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BUOYANCY EFFECTS ON A TURBULENT SHEAR FLOW

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1.0 INTRODUCTION

Experimental data invariably show that for positive values of the Richardson number all turbulent fluctuating properties are suppressed by the action of the buoyancy force, while for negative values of the Richardson number turbulent fluctuating properties are accentuated. However there is a vast difference between the behavior of $\overline{w'T'}$, $\overline{u'T'}$, and $\overline{u'w'}$ when $\partial T/\partial z < 0$ and when $\partial T/\partial z > 0$. In the stable case the heat flux is often very small or negligible despite finite gradients of temperature, yet momentum transport may still be finite. In the unstable case a large heat flux is established quite rapidly, momentum transport may be significantly smaller in proprotion, yet temperature gradients may be near zero. It appears that buoyancy-generated eddies cause relatively little momentum transport, but they are quite effective at carrying thermal energy. In other words, the rates of the associated turbulent diffusivities for heat and momentum is much larger than one, Reynolds analogy does not apply, and the idea of a simple eddy diffusivity in a stratified medium is completely wrong. Use of the diffusivity concept in calculations thus would tend to develop too rapid dissipation of inversions, and too slow a growth of turbulence in unstable situations.

These physical considerations suggest that an adequate theory for the treatment of the interaction of stratification, gravity, and a turbulent field must include transport equations for the second order correlations or their equivalent. Work by Donaldson et al. (1972), Donaldson (1973), Lewellen and Teske (1973), and Mellor (1973) do consider the second order correlation equations indluding stratification effects. Lumley (1972) has also proposed sets of equations closed at the third order correlations, while Lee (1974) has developed a set of expressions based on analogies between turbulence and Brownian motion utilizing the Fokker-Planck equations. The ability of such formulations to follow the effects of stratification on turbulence are impressive. Unfortunately one must simultaneously solve a set of at least nine to as many as twelve partial differential equations for even a one-dimensional incompressible flow situation. For the equivalent two- or three-dimensional cases the ranges required are from ten to thirteen and from fourteen to seventeen partial differential equations respectively.

Such methods must thus be limited to research areas for the great majority of cases. Those situations requiring planning or engineering information generally must consider many case permutations; thus they require a method which retains the essential physical characteristics but with a lower order of solution complexity. This report discusses the efficacy of three such solution techniques. These will be discussed under the titles of

a) An algebraic lengths scale model (ALM),
b) A differential length scale model (DLM), and
c) An algebraic stress model (ASM).

The number of partial differential equations required are of the order of six, seven, and eight for one-, two-, and three-dimensional motions.

2.0 TURBULENT MODELS

In any model developed for turbulent closure one would like to have the method possess width of applicability, accuracy, economy of computational time, and simplicity. In the search for these elusive features many closures for the turbulent equations of change have been proposed (Launder and Spalding, 1972).

Reynolds (1968) has suggested in the 1968 Stanford "Olympics" on calculational techniques a morphology for classifying methods of closure. He suggests methods which make use of eddy viscosity or mixing length concepts will be called "mean field methods," (MF) whereas methods which relate the Reynolds stress to the turbulence and hence require calculation of some aspects of the turbulence fields will be called "turbulent-field methods." (MTF) Subsequent reviewers of turbulent models have accepted this decision as a critical distinction (Bradshaw, 1972, 1973). Mellor and Herring (1973) suggest two subsets of the MRF group. Those which include a turbulent kinetic energy transport equation and some accommodation for length scales will be "mean turbulent energy" closures (MTE); whereas a "mean Reynolds stress" closure (MRS) implies a closed set of equations which include equations for all nonzero components of the Reynolds stress.

As is always the case a difficult problem soon becomes muddled again even with respect to categories such as the above. The recent work by Hanjalic and Launder (1972), Rodi (1972) and the present suggestions may lie somewhat between the MTE and MRS classifications.

2.1 Mean Turbulent Energy Methods (MTE)

A basis for both the MTF and the MTE calculations began with semiheuristic models of Kolmogarov (1942) and Prandtl (1945). They suggest the use of a turbulent kinetic energy transport equation, a turbulent-energy related eddy viscosity, and a prescribed length scale function or a differential equation for length scale.

