict β in an isotropic homogeneous uniform shear flow as long as such turbulence parameters as $[u_1^2]$, L, Γ and ${}_{S}T_{11}$ are specified in the turbulence field. Figure 11 displays the predicted β contours for i=0.1. Note that β falls between 3.0 and 5.0 for a wider range of α in the presence of shear than without the presence of shear (e.g., β = 3.0~5.0 for α= 0.35~3.3 and $\Gamma_{S}T_{11}$ = 2.0 while β=3.0~5.0 for α = 0.19~1.0 and $\Gamma_{S}T_{11}$ = 0.0). For atmospheric turbulence, $\frac{\alpha}{i}\approx\frac{1.0}{0.1}=10.0$;hence, an averaged β = 4.0 for various strain rates agrees very well with field observations.

It is difficult to provide complete information about the estimated Lagrangian statistics for a non-isotropic turbulent flow since there are four parameters involved, namely, α_1, α_2, α_3 and $\Gamma_{S}T_{11}$. However, the numerical procedure can perform an estimation for any specified combination of these four parameters. Figure 12 presents results for ${}_{L}T_{11}/{}_{S}T_{11}$ in an ideal one dimensional non-isotropic uniform turbulent flow by assuming α_2 = α_3 = 0. The magnitude of the Lagrangian autocorrelation function as well as the scale ratio is gradually reduced once the turbulence field is expanded from one-dimension to three-dimension. Figure 13 presents the resultant ${}_{L}T_{11}/{}_{S}T_{11}$ ratio for two-dimensional turbulent flow with uniform velocity. ${}_{L}T_{11}/{}_{S}T_{11}$ for a designated three-dimensional turbulence with α_2 = α_3 is given in Figure 14. Reduction of ${}_{L}T_{11}/{}_{S}T_{11}$ due to the redistribution of turbulent energy is clearly observed in Figures 12 to 14. Variation in ${}_{L}T_{11}/{}_{S}T_{11}$ resulting from different α_2 and α_3 values is plotted in Figure 15 for an α_1 = 1.0. ${}_{L}R_{11}/{}_{S}T_{11}$ may be obtained without significant error by using the averaged value of α_2 and α_3 from Figure 14. Figure 14 (Figure 13) can be used in connection with Figure 10 as the first approximation for ${}_{L}T_{11}/{}_{S}T_{11}$ in a non-isotropic uniform shear flow. Similarly, β may be estimated from Figure 10 in conjunction with Figure 7.

Both Philip (1967) and Saffman (1963) employed the Independence

Hypothesis to estimate Lagrangian autocorrelations and scales. The principal difference between Philip's analysis and the present study is the choice of a functional form for the general Eulerian space-time correlation in the mean convective frame of reference. Philip employed the following function

$$_ER_{11}(x_1,\rho,\tau) = \exp\{-\frac{\pi}{4}(\frac{x_1^2 + 3.138\ \rho^2}{L^2} + \frac{\tau^2}{_ST_{11}^{\ 2}})\}$$

where $\rho = (x_2^2 + x_3^2)^{\frac{1}{2}}$

Phillip predicted Lagrangian autocorrelation functions which appear as a family of bell shaped curves for various parametric values of α in a manner quite similar to Figure 4b of the present analysis. Philip's results for $_LT_{11}/_ST_{11}$ are reproduced and presented in Figure 6. The deviations between his results and Model II are due to the use of an averaged integral length scale regardless of the variation in the space correlation. The averaging process for L becomes more accurate in the region where the two curves intercept in Figure 6, $\alpha \simeq 1.0$.

Saffman assumed a functional form for the Eulerian spectral density function rather than an equivalent general space-time correlation function. A transform from his spectral density function to the equivalent Eulerian space-time correlation function revealed that

$$_ER_{11}(r,\theta,\tau) = e^{-\frac{1}{2}\eta^2}\{1 - \frac{1}{2}\eta^2(1-\cos^2\theta)\}(\frac{L}{\sqrt{\pi}\Lambda})^5 \ ,$$

$$\text{where} \qquad \eta = \frac{r}{\sqrt{2\Lambda}} \quad \text{and} \quad \Lambda = \{\frac{L^2}{\pi} + \frac{1}{2}[u^2]\tau^2\}^{\frac{1}{2}} \ . \tag{3.1}$$

The time dependence of the general Eulerian space-time correlation function resides in the definition of Λ. The spatial variation of the Eulerian correlation, Equation ( ), is in accordance with the Karman-Howarth equation for a Gaussian functional form, $e^{-\frac{\pi}{4}\left(\frac{x_1}{L^2}\right)^2}$, chosen as the axial space correlation. $F_1(t_*)$ is then implied in Equation 3.1 is $[1+\frac{\pi[u_1^2]}{2\,L^2}\tau]^{-\frac{5}{2}}$. A direct integration of the above expression yields the value of $_ST_{11} = 0.53\frac{L}{[u_1^2]^{1/2}}$ or α=0.53. Hence, Saffman implicitly fixed to the unique value 0.53, and this led directly to the unique proportionality which he derived between the Eulerian group α and the Lagrangian time scale. The scale ratio $_LT_{11}/_ST_{11}$ is determined as $_LT_{11}/_ST_{11}$ = 0.75 as follows from his result, $_LT_{11} = 0.40L/[u_1^2]^{1/2}$. Figure 6 includes this result for comparison purposes. By combining these results with the expression Saffman derived for the autocorrelation scale of the cross-wind components and assuming isotropy to obtain thanxial scale $_ET_{11}$, Saffman's prediction of Pasquill's may be written as

$$\frac{_LT_{11}}{_ET_{11}} = 0.355\{i[\frac{\sqrt{\pi}}{4}\ (1 + i^2)\ \mathrm{erf}(\frac{1}{2i^2})^{\frac{1}{2}} - 4(2i^2)^{\frac{1}{2}}\exp(-\ \frac{1}{2i^2})]\}^{-1} \quad (3.2)$$

This relationship labeled "Saffman" is plotted in Figure 8b for comparison with the results of the present analysis.

Favre (1965) has proposed a method for calculating the Eulerian space-time correlation from the space correlation through the Independence Hypothesis. Townsend (1976) also developed an analysis to obtain the space-time correlation starting from the space-time structure function. Both analyses took account of the mean particle displacement in the first approximation and resemble one another. Instead of using the exact form, the mean square particle displacement is assumed to have its asymptotic value as

$$[X_1^2(t)] = [u_1^2]t^2.$$

Such an approximation automatically limits the validity of the approach to short diffusion times. Their estimated Eulerian space-time correlation may be expressed in terms of the present derivation as

$$F_1(t_*) = G(\ ,t_*),$$

and (3.3)

$$\int_0^\infty G(\alpha,t_*)dt_* = 1 \ ,$$

where $G(\alpha,t_*)$ represents the { } term in Equation 20.

If one applies their estimated $F_1(t_*)$ to the present analysis, the Lagrangian autocorrelation function is found to be

$${}_LR_{11}(t_*) = G^2(\alpha,t_*)$$

and (3.4)

$$\int_0^\infty G(\alpha,t_*)dt_* = 1 \ .$$

$F_1(t_*)$ calculated from Equation 3.3 has been compared with the measured results from Favre et al. (1962) by Townsend. For a time delay of $U_1t/M = 7.57$, the effective value of $\alpha^2 I(t_*)$ is found to be 0.0648, and the estimated value of $F_1(t_*)$ is 0.48 compared to a measured maximum correlation coefficient 0.41 in Favre's experiment. The value 0.48 is different from Townsend's calculation of 0.85 but consistent with Favre's computation.

A consequence of the form of $[X_1^2(t_*)]$ adopted in such an analysis is that the predicted $F_1(t_*)$ is based on the knowledge of a known mean square particle displacement (equivalently, the Lagrangian autocorrelation function). Hence, the methodology employed in their analysis is quite similar to the present approach, except that the unknown in their analysis is the known variable in the present study and vice versa. In order to satisfy $\int_0^\infty G(\alpha,t_*)dt_* = 1$, the present analysis yields $\alpha \approx 1.1$ and ${}_LT_{11}/{}_ST_{11} = 0.335$ according to Equation 3.4. The data point marked as Favre and Townsend in Figure 6 shows their result is close to the prediction using Comte-Bellot and Corrsin's measured $F_1(t_*)$.

Lee and Stone (1983) have adopted Equation 2.9 in their Monte Carlo simulation of one-dimension turbulent diffusion. An analytical expression to predict the Lagrangian statistics from Eulerian statistics was also presented. The analytical solution for cloud growth compared favorably with the results from the Monte Carlo simulation, and both results agreed with Lagrangian statistics estimated from the present analysis (Figure 5). Lee and Stone approximated the Lagrangian autocorrelation function at a short time increment by the Eulerian space-time correlation, and they assumed that the velocity fluctuations is normally distributed with zero mean and standard deviation $[u_1^2]^{1/2}$. The Lagrangian autocorrelation function at δt can be obtained as

$$_L R_{11}(\delta t) = (\frac{2}{\pi})^{\frac{1}{2}} \exp(- \frac{\delta t}{_S T_{11}}) \int_o^\infty n^2 e^{-an} e^{-\frac{n^2}{2}} dn \qquad (3.5)$$

$$\text{where } a = \frac{[u^2]^{1/2} \delta t}{L}$$

Li and Meroney (1984) proved that Equation 3.5 can be obtained from the present analysis by replacing $_E R_{11}(x_1', x_2', x_3'; \tau)$ with $\exp\ (-t_*)\ \exp\ (-x_1'/L)$ and assuming the asymptotic approximation, $I(\delta t_*) = \delta t_*^2/2$. Therefore, the one-dimensional Monte Carlo simulation of turbulence is mathematically similar to the present analysis but with simplification in the general Eulerian space-time correlation function.

5. Conclusion

The present analysis presents a method to estimate the Lagrangian autocorrelation function from some fixed-point Eulerian measurements in a turbulence field. The shape of the predicted Lagrangian autocorrelation function strongly depends on the time dependent portion of the Eulerian space-time correlation. Furthermore, the spatial portion of the Eulerian space-time correlation and the probability density function used in the present analysis are in good agreement with the data available for approximately isotropic turbulence. The expression of the Eulerian space-time correlation as a product of space and time correlations in a convective frame provides mathematical simplicity. Experimental results in an approximately isotropic turbulence field support such a formulation.

The estimated Lagrangian autocorrelation function and its integral scale obtained from the present approach agree well with estimates reported from different analyses. The calculated Lagrangian-Eulerian scale ratios also agree with atmospheric observations. Unfortunately, most previous analyses were conducted for an isotropic uniform turbulent flow. Simultaneous measurements of turbulent diffusion and Eulerian velocity statistics have only been performed by Li and Meroney (1984). In Part II of this study, laboratory measurements of turbulent diffusion are compared with diffusion behavior calculated from Lagrangian estimates determined from Eulerian space-time correlation measured in a thick high Reynolds number boundary layer. In conclusion, the present analysis requires only two fixed-point Eulerian measurements to yield estimates for the Lagrangian autocorrelation function and the Lagrangian integral scale.

