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LETTER TO THE EDITORS

COMMENTS ON THE DISPERSION OF MATTER IN NEUTRAL AND STABLY STRATIFIED ATMOSPHERIC SURFACE LAYERS

THE CONCLUSIONS the author of [1] draws concerning the viability of the Lagrangian similarity hypothesis for stratified flows appear to be in error. The numerical calculations of the dispersion of neutrally buoyant matter in neutral and stably stratified atmospheric surface layers that were used to arrive at these conclusions may be correct; unfortunately the analysis leading to the conclusion that constant b and c of the Langrangian similarity theory are strongly dependent on dispersion time is found to be deffective as will be noted in the following paragraphs.

Chatwin [2] analytically found the constants of similarity theory using the same diffusion equation as that employed by the author. Now if the two solutions for the concentration (author's and Chatwin's) match well as in Fig. 4b, there is no reason why the statistics of $\bar{X}$ and $\bar{Z}$ of the concentration distribution should not be related to each other in the same manner. Chatwin showed that c is a definite constant (less than unity) whether the source is at the ground or elevated in neutral flow. If the velocity distribution were uniform, this constant would be unity. But because du/dz decreases with height, the constant must be less than unity (Batchelor [3]). The author's Figs. 27 and 28 suggest values of c for the same situations, which are neither constant nor less than unity. It is the integration of the similarity expressions performed by the author that leads to the ambiguous conclusions, and the conclusions pertaining to stratified flow suffer from the same defect.

For the neutral flow the constant b can be shown equal to k independent of the release height. Consider equation (2) given by the author

$$\frac{\partial C_0}{\partial t} = \frac{\partial}{\partial z}\left(K_z \frac{\partial C_0}{\partial z}\right)$$

multiplying the above equation by z, integrating, and stipulating that

$$K_z = k\, u_\tau z$$

$$C_0(z = \infty) = \frac{\partial C_0}{\partial z}(z = \infty) = 0$$

$$\int_0^\infty C_0 \, dz \equiv 1$$

gives

$$\frac{\partial \bar{Z}}{\partial t} = \frac{\partial}{\partial t}\int_0^\infty z C_0 \, dz = \int_0^\infty z \frac{\partial}{\partial z}\left(K_z \frac{\partial C_0}{\partial z}\right) dz$$

integrate twice by parts

$$= zK_z \frac{\partial C_0}{\partial z}\bigg|_0^\infty - KC_0\bigg|_0^\infty + \int_0^\infty C_0 \frac{\partial K_z}{\partial Z}\, dz$$

$$= ku_\tau \int_0^\infty C_0 \, dz = ku_\tau.$$

Therefore $b \equiv k$ with no restriction on height of release. Since these are the same equations solved by the author numerically one suspects an error from examination of Fig. 26.

The initial conditions employed to integrate the expressions for $d\bar{X}/dt$ and $d\bar{Z}/dt$ must themselves be inspired by the concepts of similarity theory. In order to clarify the point we will use only the "neutral" case in the following discussion. The hypothesis of Lagrangian similarity was advanced by Batchelor [3] for the case of a source at ground level. The hypothesis states "the statistical properties of the velocity of a marked fluid particle at a time after release at the ground depend only on u and t." The diffusion from a source at height h can only be likened to that from a ground source after a long time when the particles will have forgotten their original position. Thus the above can be used if the elevated release is considered equivalent to a ground release upwind from the elevated source at a virtual origin. This led to an extension of the original hypothesis to include initial conditions corresponding to this shift in the origin. The extended hypothesis states that "the statistical properties of the velocity of a marked particle at time after release at height h are the same as those of a particle release at the ground ($z = 0$) at the instant $-t_1$, provided $t \gg t_1$, where t_1 is expected to be of the order h/u". As shown by Batchelor [3] an approximate boundary condition on the mean position of a particle released at height h would be

$$\bar{X}_0 = u(h)t_1 = u(h)\frac{h}{u_\tau}, \qquad \bar{Z}_0 = h.$$

For a logarithmic velocity profile this condition becomes

$$\bar{X}_0 = \frac{h}{k}\ln\frac{h}{z_0}, \qquad \bar{Z}_0 = h, \qquad \text{at } t = 0.$$

This is the condition that should be employed to integrate the expressions for $d\bar{X}/dt$ to obtain an expression for c. It is thus obvious that the condition $t = 0$, $\bar{X} = 0$, as used by the author is not consistent with similarity theory and hence a test of similarity theory based on it is not valid.