Bradshaw (1973, 1972) has been very critical of methods which retain an explicit algebraic relation between stresses and the mean flow. His criticisms are related to the ad hoc nature of any eddy-viscosity transport relation, the failure to provide correct results in those cases where there is finite transport and zero velocity gradient, and the basically regressive concept of going to the trouble to solve additional transport equations and then reapplying a local-equilibrium assumption to relate stress and gradient. Mellor and Herring (1973) appear more optimistic, they try to show how MTE models derive logically from the MRS models and how both involve essentially the same empirical information. Launder and Spalding (1972) have reviewed the results of most of the effort in this area.

2.2 Mean Reynolds Stress Methods (MRS)

As noted before, MRS closure implies a closed set of equations which include equations for all nonzero components of the Reynolds stress tensor. Rotta (1951) laid the foundation for future efforts when he proposed the pressure-velocity correlation terms in the Reynolds stress equations be proportional to a deviation from isotropy. This assumption was of course an approximation and was subject to modification by subsequent investigators. Other terms in the Reynolds stress equations such as the dissipation and diffusion terms have also been modeled differently by various investigators.

In most respects the authors of MRS models agree on general points. Primary differences center around the use of algebraic or transport equations for dissipation rates, and the presence or absence of such terms as the mean strain rate in pressure strain. There are, however, numerous details over which they disagree with one another, especially philosophically in approach to model selection. Donaldson and his co-workers put much faith in the principle of invariant modeling to limit choices for pressure correlations, third order correlations, etc. Other investigators stress ad hoc empiricism and dimensional analysis.

2.3 Algebraic Stress Models: (ASM)

A very novel compromise between the simplicity of the MTE approach and the universality and greater range of predictability of the MRS method, has been proposed by Launder and Ying (1971). Transport equations for turbulent fluctuational energy and eddy dissipation (or length scale) are combined with algebraic equations for each Reynolds stress. The additional algebraic stress equations are derived directly from their exact transport equation counterparts.

Following Rodi (1972), one notes that it is the convection and diffusion terms in the $\overline{u_i u_j}$ equations which make them differential relationships. If such terms are eliminated from the transport equations for $\overline{u_i u_j}$ one produces a set of algebraic relations of the form

$$\overline{u_i u_j} = f(\overline{u_p u_q}, \frac{\partial u_\ell}{\partial x_m}, k, \epsilon)$$

Of course the simplest way to simulate such terms is to neglect them out of hand. This however produces inconsistances in other than equilibrium situations where production exactly balances dissipation. Rodi postulated that

$$(\text{Convection - Diffusion})_{\text{of } \overline{u_i u_i}} = \frac{\overline{u_i u_j}}{k} (\text{Convection - Diffusion})_{\text{of } k}$$

$$= \frac{\overline{u_i u_j}}{k} (\text{Production - Dissipation})_{\text{of } k}$$

or that $\overline{u_i u_j}/k$ varies but slowly across the flow. (An assumption closely linked to the successful suggestions of Bradshaw for thin shear layers.)

Launder and Ying (1971) applied an ASM formulation to turbulent flow in a rectangular channel. The method predicted the order of secondary notions found in channels with sharp corners and the distribution of lateral Reynolds stresses. Rodi (1972) produced profiles of $\overline{u_k' u_j'}/k$ in plane jets and wakes where conventional two equation models undergo both strong and weak strain. Launder et al. (1972) in the NASA "free-shear flow computational olympics" compared six turbulence models and concludes MRS and ASM models produced results of comparable quality. Finally Date (1972) has produced shear and heat flux results for flow in a tube containing a twisted tape by means of ASM type approximations. One concludes therefore that the algebraic stress models may combine the most important features of the MRS type (the influence of complex strain fields on the stresses) with (almost) the numerical simplicity of a MTE model.