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Table 1.

x_2/M	$E^R_{11}(\frac{x_1}{M} = 4, \frac{x_2}{M}, 0; \tau_m)$ Measured* Comte-Bellot and Corrsin (1971)	Calculated
0	0.72	0.72
0.05	0.70	0.61
0.125	0.58	0.48
0.225	0.48	0.34
0.40	0.30	0.18

*L = 2.40 cm
M = 5.08 cm, Mesh size.

x_3 (cm)	$E^R_{11}(x_1 = 30.5$ cm, $0, x_3; \tau_m)$		$E^R_{11}(x_1 = 61.0$ cm, $0, x_3; \tau_m)$	
	Measured** Champagne et al. (1970)	Calculated	Measured Champagne et al. (1970)	Calculated
0	.75	.75	.68	.68
1.27	.54	.51	.56	.46
2.54	.36	.34	.43	.31
3.81	.23	.22	--	--
5.08	--	--	.20	.13

*L = 5.08 cm

LIST OF FIGURES

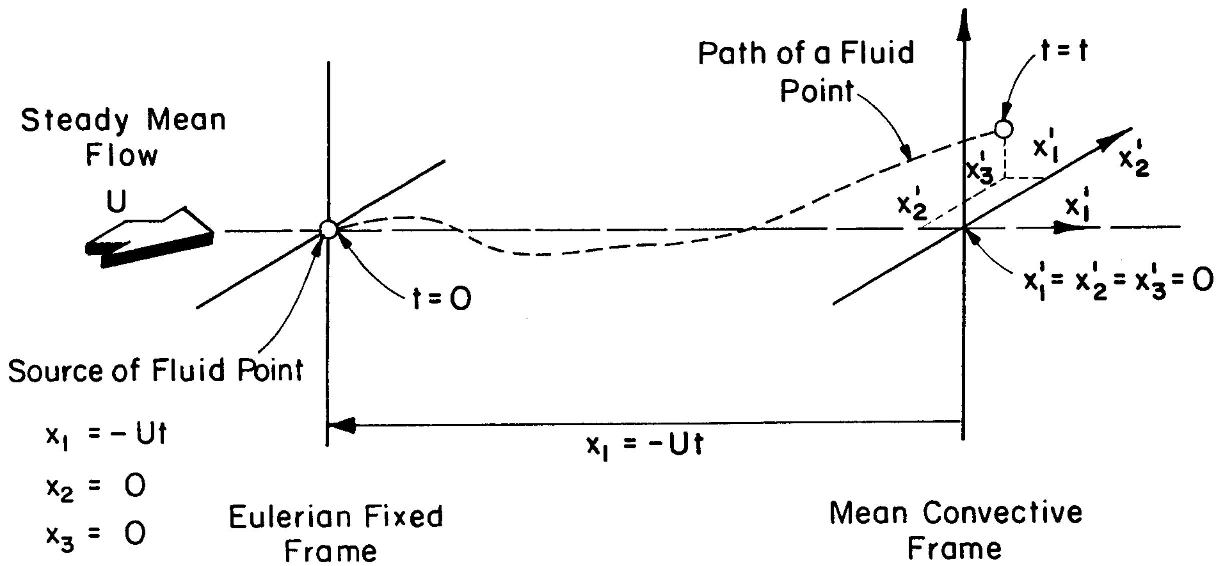


Figure 1 Mean Convective Frame of Reference

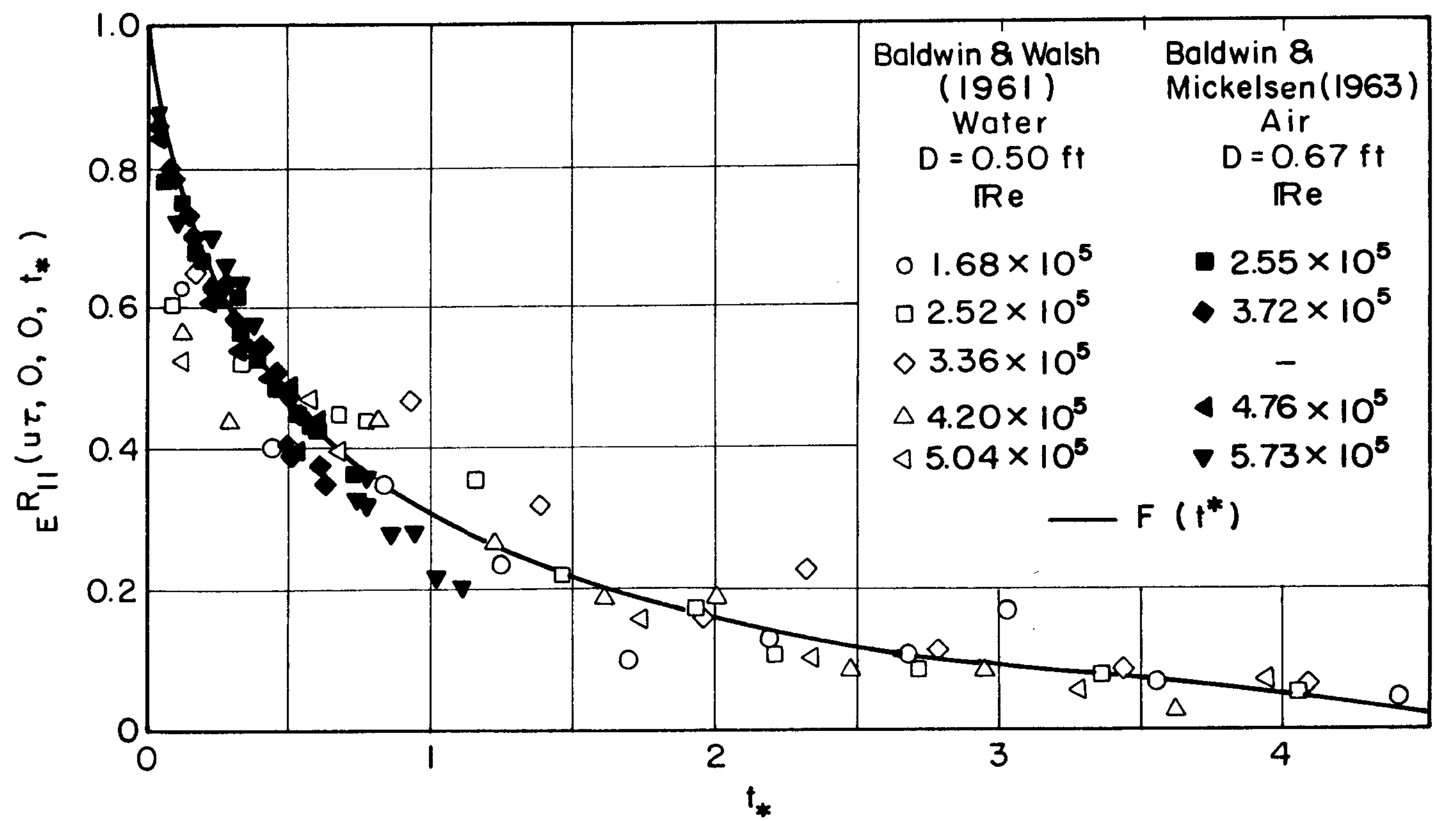


Figure 3 Space-Time Correlations Measured Along Centerline of Fully Developed Pipe Flow

