A WIND-TUNNEL STUDY OF PLUME RISE AND DISPERSION UNDER STABLE STRATIFICATION

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1. INTRODUCTION

The discharge of effluent into the atmosphere from industrial stacks has received much attention over the last half century. More intense interest has developed since the promulgation of the Clean Air Act whereby concentration levels of certain pollutants are regulated.

The primary method recommended for meeting air-quality standards at the present time is control or recovery of the pollutant of concern. In general, this control is expensive and not favored by industry. A less expensive method is the utilization of optimum stack design so as to minimize ground-level concentrations. The latter approach may include building a taller stack and/or selecting proper stack exit parameters (temperature, velocity and diameter) to obtain a higher effective plume altitude.

The design or modification of an industrial stack may be optimized so it can meet ambient air quality standards by employing numerical and/or experimental techniques. Petersen et al. (1975) described a method consisting of numerical and full-scale field measurements to arrive at the optimum stack height for a smelter complex. The basic drawbacks to full-scale measurement studies are 1) many of the variables of importance are difficult to control and measure and 2) the studies are very expensive.

The standard numerical methods employed include plume-rise predictions using equations such as are outlined in Briggs (1975) or Hoult et al. (1969). The concentration patterns are then estimated using the Gaussian model as described in Turner (1969).

The basic drawback to physically based plume-rise models is the method employed to close the system of governing equations. Typically the ambient air entrainment rate is set proportional to the plume centerline velocity (Briggs, 1975; Hoult et al., 1969; Rajasekaramurthy, 1968; and Fan, 1967). Most of the theoretical investigators have assumed that the ambient flow is a steady, horizontal laminar cross wind with no variation of temperature or wind with altitude. Rajasekaremurthy (1968) and Murthy (1970) did include in their numerical analysis a power-law wind profile while Briggs (1975) and Hewett et al. (1970) allow for vertical temperature variations in a stable atmosphere.

A logical alternative to full-scale measurements is to employ physical modeling in a wind tunnel to simulate the desired flow condition. A scale model of a stack and surrounding topography or roughness, positioned in a long test section wind tunnel, would allow for measurements of plume rise and dispersion in a turbulent cross flow with vertically varying temperature and wind speed (Cermak, 1971; Synder, 1972).

Numerous investigations of plume transport and dispersion have been conducted in wind tunnels over a limited range of simulated plant operating conditions. In most of the studies similarity between atmospheric motion in the wind tunnel was relaxed to some degree. Typically the Reynolds number cannot be maintained in the prototype and model. Golden, as cited by Halitsky (1963) found that concentration patterns were invariant in the wake and at the surface of the building for certain Reynolds numbers while Weil (1962) found that plume trajectories were invariant as long as the plume was turbulent at the stack exit. Another dimensionless parameter which is occasionally relaxed in the wind tunnel is the density ratio $(\rho_s - \rho_a)/\rho_a$. To date there is no extensive experimental research on the dependence of plume rise and dispersion on the density ratio.

The purpose of this study is to determine the effect of stack-gas emission temperature and velocity on the trajectory and concentration patterns of the resulting plume under stable stratification. The intent being to provide a data base over a wide range of plant operating conditions that will establish the feasibility of increasing stack-gas emission speed or temperature to achieve greater plume rise and lower concentrations through enhanced dispersion.

Neutral thermal stratification and ground-level inversions were simulated in the Colorado State University (CSU) meteorological wind tunnel (MWT). A 1:300 scale model of a 60.8 m stack was placed in the wind tunnel. Plume rise was assessed by photographing a visual simulation of the plume. The concentration patterns were determined by traversing the plume at various locations and measuring the concentrations of tracer gas (He) released from the stack. While the exit velocity and temperature varied during the study, the volume of gas flow remained a constant for all tests. A summary of the test scenario follows:

Test Scenario

Richardson Number	Stack-Gas Temperature (°K)	Reference Wind Speed (m/s)	Stack-Gas Exit Speed (m/s)
0.0	366, 422	2, 4, 8, 16	12.5, 30, 60 120, 240
	388	2, 4, 8, 16	30, 60
0.57	366, 388, 422	5.9	12.5, 30, 60 120, 240
0.31	366, 388, 422	3.0	

Included in this report are a complete description of similarity requirements for wind-tunnel modeling, the experimental program, the results and conclusions. A comparison of the observed plume-rise with the Hoult et al. (1969) and Briggs (1975) equation is given. The maximum centerline concentrations are compared with those predicted using the Gaussian model (Turner; 1969).