3.0 A TURBULENT MODEL FOR STRATIFIED FLOW

In order to close the turbulence equation system, some of the correlations in u_i', p', and T' must be approximated in terms of quantities that can be calculated. Model assumptions about turbulence are thereby introduced which may not be entirely realistic. These assumptions relate the chosen higher order correlations in u_i', p', and T' to other time-averaged quantities; they are expressed in differential and/or algebraic equations which help produce a mathematically closed set.

The models developed required the solution of partial differential equations for total turbulent kinetic energy, k, total turbulent temperature fluctuations, k_t, eddy dissipation, ϵ, and thermal eddy dissipation, ϵ_t. Three separate versions of this model are discussed--an algebraic length scale version, a Prandtl-Kolmogarov eddy viscosity version, and an algebraic stress and heat flux model. For purposes of demonstration simple time dependent one-dimensional versions of the governing equations will be applied to a set of free shear flow test cases for which a complete MRS solution is available. Details of the model building process followed here are found in Meroney (1974). The partial differential equations and algebraic relations solved are in a dimensionless format (scales of u_{max}, ΔT_{max}, and L).

Turbulent Model Equations (ALM) and (DLM)

$$\frac{\partial u}{\partial t} = \frac{1}{Re} \frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial z}(C_D \frac{k^2}{\epsilon} \frac{\partial u}{\partial z}) + X(t) \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{1}{RePr} \frac{\partial^2 T}{\partial z^2} + \frac{\partial}{\partial z}(C_H \frac{k\, k_t}{\epsilon_t} \frac{\partial T}{\partial z}) \tag{2}$$

$$\frac{\partial k}{\partial t} = \frac{1}{Re} \frac{\partial^2 k}{\partial z^2} + \frac{\partial}{\partial z}(\frac{C_D}{\sigma_k} \frac{k^2}{\epsilon} \frac{\partial k}{\partial z}) + C_D \frac{k^2}{\epsilon}(\frac{\partial u}{\partial z})^2 - \epsilon + RiC_H \frac{k\, k_t}{\epsilon_t}(\frac{\partial T}{\partial z}) \tag{3}$$

$$\frac{\partial k_t}{\partial t} = \frac{1}{RePr} \frac{\partial^2 k_t}{\partial z^2} + \frac{\partial}{\partial z}(\frac{C_H}{\sigma_{kt}} \frac{k\, k_t}{\epsilon_t}) + C_H \frac{k\, k_t}{\epsilon_t}(\frac{\partial T}{\partial z})^2 - \epsilon_t \tag{4}$$

$$\frac{\partial \epsilon}{\partial t} = \frac{1}{Re} \frac{\partial^2 \epsilon}{\partial z^2} + \frac{\partial}{\partial z}(\frac{C_D}{\sigma_\epsilon} \frac{k^2}{\epsilon} \frac{\partial \epsilon}{\partial z}) + C_{E1} C_D k(\frac{\partial u}{\partial z})^2 - C_{E2} \frac{\epsilon^2}{k} - F \cdot Ri\, C_H k_t \frac{\epsilon}{\epsilon_t}(\frac{\partial T}{\partial z}) \tag{5}$$

$$\frac{\partial \epsilon_t}{\partial t} = \frac{1}{RePr} \frac{\partial^2 \epsilon_t}{\partial z^2} + \frac{\partial}{\partial z}(\frac{C_H}{\sigma_{\epsilon t}} \frac{k\, k_t}{\epsilon_t} \frac{\partial \epsilon_t}{\partial z}) + C_{Et1} C_H k_t \frac{\epsilon}{\epsilon_t}(\frac{\partial T}{\partial z})^2 - C_{Et2} \frac{\epsilon\, \epsilon_t}{k} \tag{6}$$