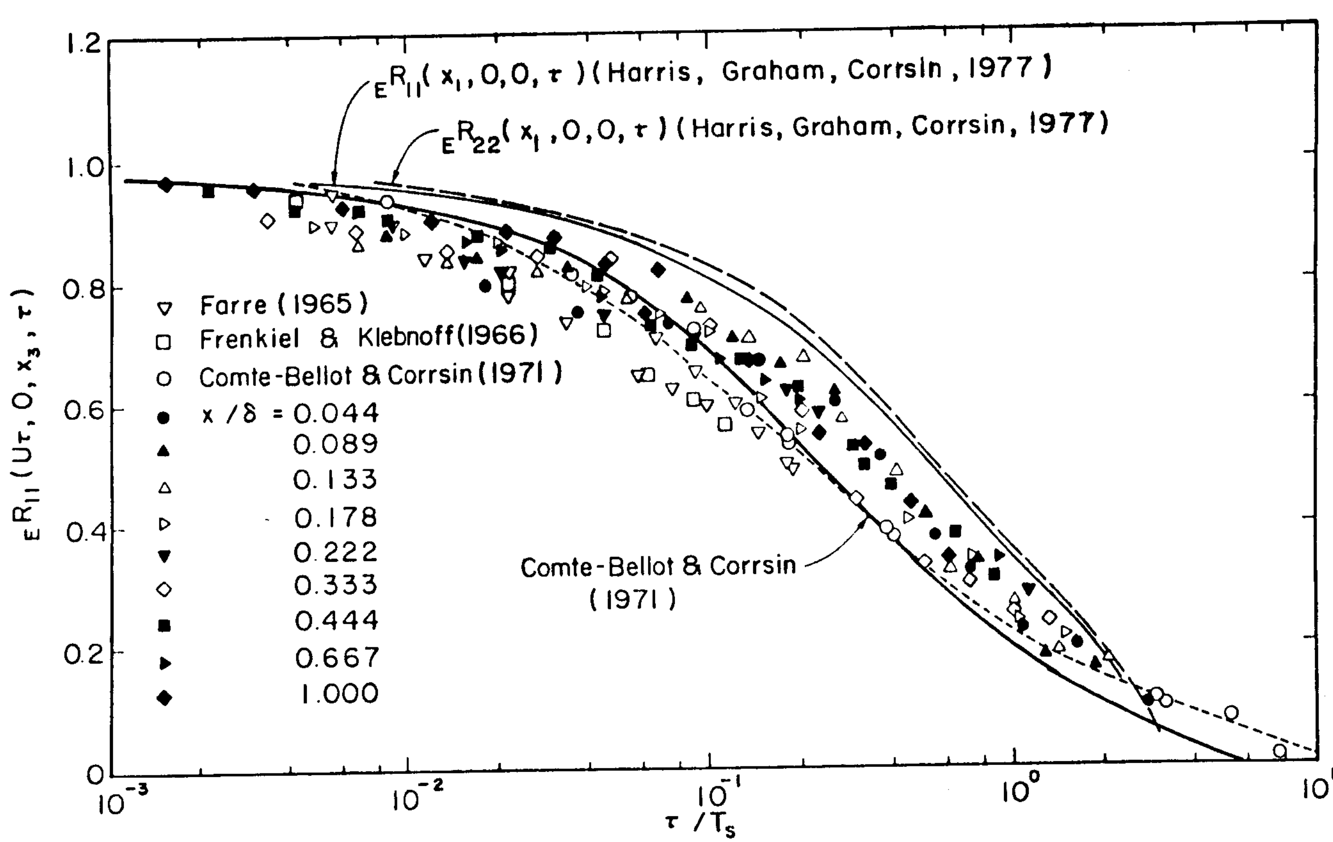
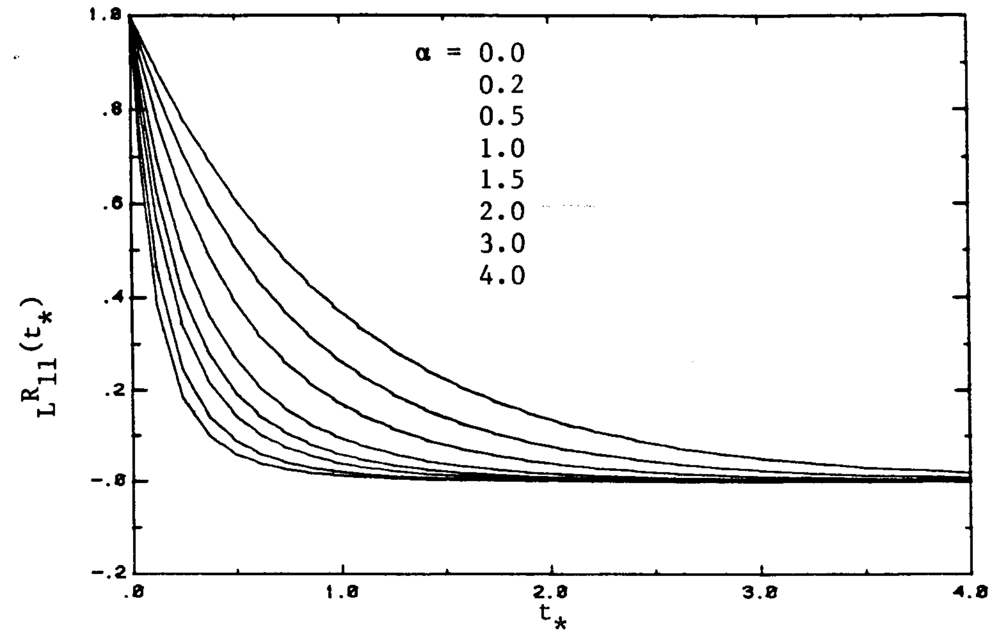
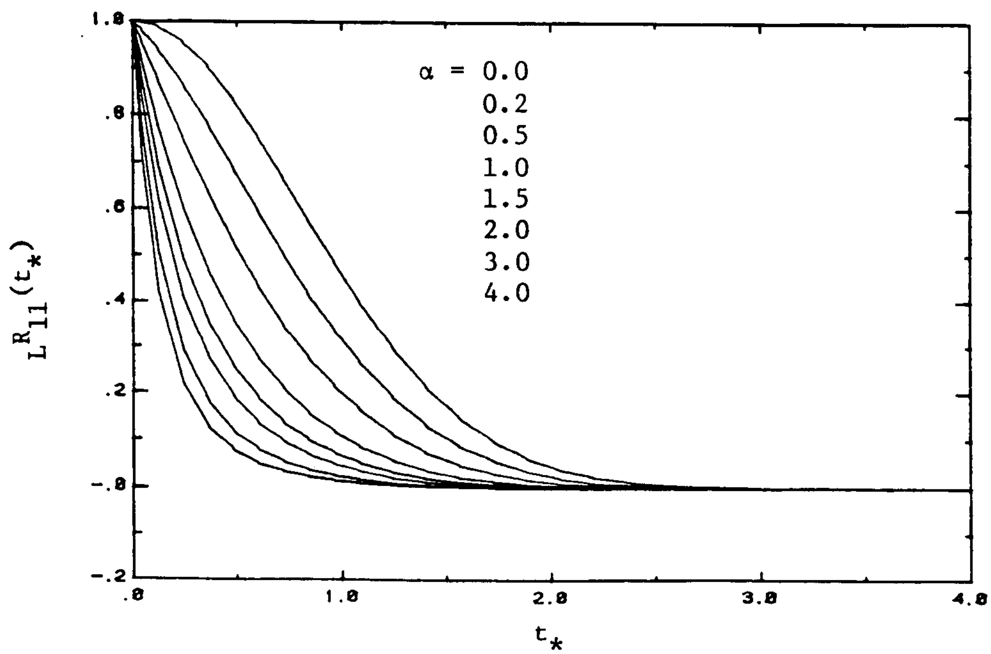


Figure 2 Normalized longitudinal space-time correlation in the boundary-layer (filtered)

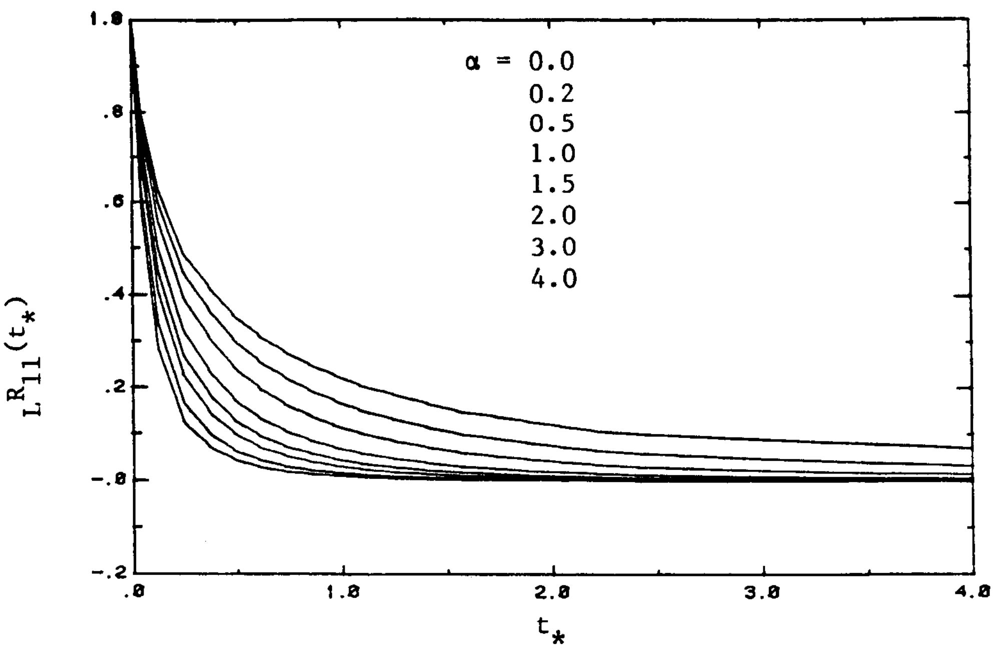


(a) Model I: $F(t_*)=\exp(-t_*)$

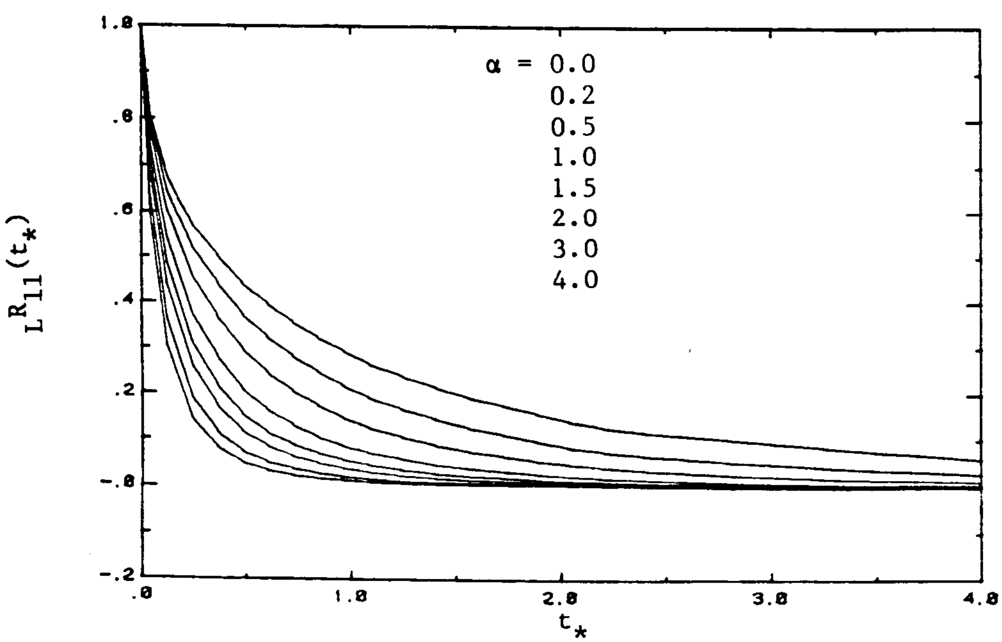


(b) Model II: $F(t_*)=\exp(-\frac{\pi t_*}{4}$

Figure 4 Lagrangian autocorrelation functions predicted by various model curves for $F_1(t_*)$