2.0 THEORETICAL CONSIDERATIONS

2.1 Simulation of Atmospheric Motion

The basic equations governing atmospheric motion (conservation of mass, momentum and energy) are given in Cermak (1971) and Snyder (1972). They conclude that for exact similarity, the non-dimensional quantities in the governing equations and boundary conditions must be the same in the wind tunnel and for the corresponding atmospheric flow. The complete set of requirements for "exact" similarity is

1) Undistorted geometry
2) Equal Rossby number: $Ro = u_o/(L_o \Omega_o)$
3) Equal gross Richardson number:

$$Ri = \frac{\Delta T_o g_o L_o}{T_o u_o^2}$$

4) Equal Reynolds number: $Re = u_o L_o / \nu_o$
5) Equal Prandtl number: $Pr = (\nu_o \rho_o C_{p_o})/k_o$
6) Equal Eckert number: $Ec = u_o^2/[C_{p_o} (\Delta T)_o]$
7) Similar surface-boundary conditions
8) Similar approach-flow characteristics

All of the above requirements cannot be simultaneously satisfied in the wind tunnel and atmosphere. Strict equality of Ro, Ec and Re in model and prototype is not essential to obtain a good approximation of boundary-layer characteristic in flows of common occurrence (Cermak; 1971, 1974). The parameter which is most relevent to this study will now be discussed.

For simulating thermally stratified conditions, equality of the Richardson number between model and prototype is required. The bulk Richardson number is defined by

$$Ri_b = \frac{\frac{g}{T}\left[\frac{\Delta T}{\Delta z} + \Gamma\right]}{\left(\frac{\Delta u}{\Delta z}\right)^2} = \frac{\frac{g}{T}\frac{\Delta \theta}{\Delta z}}{\left(\frac{\Delta u}{\Delta z}\right)^2} \quad .$$

Hence, if $Ri_m = Ri_p$ the following relation results

$$\left(\frac{\Delta T}{\Delta z}\right)_m = 300\left(\frac{\Delta \theta}{\Delta z}\right)_p \quad .$$

2.2 Simulation of Source Characteristics

The Froude number, Fr, for the stack-gas is defined by

$$Fr = \frac{u_s}{\sqrt{g\gamma D}}$$

where

$$\gamma = \frac{\rho_a - \rho_s}{\rho_s} \quad .$$

Although Fr does not specifically appear in the list of similarity parameters, it is the reciprocal of the Richardson number squared. Dimensional analysis reveals that the parameter γ is also important for simulating plume motion. Thus, if Fr and γ are set equal for model and prototype, the following relation between model and prototype velocity is obtained

$$\frac{(u_a)_m}{(u_a)_p} = \left(\frac{D_m}{D_p}\right)^{1/2} \quad .$$

Since $D_m/D_p = 300$,

$$(u_a)_m = 0.058\ (u_a)_p \quad .$$

To properly scale the stack-gas exit velocity with the approaching wind speed at stack height the ratio $R = u_s/u_h$ is set equal for model and prototype.