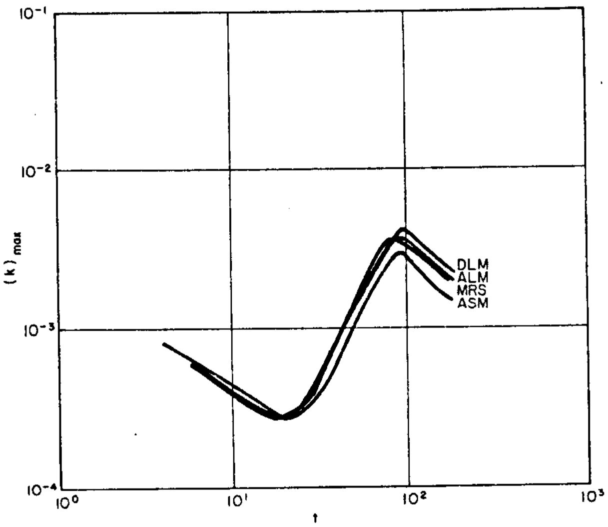

Fig. 2. Maxima of velocity correlations: Case I.

closure trials. One perceives the initial decay of turbulence, followed by the production of the correlations as a result of the interaction of turbulence and mean flow, and finally the decay after the forcing function is removed.

5.2 Case II: Unstable-Stable-Unstable Atmosphere

The mean velocity and shear are changed somewhat but not drastically by the effect of the temperature profile. In the thermally unstable regions the development of turbulence is accelerated. At large times the production by $\partial u/\partial z$ becomes significant and a maxima in k occurs at t = 90. There is a large influence on k, k_T, and $w'T'$ in the regions where $\partial T/\partial z < 0$ versus those regions where $\partial T/\partial z > 0$. If one insists upon an eddy diffusivity model a large variation in turbulent Prandtl number with stability is indicated. In regions of high stability the existence of a single length scale in the ALM method is not sufficient to develop the necessary degrees of temperature fluctuation dissipation. Thus k_T is excessively large near z = 0. (See Fig. 3)

The ALM and DLM models fail to keep pace with the changes recorded by the ASM and MRS relations. In addition the ALM and DLM models inherently require $\overline{w'T'}$, $\overline{u'T'}$, and $\overline{u'w'}$ be identically zero where the respective gradients, $\partial u/\partial z$ and $\partial T/\partial z$, are zero. It appears that failure to follow the time rate of change in k and other correlations is due to the inherent assumption that Production equals Dissipation in the ALM and DLM formulations. This effect is most marked in the logarithmic plot of the maxima of the turbulent kinetic energy as shown in Fig. 4. At small times the major production of turbulence is by thermal instability. Near the turbulence maxima there is finite contribution due to mean shear, $\partial u/\partial z$; but, after t = 90, the thermal instabilities in the outer regions dominate.

5.3 Case III: Stable-Unstable-Stable Atmosphere

This case was utilized to specify the magnitude of constant C_{Et2}. In Cases I and IV the constants were unchanged for all models. Again we do not detect major influence of stability upon the production of a shear layer. There is intense production of k, k_T, and $\overline{w'T'}$ in the center of the

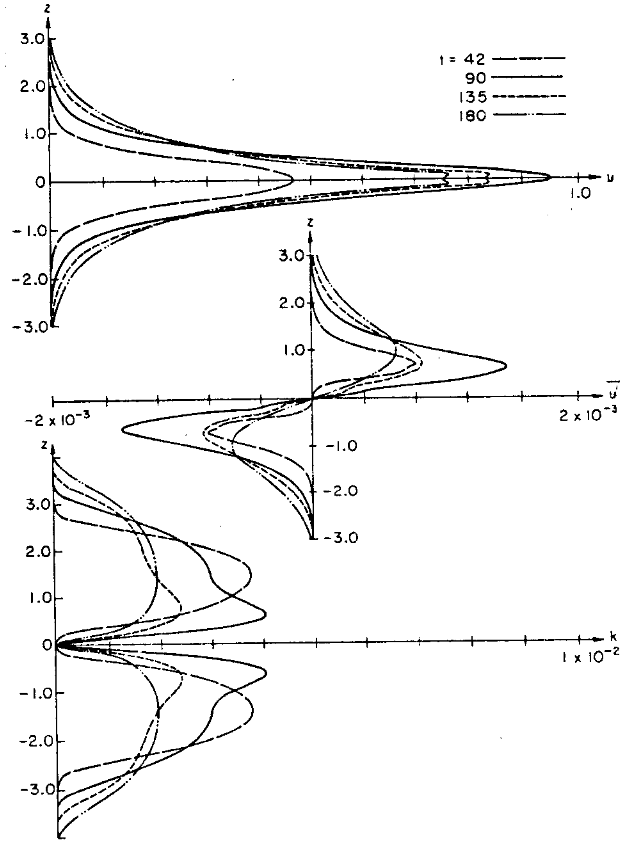

Fig. 3a. Case II: ASM model.