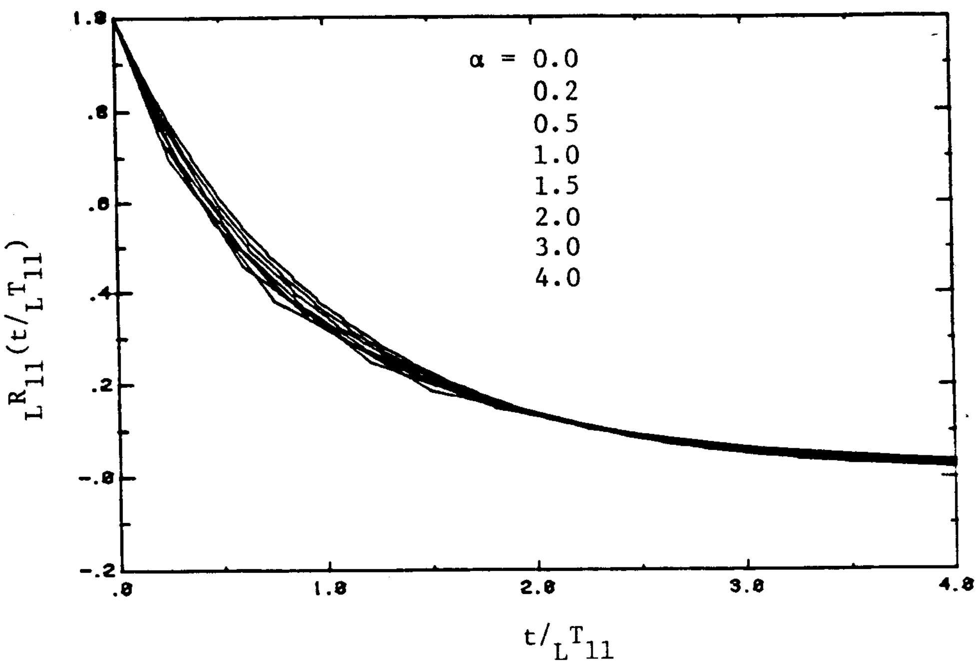


(c) Model III: Solid line in Figure 2

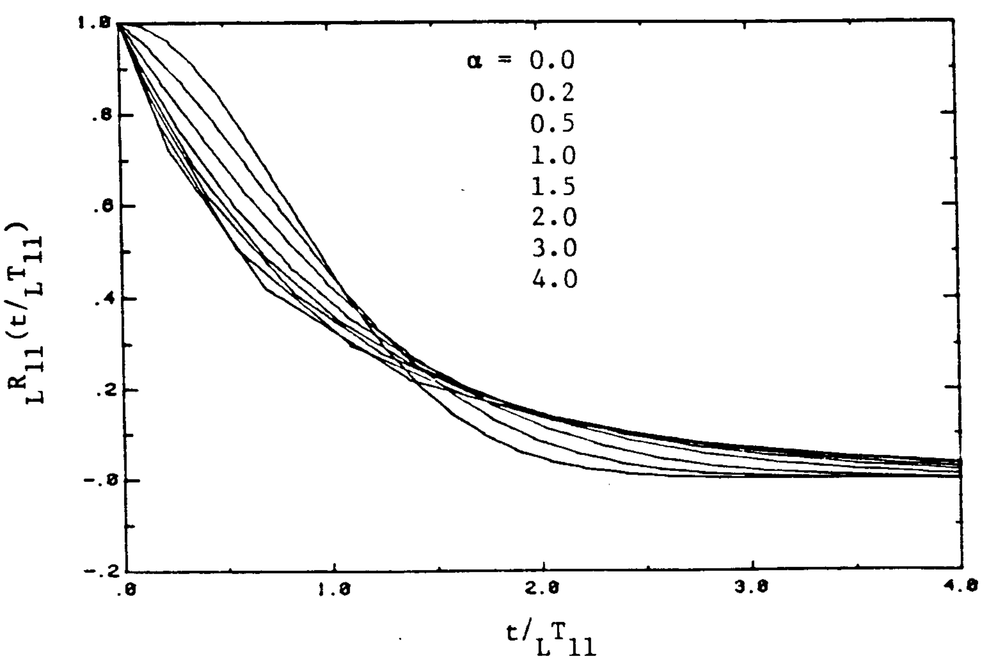


(d) Model IV: Dotted line in Figure 2

Figure 4 Lagrangian autocorrelation functions predicted by various model curves for $F_1(t_*)$

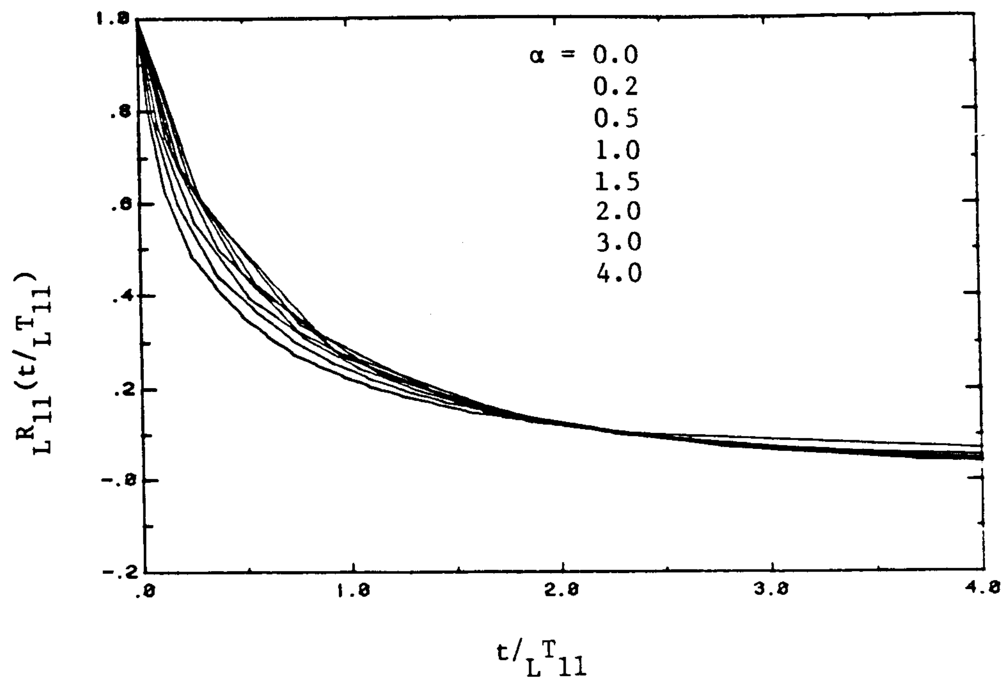


(a) Model I

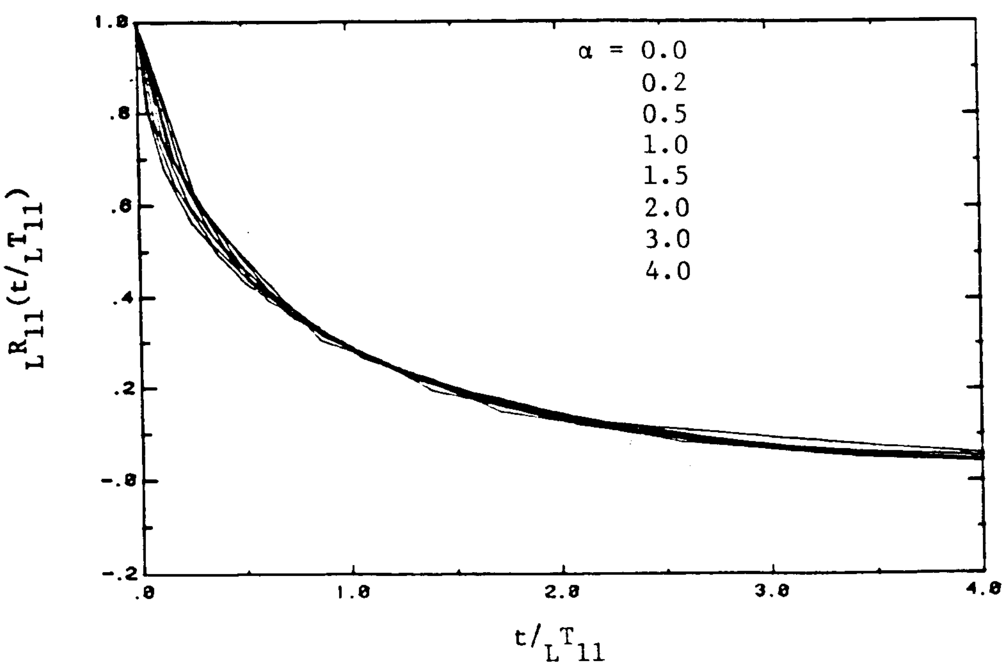


(b) Model II

Figure 5 Normalized Lagrangian autocorrelation functions predicted by various model curves for $F_1(t_*)$



(c) Model III



(d) Model IV

Figure 5 Normalized Lagrangian autocorrelation functions predicted by various model curves for $F_1(t_*)$