2.3 Simulation of Plume Characteristics

To summarize, the following scaling criteria were applied to obtain similarity of source characteristics and atmospheric flow:

1) $Fr = \frac{u_s}{\sqrt{g \gamma D}}$; $Fr_m = Fr_p$

2) $R = \frac{u_s}{u_h}$; $R_m = R_p$

3) $\gamma = \frac{\rho_a - \rho_s}{\rho_s}$; $\gamma_m = \gamma_p$

4) $Ri = \frac{g\left(\frac{\Delta\theta}{\Delta z}\right)}{T\left(\frac{\Delta u}{\Delta z}\right)^2}$; $Ri_m = Ri_p$

5) Similar geometric dimensions

6) Sufficiently high Reynolds number to insure proper flow conditions.

Those criteria form a necessary set of conditions for similarity of the plume behavior. The parameters for each test considered are summarized in Table 1.

Table 1 Range of Operating Conditions for Wind-Tunnel Tests

Parameter	Prototype	Model
Stack height (h)	60.8m	0.2m
Volumetric Emission Rate (V)	320 m^3/s	$2.14 \times 10^{-4} m^3/s$
Ambient Temperature (T_a)	293°K	293°K
Wind Speed at Stack Height (u_h)	2-16 m/s	0.12 - 0.92 m/s
Stack Exit Velocity (u_s)	12.5-240 m/s	0.72 - 13.9 m/s
Stack Exit Temperature (T_a)	386 - 422°K	293 °K
Stack Diameter (D)	5.7 - 1.3m	1.93 - 0.43 cm
Velocity Ratio (R)	0.94 - 120	0.94 - 120
Froude Number (Fr)	3 - 150	3 - 150
Density Ratio (γ)	0.20 - 0.31	0.20 - 0.31
Stack Reynolds Number (Re_s)	5.0×10^6-19.7×10^6	960 - 3800
Richardson Number (Ri)	0 - 0.57	0 - 0.57

2.4 Plume Rise Predictions

The observed plume rise in the wind-tunnel was compared with the models of Briggs (1975) and Hewett et al. (1970). Both models are nearly identical when the initial momentum flux is small compared to the buoyancy flux.

For a rise limited by stable stratification, Briggs (1975) gives the following equation for the equilibrium height of a bent-over plume.

$$z_{eq} = \left[\frac{3F(2.25)}{\beta^2 u_a s}\right]^{1/3} \left(1 + \sqrt{\frac{s}{2.25}}\ \frac{Fm}{F}\right)^{1/3}$$

where

$$F = g\left[\frac{T_s - T_a}{T_s}\right] r_o^2 u_s$$

$$F_m = r_o^2 u_s^2 T_a/T_s$$

$$s = \frac{g}{T}\frac{\partial T}{\partial z}$$

$$\beta \approx 0.7$$

The rise of the plume prior to bending-over is given by

$$z = \left[\frac{3}{\beta_1^2}\frac{F_m x}{u_a^2} + \frac{3}{2\beta_2^2}\frac{Fx^2}{u_a^3}\right]^{1/3}$$

where

$$\beta_1 \approx 1/3 + R^{-1}$$

$$\beta_2 \approx 0.5$$

Hewett et al. (1970) gives the following equation for the equilibrium height of a bent-over plume

$$z_{eq} = \left(\frac{6\ F}{\beta^2 s u_a}\right)^{1/3}$$

where he assumes $sF_m^2 << F^2$ and the momentum dominated region of rise is small. Prior to reaching z_{eq} the equations utilized are

$$z = \left(\frac{3}{2\beta^2}\right)^{1/3} \frac{(Fx^2)^{1/3}}{u_a} \qquad x > x_c$$

and

$$z = \left(\frac{R}{(\beta+\alpha R)}\right)^{1/2} (x\ r_o R)^{1/2} \qquad x < x_c$$

where

$$x_c = \left(\frac{2\beta^2}{3}\right)^2 \left(\frac{R}{\beta+\alpha R}\right) \left(\frac{r_o R u_a^2}{F^{2/3}}\right)^3$$

$$\alpha = 0.15$$

3.0 EXPERIMENTAL PROGRAM

3.1 Meteorological Wind-Tunnel

The Colorado State University meteorological wind-tunnel (MWT) was used for this study. This wind-tunnel, espeically designed to study atmospheric flow phenomena--Cermak (1958), Plate and Cermak (1963), incorporates special features such as an adjustable ceiling, a rotating turntable, temperature-controlled boundary walls, and a long test section to permit adequate reproduction of micrometeorological behavior. Mean wind speeds of 0.96 to 39.6 m/s in the MWT can be obtained. Boundary-layer thicknesses up to 1.2 m can be developed "naturally" over the downstream 6.1 m of the MWT test section. Thermal stratification in the MWT is provided by the heating and cooling systems in the return-section passage and the test-section floor.