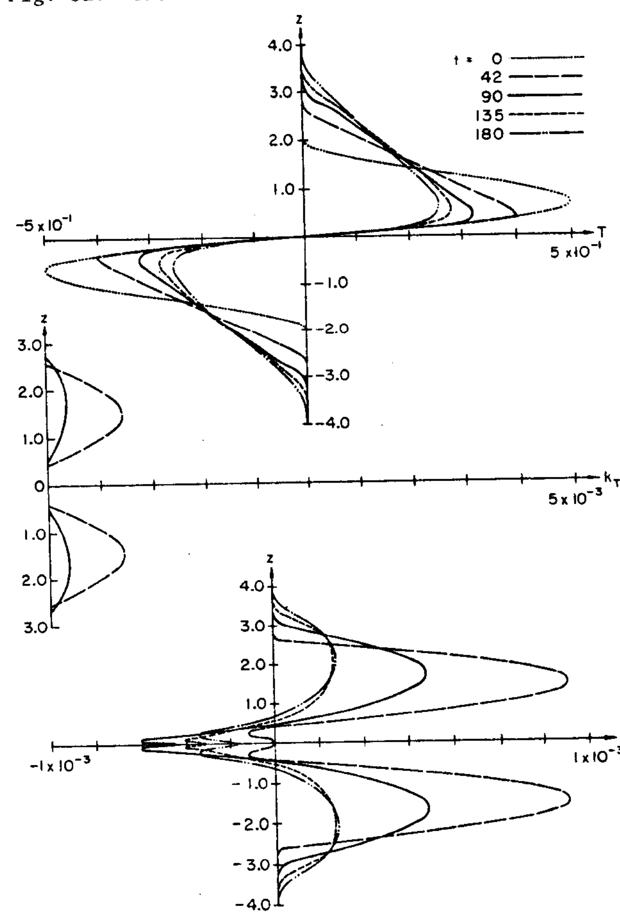

Fig. 3b. Case II: ASM model.

Turbulent Model Equations (ASM)

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\frac{\partial^2 u}{\partial z^2} - \frac{\partial}{\partial z}(\overline{u'w'}) + X(t) \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{1}{RePr}\frac{\partial^2 T}{\partial z^2} - \frac{\partial}{\partial z}(\overline{w'T'}) \tag{2}$$

$$\frac{\partial k}{\partial t} = \frac{1}{Re}\frac{\partial^2 k}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_D}{\sigma_k}\frac{k^2}{\varepsilon}\frac{\partial k}{\partial z}\right) - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'} \tag{3}$$

$$\frac{\partial k_t}{\partial t} = \frac{1}{RePr}\frac{\partial^2 k_t}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_H}{\sigma_{kt}}\frac{k\,k_t}{\varepsilon_t}\frac{\partial k_t}{\partial z}\right) - \overline{w'T'}\frac{\partial T}{\partial z} - \varepsilon_t \tag{4}$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{Re}\frac{\partial^2 \varepsilon}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_D}{\sigma_\varepsilon}\frac{k^2}{\varepsilon}\frac{\partial \varepsilon}{\partial z}\right) - C_{E1}\,\overline{u'w'}\left(\frac{\varepsilon}{k}\right)\left(\frac{\partial u}{\partial z}\right) - C_{E2}\frac{\varepsilon^2}{k} \tag{5}$$

$$+ F\,Ri\,\overline{w'T'}\frac{\varepsilon}{k}$$

$$\frac{\partial \varepsilon_t}{\partial t} = \frac{1}{RePr}\frac{\partial^2 \varepsilon_t}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_H}{\sigma_{\varepsilon t}}\frac{k\,k_t}{\varepsilon_t}\frac{\partial \varepsilon_T}{\partial z}\right) - C_{Et1}\,\overline{w'T'}\left(\frac{\varepsilon}{k}\right)\left(\frac{\partial T}{\partial z}\right) - C_{Et2}\frac{\varepsilon\,\varepsilon_t}{k} \tag{6}$$