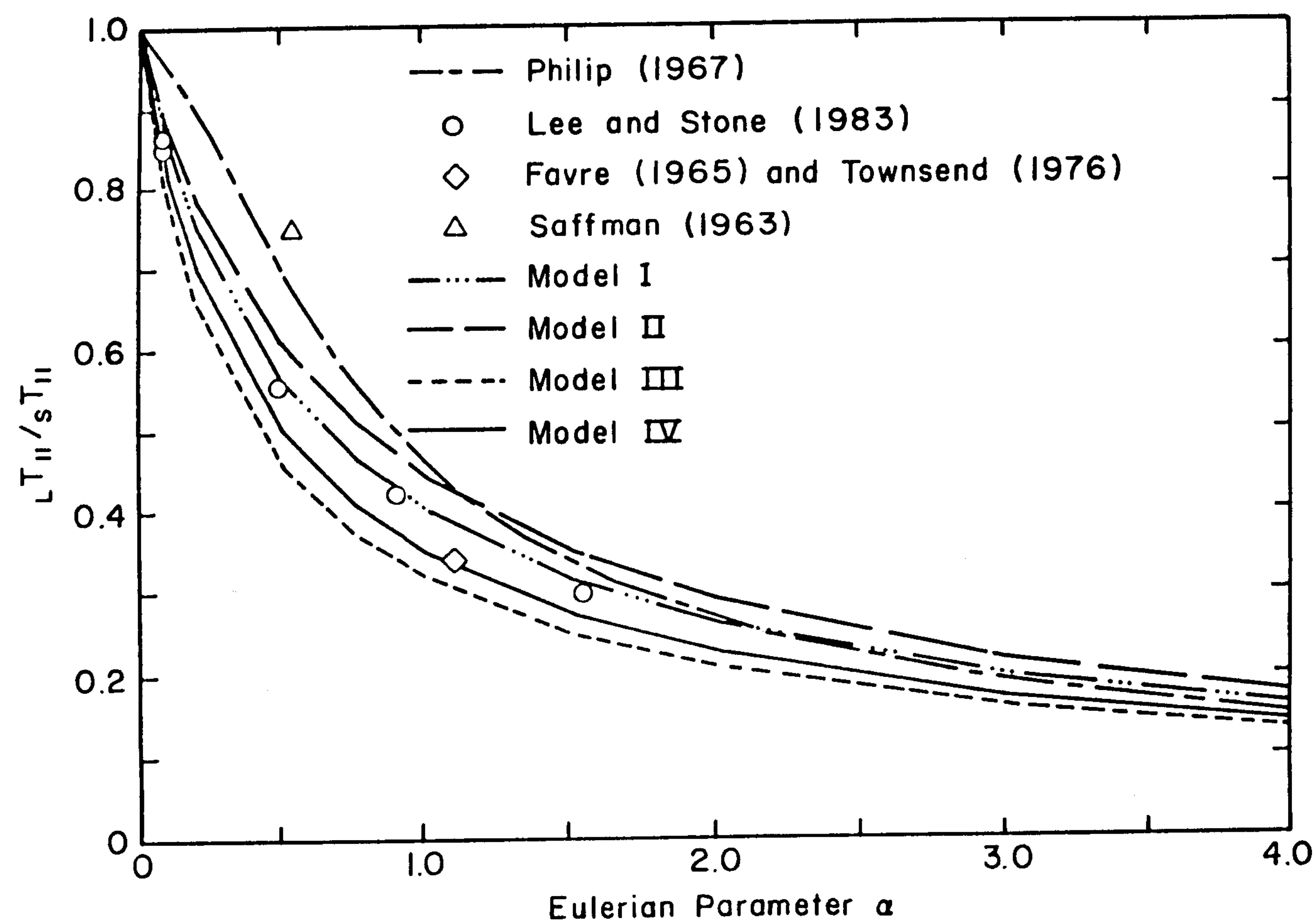


Figure 6 Lagrangian autocorrelation function parameterized by α

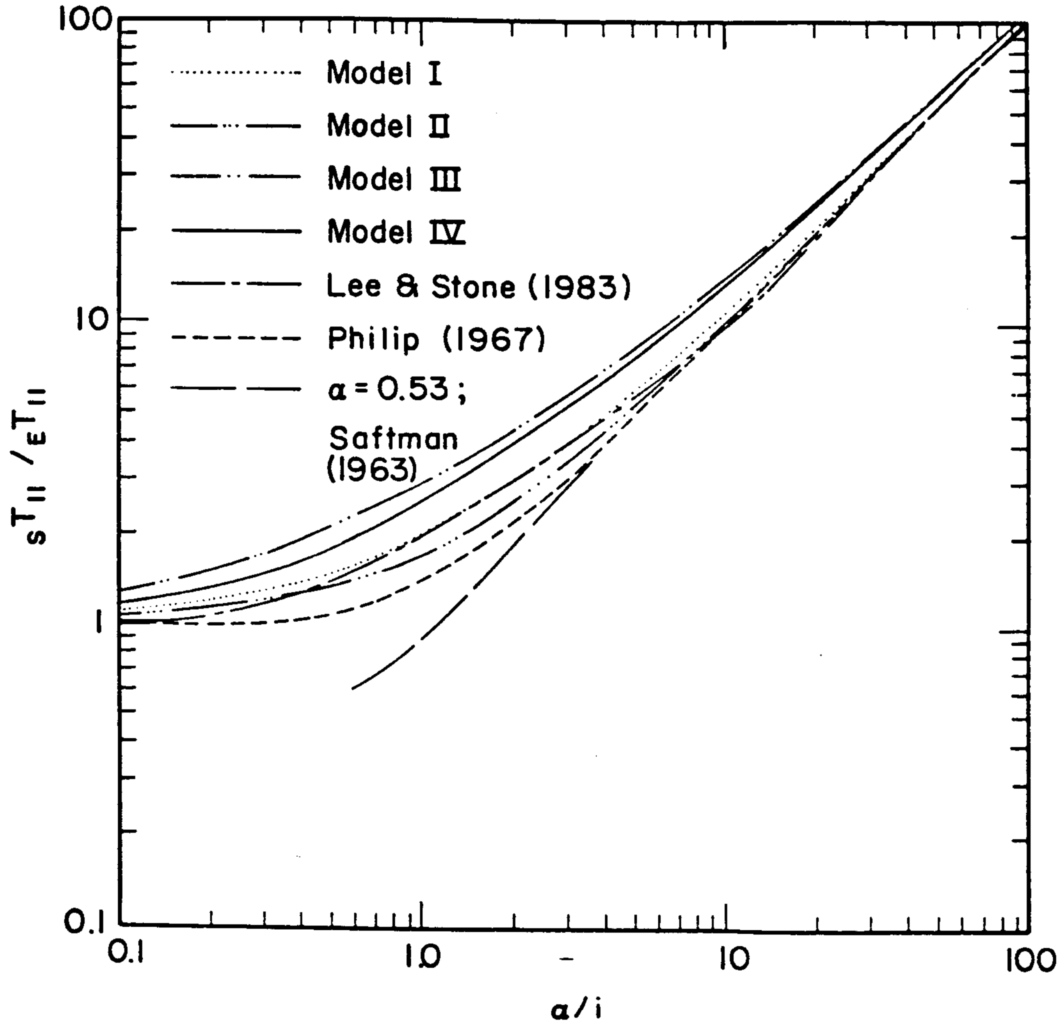


Figure 7 Ratio of space-time to Eulerian Integral time scales versus α