A set of vortex generators was installed 0.6 m downwind of the entrance to give the simulated boundary an ititial impulse of growth. From 1.8 to 6.1 m a set of 12 roll-bond aluminum panels were placed on the tunnel floor. These panels were connected to the facility refrigeration system and cooled to approximately 0°C for F stability tests and 14°C for E stability tests. The free-stream air was maintained at about 24°C for both tests. For neutral condition, no heating or cooling of the flow was used.

3.2 Model

The power-plant stack was simulated by constructing a 1:300 model. To simulate various exit velocities while maintaining a constant volume flow, five nozzles were constructed for positioning on top of the model stack. The relevant stack parameters are summarized in Table 1.

Metered quantities of gas were allowed to flow from the model stack. Helium (the tracer) and compressed air were mixed in the appropriate proportion to simulate the densities associated with prototype exit temperatures of 366, 388 and 422°K. Fischer-Porter flow-meter settings were adjusted for pressure, temperature, and molecular weight effects as necessary.

3.3 Flow-Visualization Techniques

Smoke was used to define plume behavior from the model stack. The smoke was produced by passing the air mixture through a container of titanium tetrachloride located outside the wind-tunnel and transported through the tunnel wall by means of a tygon tube terminating at the stack inlet.

The plume was illuminated with arc-lamp beams and a visible record was obtained by means of pictures taken with a Speed Graphic camera. Additional still pictures were obtained with a Hasselblad camera. Stills were taken with a camera speed of one second to identify mean plume boundaries. A series of 16mm color motion pictures were also taken with a Bolex motion-picture camera.

The color slides of the plume visualization were used to identify the plume centerline trajectory. To determine the centerline trajectory the color slides were projected onto a gridded screen; after which, the centerline coordinates were key-punched for subsequent analysis. Using trigonometric relations, the photo-coordinates were corrected for optical distortion according to Halitsky (1961).

3.4 Gas-Tracer Techniques

After the desired wind-tunnel conditions were obtained, a mixture of helium and air of predetermined concentration was released from the model stack at the required rate to simulate prototype plume rise. The flow rate of the helium mixture was controlled by a pressure regulator at the supply cylinder outlet and monitored by a Fisher-Porter precision flow meter.

Gas samples of the plumes were obtained by drawing the gas through a sampling train which ended at the outlet of a thermal-conductivity gas chromatograph (TCGC). A 40 cm telescoping brass tube (0.16 cm ID) was mounted on a traversing mechanism in the wind-tunnel. PVC tubing connected the probe and the TCGC. The sampling probe was positioned at 10, 41 and 203 cm downwind of the stack. At each downwind position horizontal traverses through the plume were made at incremental heights above the tunnel floor. The horizontal traverse speed was approximately 4.7 cm/min. This traverse speed was determined by experimentation to be slow enough to prevent smoothing of the peak values.

Concentrations of the tracer gas (He) were determined by using the TCGC. The TCGC was modified so that continuous sample analysis was possible. The flow rate through the TCGC was maintained at 2.5 cc/min and the carrier gas was ambient air.

The voltage output readings from the TCGC were transformed into a helium concentration using the relation

$$\chi(\mathrm{ppm}) = C(\mathrm{ppm/mvs})\, E(\mathrm{mvs})$$

where C was determined from the daily calibrations with 100 percent helium.