$$\overline{u'w'} = \frac{\{-\frac{2}{3}(1 - A_1)\,k^2\frac{\partial u}{\partial z}\,(C_{P1}\varepsilon - \overline{u'w'}\,A_1\frac{\partial u}{\partial z} - \varepsilon + 3\,Ri\,\overline{w'T'})\}}{\{C_{P1}\varepsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'}\}^2} \tag{7}$$

$$+ \frac{Ri\,\overline{u'T'}\,k}{\{C_{P1}\varepsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'}\}}$$

$$\overline{w'T'} = \frac{\{-\frac{2}{3}(1 - A_2)\,k^2\frac{\partial T}{\partial z}(C_{P1}\varepsilon - \overline{u'w'}\,A_1\frac{\partial u}{\partial z} - \varepsilon + 3\,Ri\,\overline{w'T'})\}}{\{C_{P1}\varepsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'}\}\{C_{P2}\varepsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'}\}} \tag{8}$$

$$+ \frac{2\,Ri\,k_t k}{\{C_{P2}\varepsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'}\}}$$

$$\overline{u'T'} = \frac{\{(1 - A_3)(-\overline{u'w'}\frac{\partial T}{\partial z} - \overline{w'T'}\frac{\partial u}{\partial z})k\}}{\{C_{P3}\varepsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \varepsilon + Ri\,\overline{w'T'}\}} \tag{9}$$

4.0 TEST CALCULATIONS FOR STRATIFIED TURBULENCE MODELS

Mellor and Herring (1973) recommend that investigators evaluate MTE and MRS models in tandem in order that critical limitations of the MTE approach are identified. Bradshaw (1971) has also proposed such tuning of a "simple" calculation method by a "refined" calculation technique.

4.1 Simple Case of Atmospheric Shear

The method chosen for comparison with the ALM, DLM, and ASM methods proposed herein was the MRS technique developed by Donaldson and Rosenbaum (1968). Their "invariant" modeling closure was applied to a hypothetical free-shear clear air turbulence case in Donaldson, Sullivan, and Rosenbaum (1972). This was a simple, time-dependent, one-dimensional example which characterizes the important effects of buoyancy dominated turbulent shear flows.

The atmospheric test case is assumed to have an initially 4000 ft band of turbulence that is isotropic with $\overline{u'^2} = \overline{v'^2} = \overline{w'^2} = 1$ (fps)^2. The band is centered at an altitude of 20,000 ft; however the effects of altitude upon p_o, T_o, and ρ_o are neglected for purposes of the example. The atmosphere is initially at rest, i.e., at $t = 0$, $u = 0$. A body force acts on the atmosphere to create a mean motion. The dimensionless driving function and initial temperature distributions imposed are shown in Fig. 1.

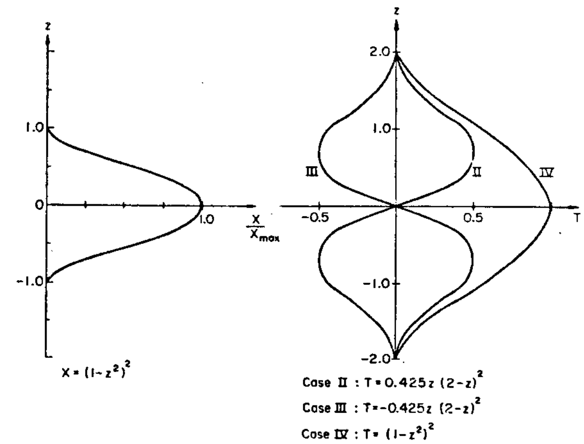

Fig. 1. Driving force function and initial temperature conditions clear-air turbulence model.