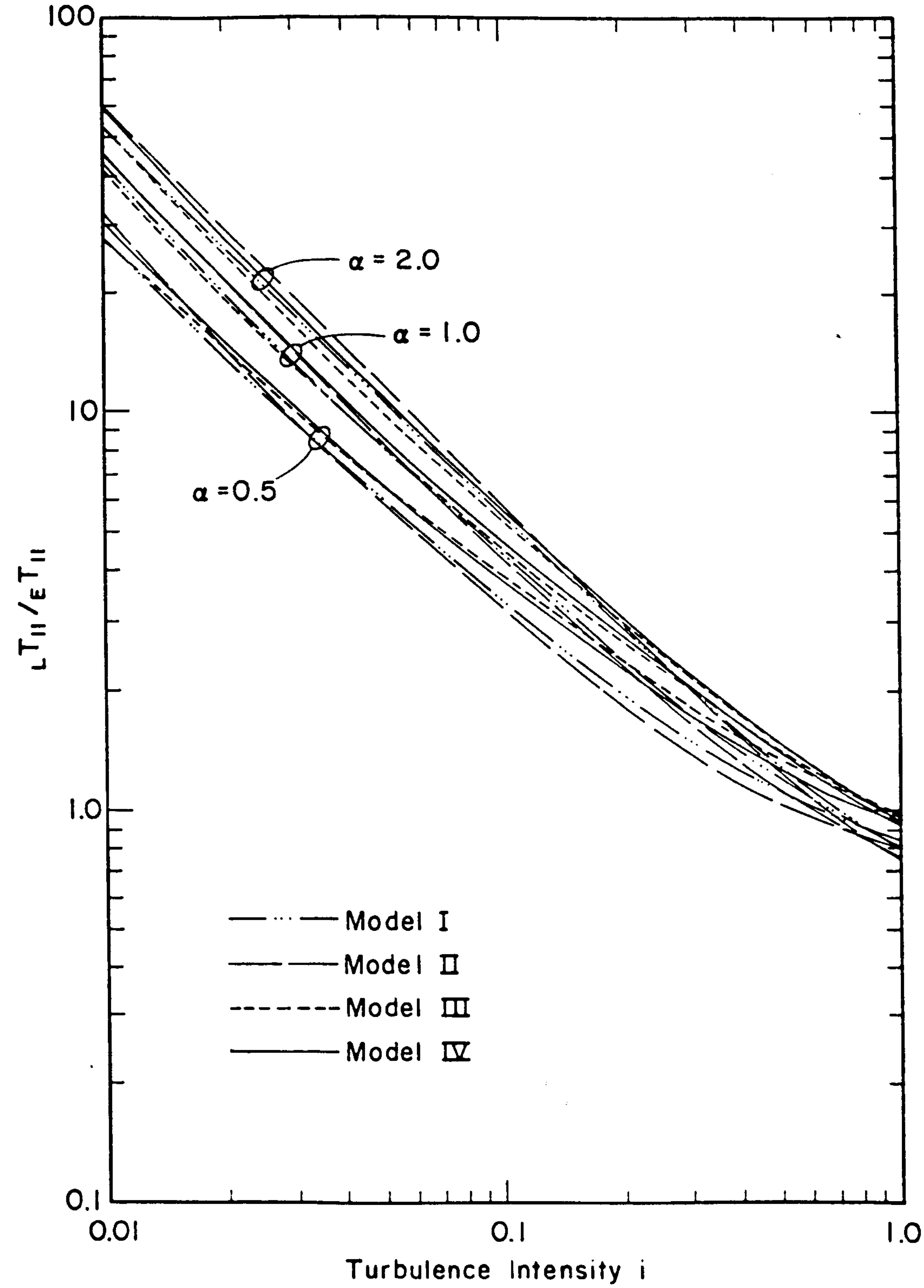


Figure 8a Ratio, β, of Lagrangian to Eulerian Intergral time scales versus α

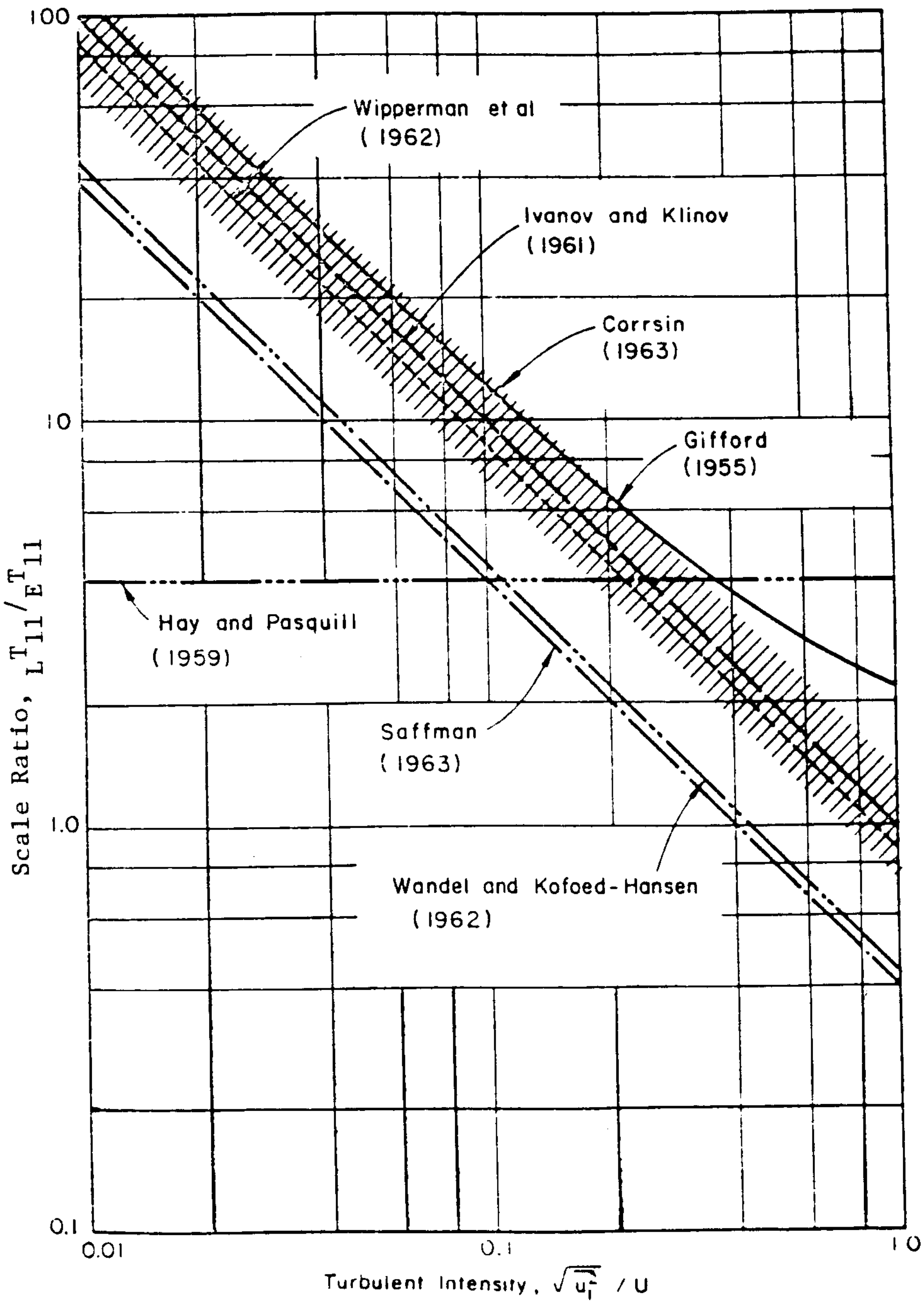
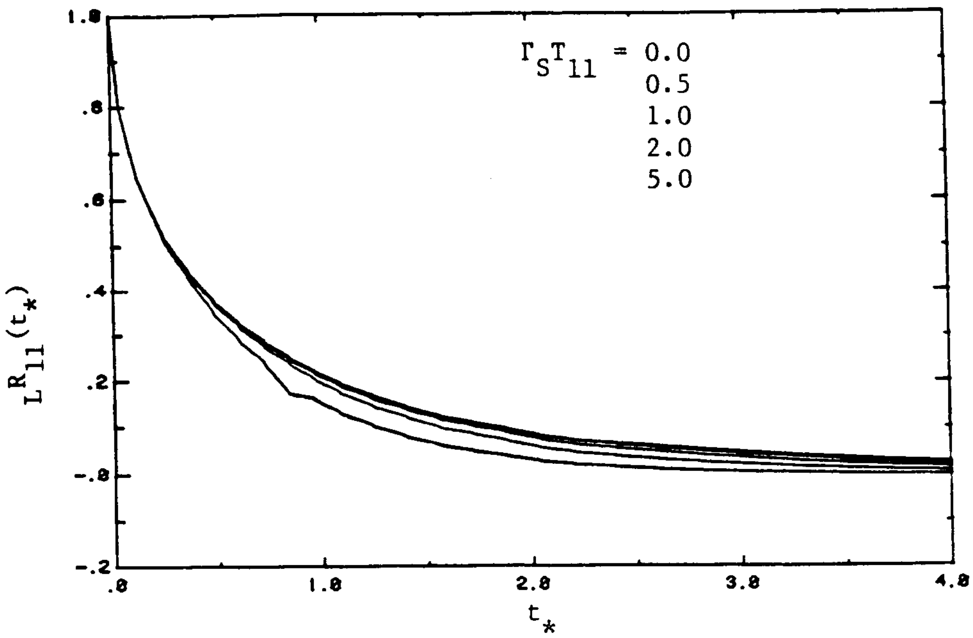
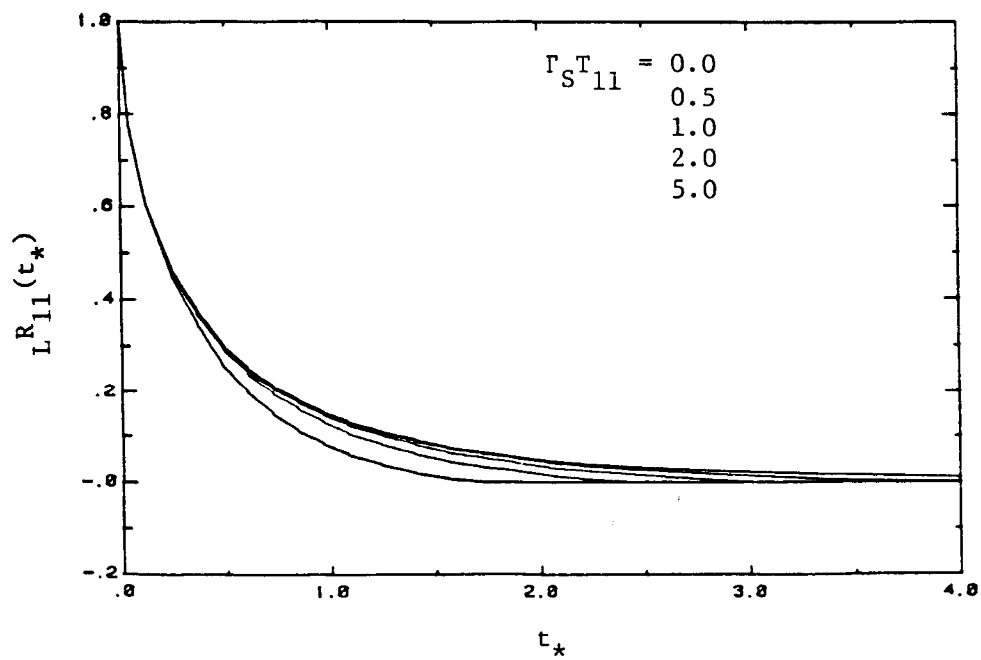