The values of the concentration parameter initially determined apply to the model and it is desirable to express these values in terms of the field. The simplest and most straightforward procedure is to make this transformation using the scaling factor of the model. Since

$$1\mathrm{m}\big|_m = 300\mathrm{m}\big|_p ,$$

one can write

$$\frac{\chi u_a}{Q}\Big|_p (\mathrm{m}^{-2}) = \frac{1}{300^2}\,\frac{\chi u_a}{Q}\Big|_m (\mathrm{m}^{-2}) \quad .$$

3.5 Velocity and Temperature Measurements

A hot-wire velocity probe was positioned upwind of the stack, a height of 0.2 m to set and monitor tunnel-flow conditions. This probe has a range capability of 0.03 to 5.1 m/s. The same probe was used to measure velocity profiles at the stack location. The probe was attached to a vertically traversing carriage and average velocities (60 sec) were obtained at incremental altitudes. The power-law exponents for the stable cases were 0.27 and 0.83.

The temperature conditons in the wind-tunnel were set using a rack of twelve thermistors (positioned at incremental altitudes) placed upwind of the stack. The rack was taken out of the tunnel once the desired condition stabilized.

4.0 RESULTS

4.1 Plume Rise

The photographs of the simulated plumes over the range of conditions summarized in Table 1 were analyzed to determine the plume centerline trajectories. The results showed the expected trend of increased rise with increasing exit velocity and decreasing stack-gas density. Figure 1 shows plots of plume rise versus distance for $Ri = 0.31$, $\gamma = 0.245$ and $R = 2.0$, 9.8 and 39.0. For this test the velocity power-law exponent was 0.33. Also shown in the figure are the calculated plume-rise values using the Briggs and Hewett equations.

Overall the "general" Briggs equations agreed best (17% error compared to a 21% error of the Hewett equation) with the experimental data. That is because the Briggs equations do not neglect the momentum flux term, F_m. The Hewett equations (and the simplified Briggs equations) neglect the momentum flux and consequently give a constant maximum plume rise when the buoyancy flux is constant. In fact the momentum flux term can contribute significantly to the rise as is evident from Fig. 1 and should be considered in many cases. It is surprising that the Briggs and Hewett equations agree as well as they do, since in the development of the models uniform flow is assumed.

Further analysis of the data revealed that the Hoult et al. (1969) and Hewett (1970) entrainment coefficient α varied with R and Fr over a range from 0.1 to 1.0. On the other hand β did not vary significantly and averaged 0.7 for Ri equal to 0 and 0.57

4.2 Concentrations

The model concentration data were analyzed to obtain peak centerline concentrations for each downwind distance and atmospheric condition studied. To quantitatively assess the dependence of maximum centerline concentration on exit temperature and exit velocity, the dilution $(\chi u_a/Q)$ was plotted versus downwind and trajectory distance. The velocity, u_a, is at the height of the maximum concentration and was calculated from the measured velocity power-law relation. The trajectory distance (the centerline travel distance) to each downwind position was computed from the experimentally measured plume trajectories.

Figure 2 shows the plume dilution versus downwind distance for $Ri = 0.57$, $\gamma = 0.245$ and $R = 2.1$, 5.1, 10.2, 20.2 and 40.7. All dilution values fell below the Pasquill E curve but appear to approach the curve beyond 600m. Also evident from the figure is the decrease in dilution with increasing R. This decrease is due to the increased travel distance to each downwind location as R increases.

If plume dilution is plotted versus trajectory distance, the data should fall along a straight line unless the rate of diffusion is affected by the changing stack conditons. Figure 3 shows a plot of dilution versus trajectory distance for the same conditions as in Figure 2. In this figure the dilution values fall along a straight line and scatter about the Pasquill E curve. This leads one to the conclusion that changes in the stack exit conditions do not significantly affect plume dispersion. The main advantage to be gained by an increase in R is an increase of the plume travel distance. Hence at a given downwind position the concentration will decrease as R increases.