The algebraic length method of the MTE type approach suggested here still requires an expression to specify ε and ε_t in the governing partial differential equations. The algebraic relation formulated by Donaldson et al. (1972) can be re-expressed in terms of equivalent values of dissipation; hence for large Re_Λ one can find

$$\varepsilon = 5.52\,k^{1.5}/z_\delta$$
$$\varepsilon_T = 5.52\,k_t k^{0.5}/z_\delta.$$

where $z_\delta \equiv \delta^*/L^*$ and δ^* is the breadth of the mean shear layer at the half-maximum velocity points.

Although the MRS method compared to here did not require specification of the series of constants used in the ALM, DLM, and ASM models, consideration of the algebraic length scale equation plus asymptotic forms of the governing equations in those regions where production or dissipation dominate allows one to specify the equivalent constant values. After consideration of the results obtained from the original constants it was only found necessary to adjust C_{E2} and C_{Et2} slightly to arrive at a reasonable comparison to the Donaldson et al. (1972) results.

Investigator	C_D	C_H	C_{E1}	C_{E2}	C_{Et1}	C_{Et2}	F	σ_k	σ_ε	σ_{kt}	$\sigma_{\varepsilon t}$	A_1	A_2	A_3	C_{P1}	C_{P2}	C_{P3}
Donaldson et al. (1972)	0.1	0.1	1.5	1.5	1.0-1.5	1.5	1.5	0.1		0.2		0.0	0.0	0.0	5.0	5.0	5.0
Meroney (1974)																	
ALM	0.1	0.1						0.1		0.2							
DLM	0.1	0.1	1.5	1.59	1.5	1.59	1.5	0.1	0.1	0.2	0.2						
ASM	0.1	0.1	1.5	1.55	1.5	1.5	1.5	0.1	0.1	0.2	0.2	0.0	0.0	0.0	5.0	5.0	5.0

5.0 RESULTS

5.1 Case I: Neutrally Stable Atmosphere

For each model proposed this neutral case was used to adjust C_{E2} to obtain optimum time behavior of k when compared to the MRS results. At very small times, $t < 20$, there is still very little turbulence created by the mean action and most turbulence is left over from the decaying initial turbulent layer. When the forcing function, X, is removed at $t = 90$, the turbulent kinetic energy is a maximum. The influence of the value $\partial u/\partial z = 0$ at $z = 0$ on the production of new turbulence is evident. Finally at $t = 180$ one sees the effect after decay has set in.

By plotting the maxima of k for the various models versus time in Fig. 2 one can perceive the degree of agreement between the

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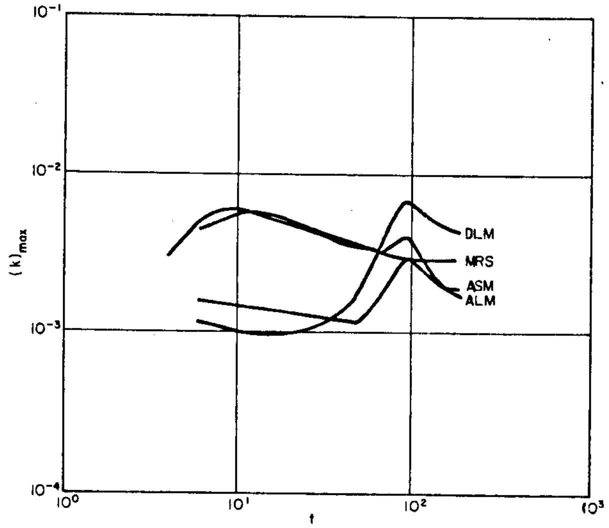

Fig. 4. Maxima of velocity correlations: Case II.

unstable region. Again only the ASM method can track the nonequilibrium behavior displayed by the MRS relations. The outlying stable regions limit the vertical dispersion of this intense turbulence. The temperature inversion is extremely persistent at large $\pm z$. In the time dependent plot of k, Fig. 5, we detect the initial production of turbulence by thermal instability, the modification of this by shear generated turbulence near $t = 90$, and the final phase where production of turbulence by thermal instability just about balances dissipation.