Figure 8b Measured values of ratio β

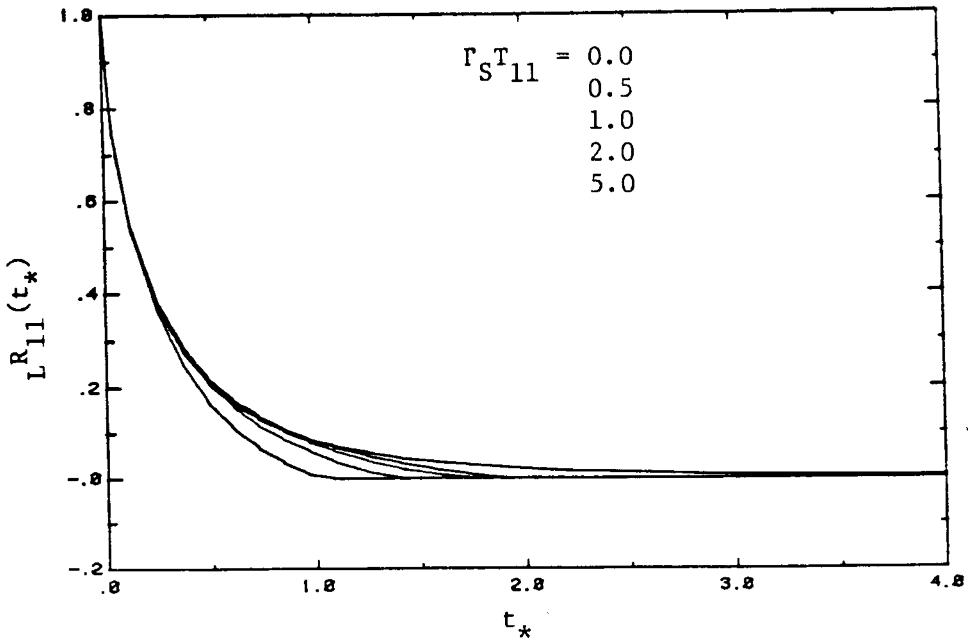


(a) $\alpha = 0.2$, $\Gamma_S T_{11} \neq 0$

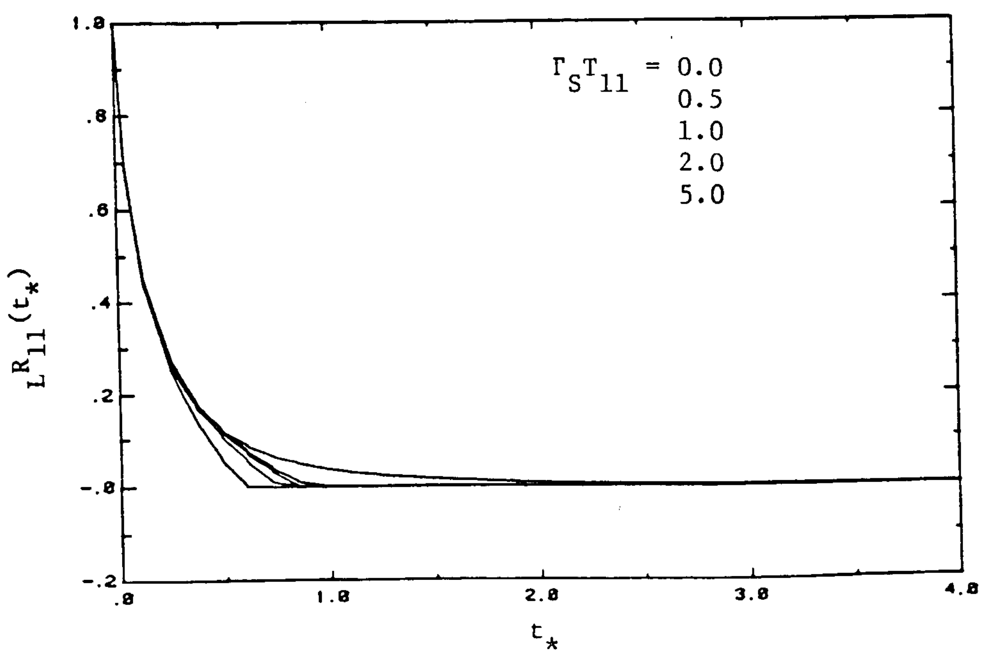


(b) $\alpha = 0.5$, $\Gamma_S T_{11} \neq 0$

Figure 9 Lagrangian autocorrelation versus α and $\Gamma_s T_{11}$



(c) α = 1.0 , $\Gamma_S T_{11} \neq 0$



(d) α = 2.0 , $\Gamma_S T_{11} \neq 0$

Figure 9 Lagrangian autocorrelation versus α and $\Gamma_s T_{11}$

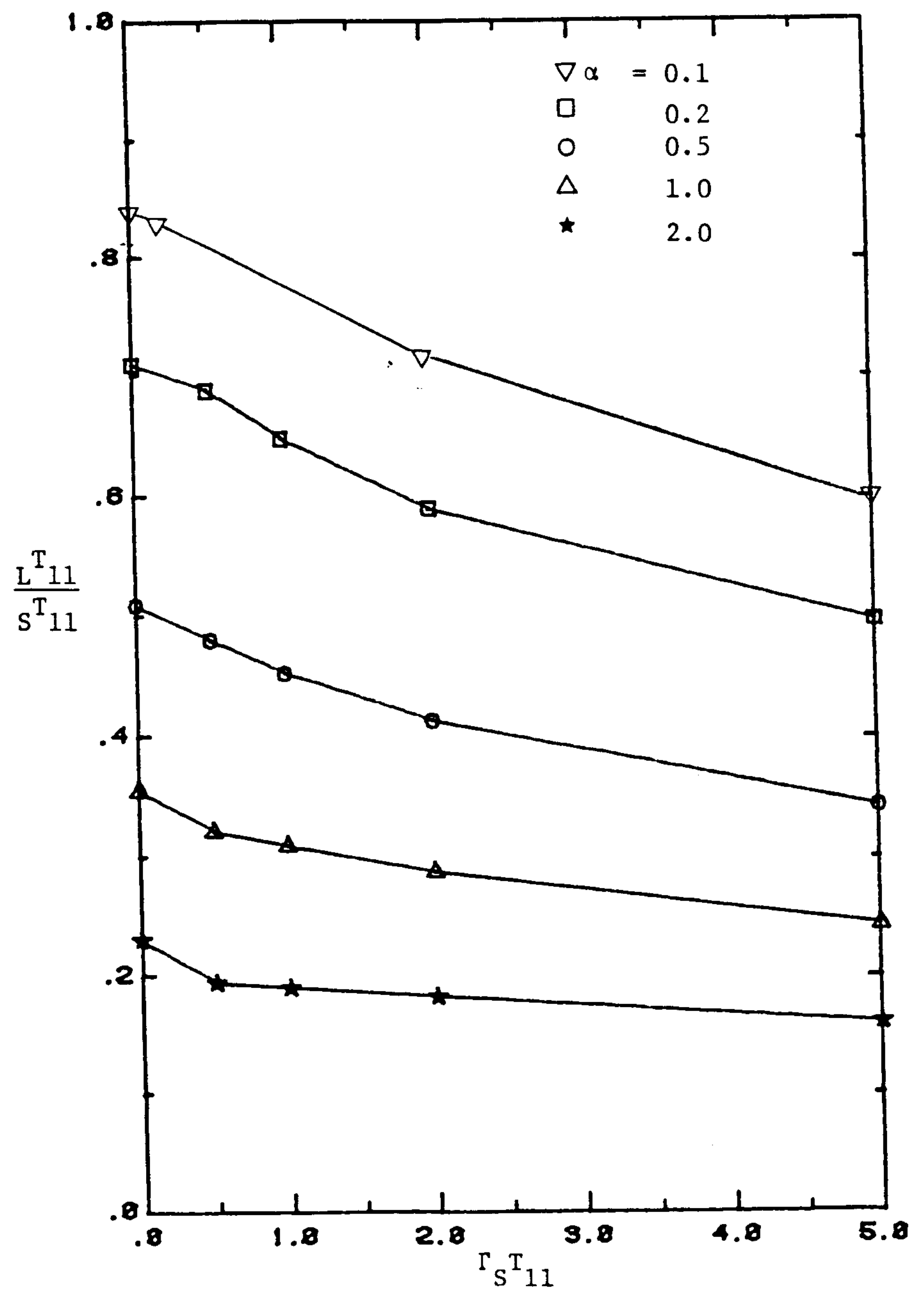


Figure 10 Ratio of sheat modified Lagrangian to Space-time integral time scales, $\Gamma_L T_{11}/_s T_{11}$, versus $\Gamma_s T_{11}$ and α

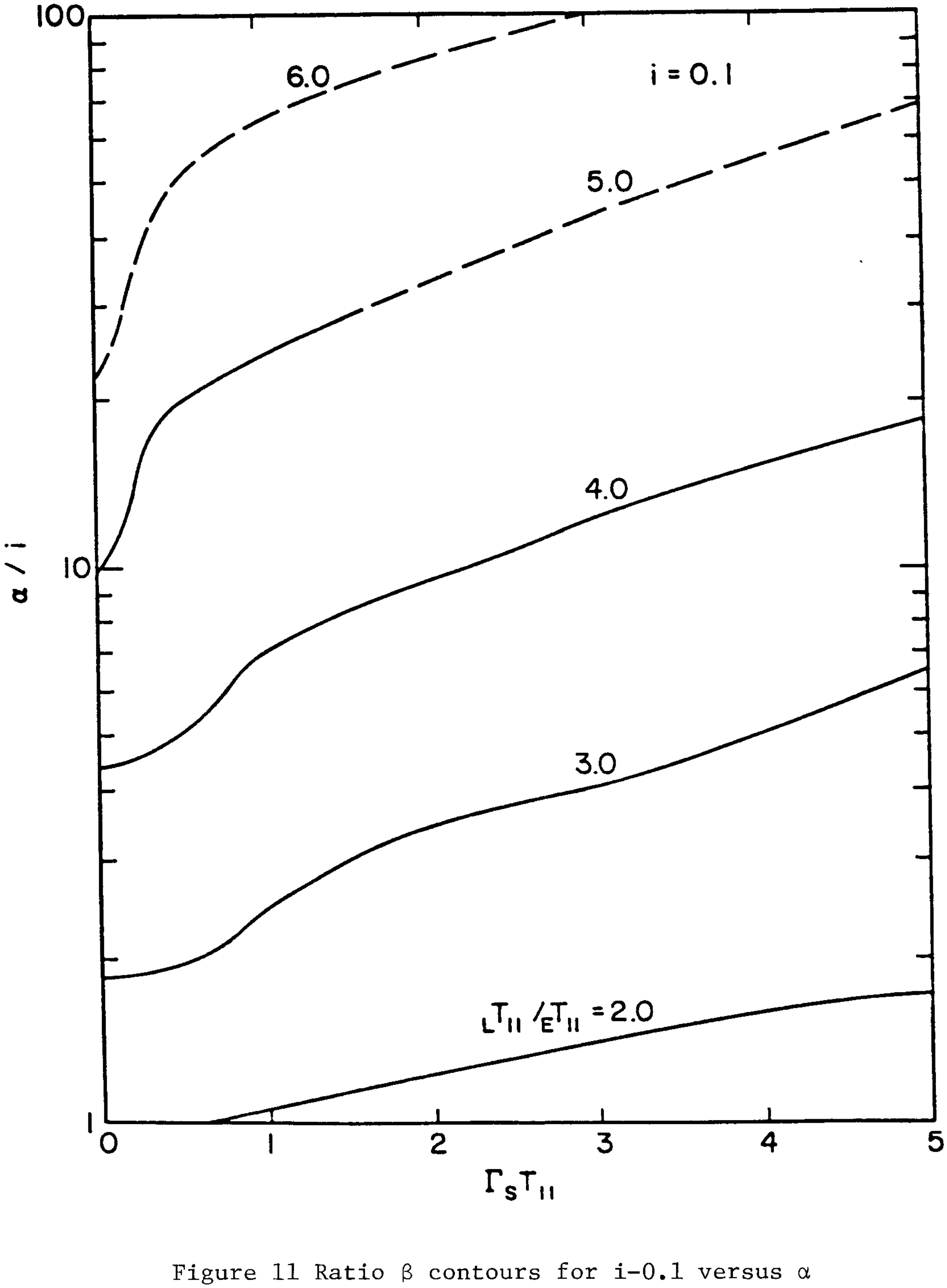


Figure 11 Ratio β contours for i-0.1 versus α

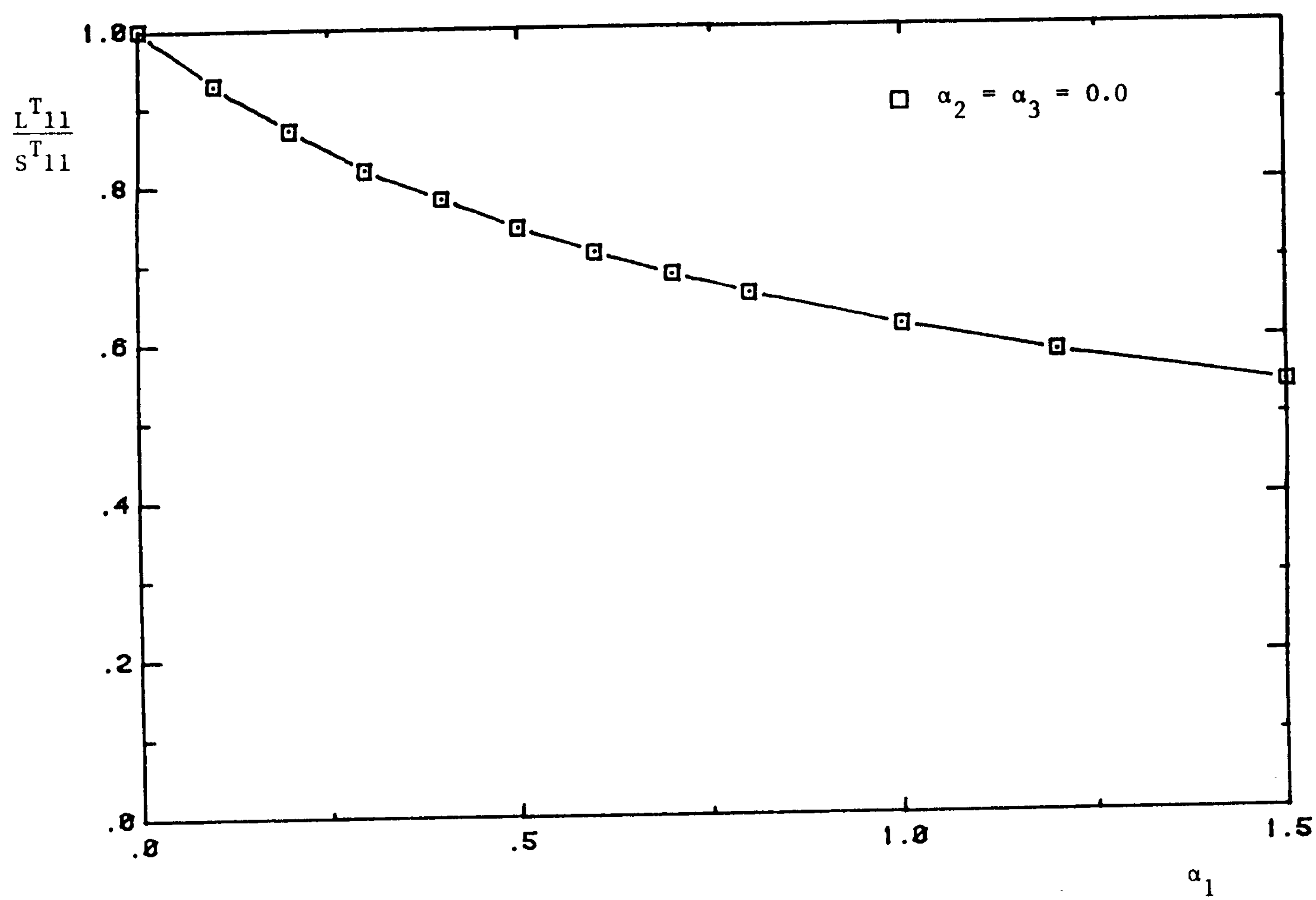


Figure 12 Ratio of Lagrangian to Space-time integral time scales for one dimensional non-isotropic uniform turbulent flow assuming $\alpha_2=\alpha_3=0$ versus α_1