If the integration is performed an expression for c can be obtained to compare with equation (21) of [1]. Following the author's notation

$$\frac{k^2}{2}\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}\tau_n} = \ln c + \ln \bar{\zeta} + 1$$

$$\frac{k^2}{2}(\bar{\xi} - \bar{\xi}_0) = (1 + \ln c)\,\tau_n + \int_0^{\tau} n \ln\left(\frac{2b}{k}\tau_n + \frac{\bar{Z}_0}{Z_n}\right)\mathrm{d}\tau_n$$

$$= (1 + \ln c)\,\tau_n + \frac{k}{2b}\left[\left(\frac{2b}{k}\tau_n + \frac{\bar{Z}_0}{Z_n}\right)\ln\left(\frac{2b}{k}\tau_n + \frac{\bar{Z}_0}{Z_n}\right) - \left(\frac{2b}{k}\tau_n + \frac{\bar{Z}_0}{Z_n}\right) - \frac{\bar{Z}_0}{Z_n}\ln\frac{\bar{Z}_0}{Z_n} + \frac{\bar{Z}_0}{Z_n}\right]$$

but

$$\bar{\xi}_0 = \frac{\bar{X}_0}{Z_n} = \frac{1}{k}\left(\frac{\bar{Z}_0}{Z_n}\right)\ln\frac{\bar{Z}_0}{z_0}.$$

The corrected expression for c in terms of the author's notation should be

$$c = \exp\left\{\frac{k^2\bar{\xi}}{2\tau_n} - \frac{k}{2b\tau_n}\left[\left(\frac{2b}{k}\tau_n + \frac{\bar{Z}_0}{Z_n}\right) \times \ln\left(\frac{2b}{k}\tau_n + \frac{\bar{Z}_0}{Z_n}\right) - \left(\frac{\bar{Z}_0}{Z_n}\ln\frac{\bar{Z}_0}{Z_n}\right)\right] - \frac{k}{2\tau_n}\left(\frac{\bar{Z}_0}{Z_n}\right)\left[\ln\frac{\bar{Z}_0}{z_0}\right]\right\}.$$

The choice of scales selected by the author did not make clear the values of u_τ or z_0 actually utilized. Hence it is not possible to calculate whether the additional term fully compensates for the excursion in c found in Fig. 28. The correction would appear to be large and in the right direction.

In the case of stably stratified flow, an extension of the Lagrangian similarity for elevated sources such as is found in [4] should be used to develop the equation. It is found therein that $\mathrm{d}\bar{X}/\mathrm{d}t - \bar{U}(\bar{Z})$ is always in an excess by an amount which has between u_τ and $2u_\tau$ for extreme values of stability. Because of its small magnitude, the error is significant only near the point of release and hence the assumption $\mathrm{d}\bar{X}/\mathrm{d}t \simeq \bar{U}(\bar{Z})$ is reasonable when considering diffusion at large distances.

It must, however, be emphasized that the expression is still approximate and to use this as a test is only appropriate for large diffusion times. Again the author's calculations fail due to the limited time considered. In the author's notation the dimensionless diffusion times required must be

$$t \gg \frac{h}{u_\tau} \text{ or } \tau_n = \frac{D_n t}{Z_n^2} \gg \frac{D_n h}{u_\tau Z_n^2} = 0{\cdot}2\,h \simeq 0{\cdot}02 \qquad \text{say } \tau_n \simeq 0{\cdot}2;$$

whereas the maximum times displayed are $\tau_n = 0{\cdot}1$. The situation is further frustrated by the fact that the numerical solution provided is developed for the bounded case of matter diffusing between two solid walls. This case is essentially different from the case of an unbounded atmosphere, but an equivalence may exist for short diffusion times. Saltzman (1962) has shown that for a finite upper bound to be insignificant $t \ll Z_n^2/2D_n$ or $\tau_n \ll 0{\cdot}5$. Dr. Atesman comments on effects of the wall detected for $\tau_n > 0{\cdot}15$. Hence the range of dimensionless dispersion time used by the author is necessarily inadequate to criticize the results of the Lagrangian similarity hypothesis.

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