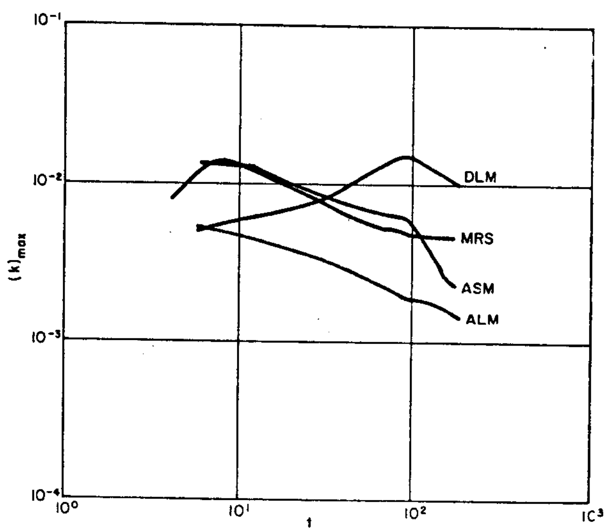

Fig. 5. Maxima of velocity correlations: Case III.

5.4 Case IV: Stable-Unstable Atmosphere

This example is marked by its asymetric appearance, the persistence of the inversion region, $\partial T/\partial z > 0$, and the rapid vertical diffusion of energy into the neutral upper regions where initially $\partial T/\partial z = 0$. No existing eddy diffusivity model would be expected to perform adequately. Figure 6 records the time rate of change of k maxima.

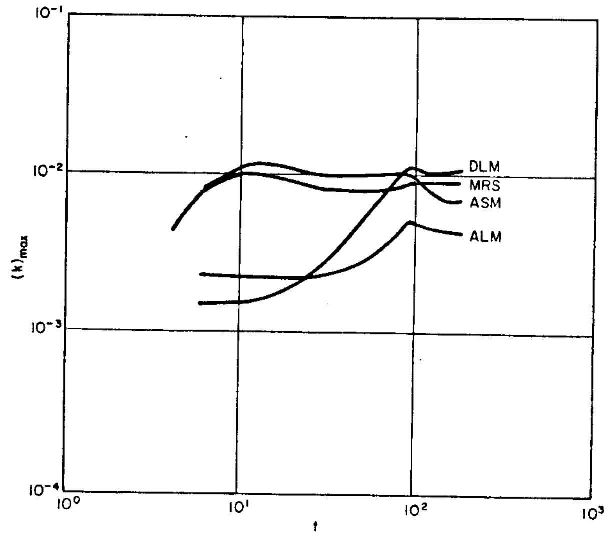

Fig. 6. Maxima of velocity correlations: Case IV.

5.5 Discussion of Results

Of course one of the most interesting aspects of these results is the duplication of Donaldson et al. (1972) conclusion that there is a radically different behavior of the heat flux correlation $\overline{w'T'}$ depending on whether $\partial T/\partial z$ is greater or less than zero. In fact, the ASM equations (7), (8), and (9) indicate that for small values of $\partial T/\partial z$ and $\partial u/\partial z$ there may be transport of heat and momentum up the gradients due to finite values of k_T or $\overline{u'T'}$! This effect has often been observed by experimentalists in atmospheric transport. In addition if one considers only production terms and neglects pressure scrambling and dissipation terms in equations for k, k_T and $\overline{w'T'}$ it is not difficult to show that when $\partial T/\partial z < 0$, there is an exponential development of $\overline{w'T'}$. However when the atmosphere is stable, i.e., when $\partial T/\partial z > 0$, the heat flux correlation $\overline{w'T'}$ is oscillatory about the Brant-Vassala frequency.

One must conclude on the basis of these results that:

1) An algebraic length scale version of a MTE model closure is not able to replicate the behavior of thermally stratified flow, especially in regions where production and dissipation of turbulence are not in equilibrium. A single dissipation length scale does not appear sufficient here to develop the expected degree of damping in stable regions.

2) Addition of transport equations for length scales does not suffice to solve the above problem. Such MTE models are still inadequate.

3) Addition of algebraic relations for stress and heat flux which incorporate the influence of stability do appear to incorporate the physics of the phenomena to the extent that results are similar to the MRS test case.

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