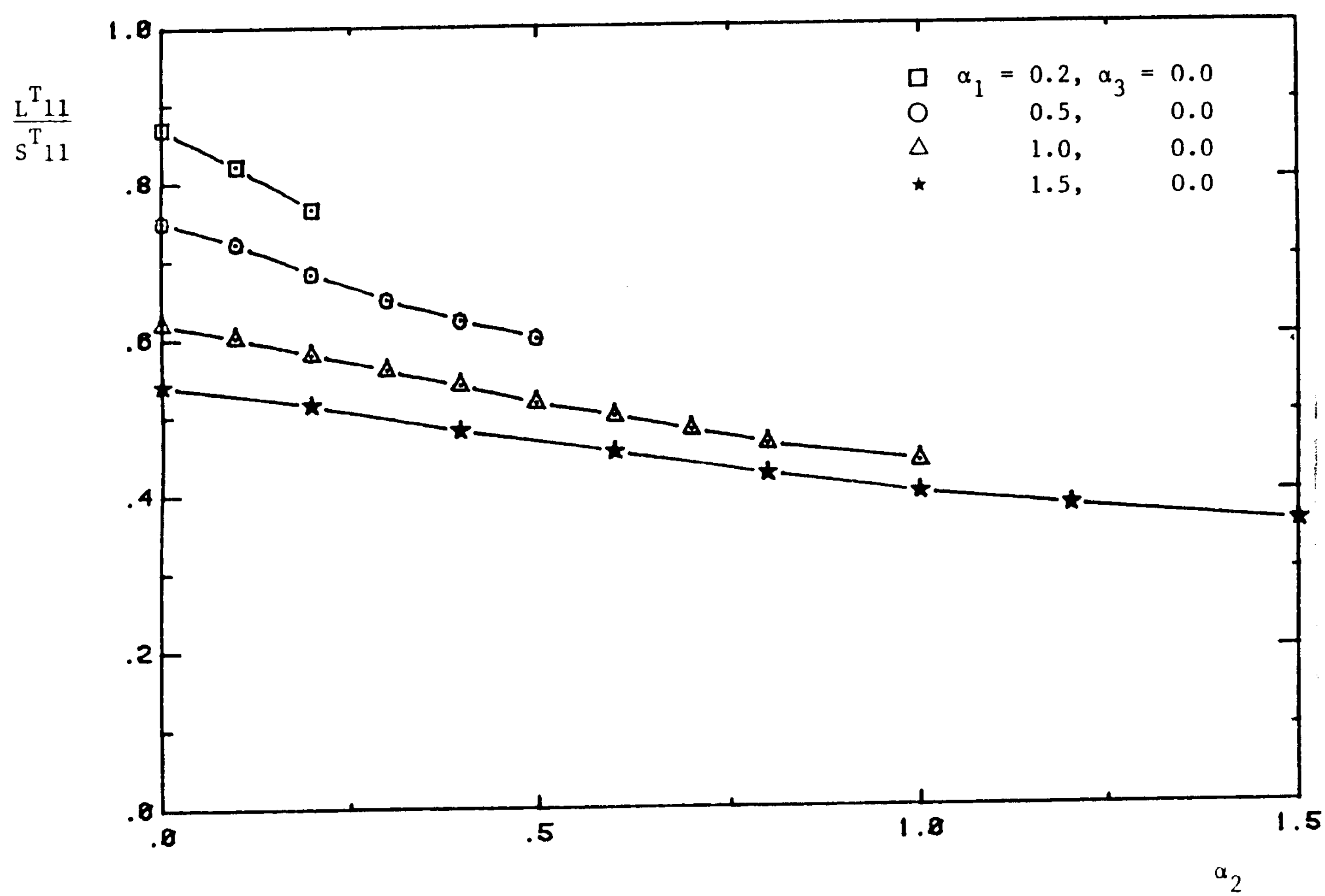


Figure 13 Ratio of Lagrangian to Space-time integral time scales for two-dimensional turbulent flow with uniform velocity

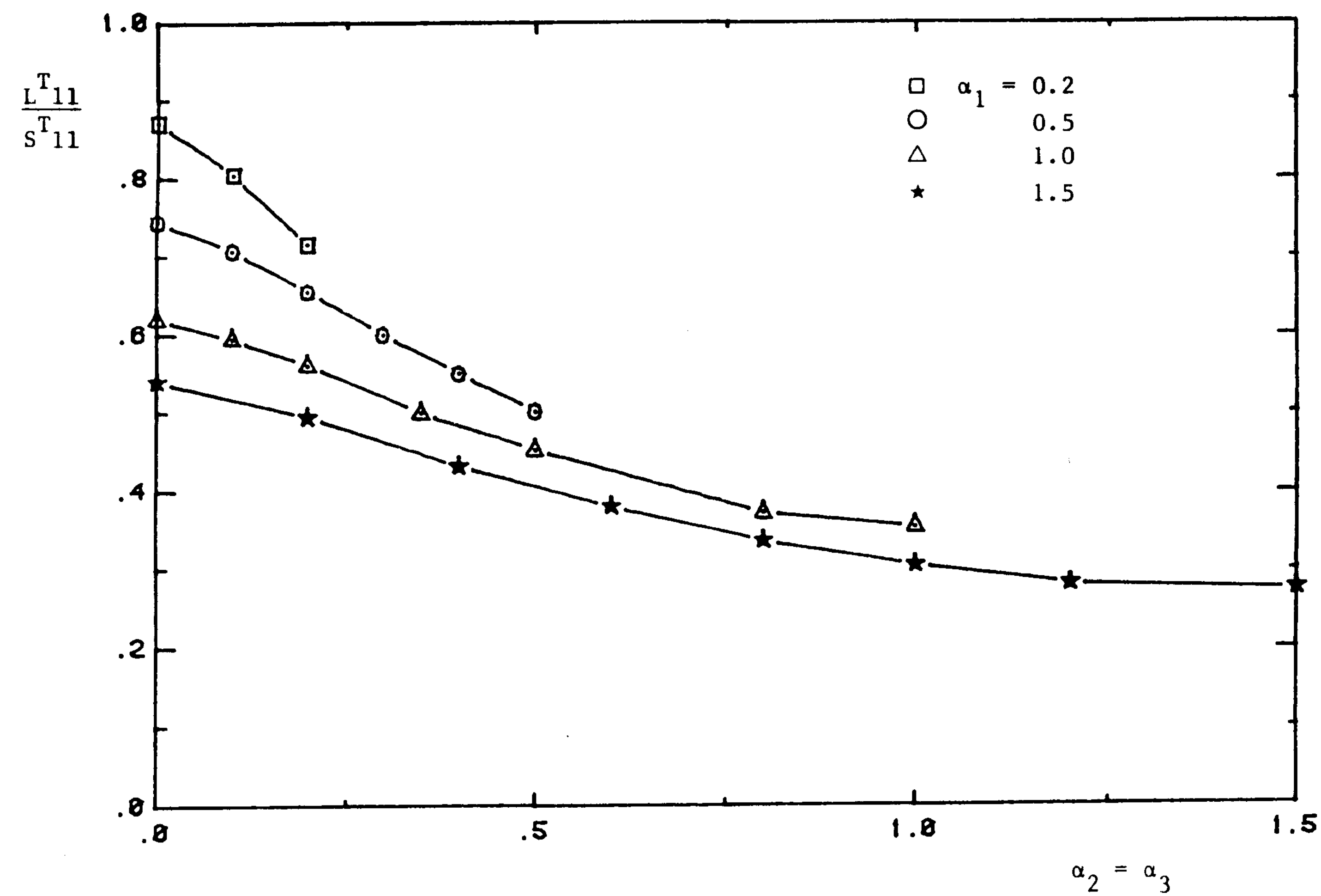


Figure 14 Ratio of Lagrangian to Space-time integral time scales for three-dimensional turbulent flow with α_2=α_3

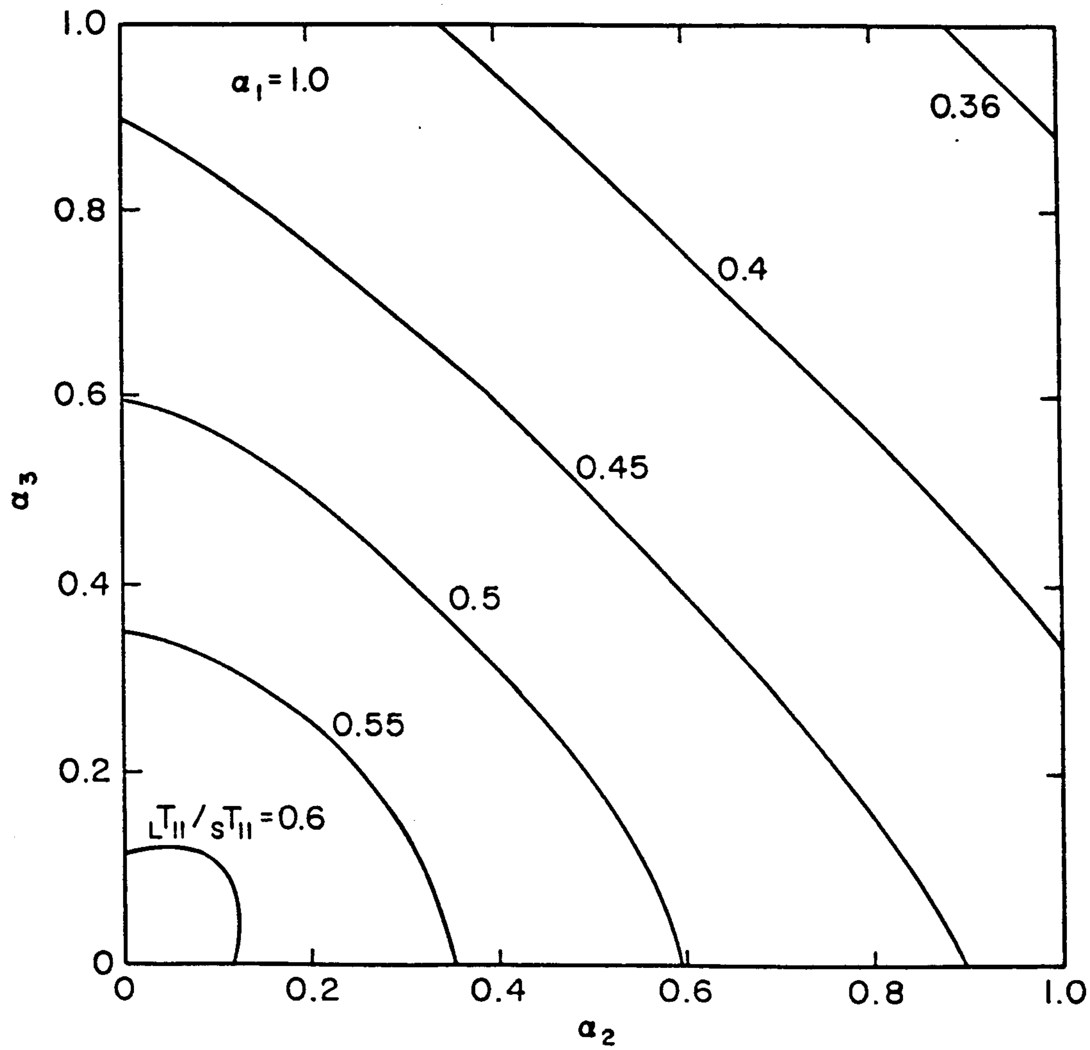


Figure 15 Variation in Lagrangian to Space-time integral time scale ratio for different α_2 and α_3 values when α_1=1.0