5.0 SUMMARY

The results of the wind-tunnel tests of plume rise and dispersion under stable stratification can be summarized as follows:

1) The generalized Briggs plume rise equations for stable conditions give better agreement with the wind-tunnel results than the simplified Hewett equations.

2) Increasing the momentum flux is a viable strategy for enhancing plume rise and decreasing the maximum centerline concentration at downwind locations.

3) Wind-tunnel plume centerline dilution $(\chi u_a/Q)$ when plotted versus trajectory distance falls along the equivalent Pasquill curves.

4) The Briggs and Hewett equations compare well (average error of approximately 18%) even when strong wind shear exists.

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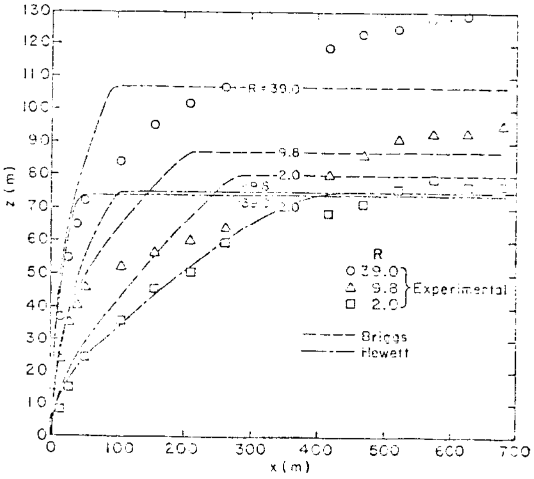

Figure 1. Comparison of predicted and observed plume rise for Ri = 0.31, $\gamma = 0.245$ and $u_h = 3.0$ m/s.

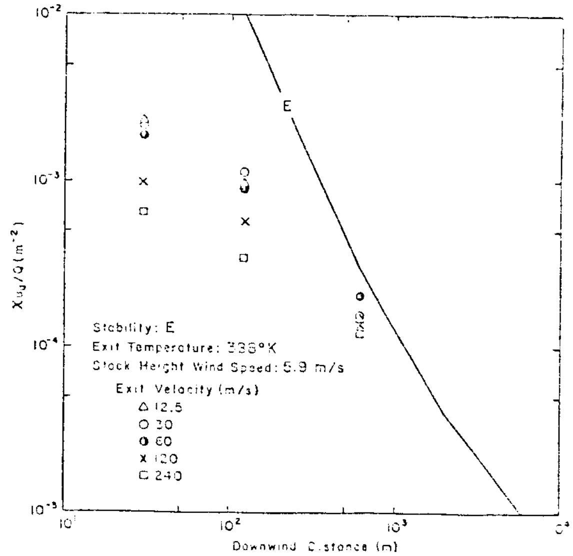

Figure 2. Plume dilution $(\chi u_a/Q)$ versus downwind distance for Ri = 0.57.

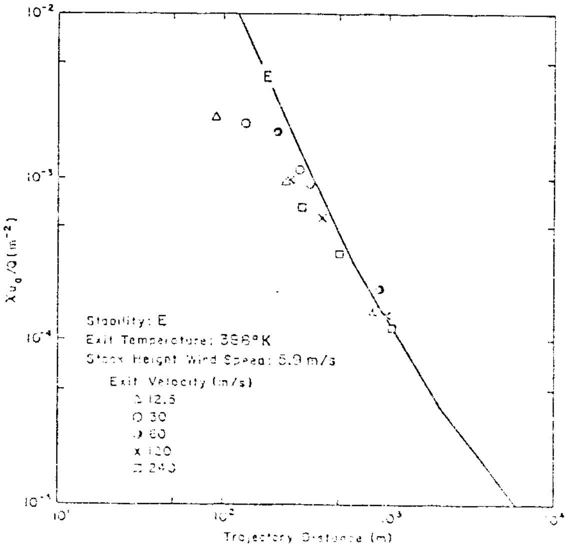

Figure 3. Plume dilution $(\chi u_a/Q)$ versus trajectory distance for Ri = 0.57.