Atmospheric Environment Pergamon Press 1973. Vol. 7, pp. 443–454. Printed in Great Britain.

A LABORATORY STUDY OF DIFFUSSION IN STABLY STRATIFIED FLOW

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(*First received* 9 *August* 1971 *and in final form* 3 *July* 1972)

Abstract—This paper investigates the usefulness of the similarity that exists between the stably stratified atmospheric boundary layer and a wind tunnel boundary layer developing over a cold plate in regard to mass diffusion phenomenon. Detailed observations of the concentration field downwind of a point source of Krypton-85 were made under different stability conditions in the wind tunnel boundary layer. The concentration characteristics obtained from diffusion experiments show excellent agreement with those observed in the atmosphere. The index *m*, describing the power law variation of ground level concentration with distance, matches reliable field estimates. The data compares well with the predictions of Lagrangian similarity theory. It appears that the parameters evaluated in the field by KLUG (1968) hold also for the wind tunnel data. A comparison of detailed observations with an available numerical solution of the three-dimensional diffusion equation suggests that the detailed diffusion patterns obtained from the wind tunnel experiments may be preferable to numerical solutions requiring arbitrary specification of lateral diffusivity.

1. INTRODUCTION

ATMOSPHERIC diffusion requires experimental data to provide a pragmatic datum for analysis. Although a number of diffusion experiments were undertaken in the atmosphere in the early sixties, their motivational differences work against a simple comparison of experimental results (SLADE, 1968). Moreover, the pace of these full scale studies has been slowing down due to their prohibitive cost. The prospect that the dynamical features of air flow in the surface layer of the atmosphere can be simulated in a wind tunnel leads to an additional means of studying atmospheric diffusion experimentally. The specific problem, studied here by means of a wind tunnel, is the turbulent diffusion from a point source at ground level in a stably stratified atmospheric boundary layer.

A number of laboratory studies of the basic diffusion phenomenon have been reported in the literature (for example, MALHOTRA and CERMAK 1964; DAVAR and CERMAK 1964; POREH and CERMAK 1964; etc.). These investigations were mostly made in a developing boundary layer which cannot be regarded as a correct analog of the atmospheric surface layer. The present work is distinguished from all these by having been conducted in a wind tunnel boundary layer naturally developed over a very long test section.

PLATE and LIN (1966) and ARYA (1968) have demonstrated the similarity that exists between the stably stratified atmospheric boundary layer and a wind tunnel boundary layer developing over a cold plate. They showed that the mean flow and turbulence characteristics in the near wall region of the laboratory boundary layer are well described by Monin and Obukhov's similarity theory and that this theory provides a good basis for modeling the similar characteristics of the atmospheric surface layer.

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CEP 70-71-63

This paper investigates the usefulness of the similarity in regard to the diffusion phenomenon.

A continuous point source of Krypton-85 was placed at the surface of a smooth cold plate over which a stably stratified shear layer was produced by blowing hot air. The diffusion patterns were observed by sampling the gas and evaluating sample strength with Geiger-Mueller tubes.

The data are analyzed to display the effects of thermal stratification on plume spread. As the flow characteristics are well described by Monin and Obukhov's similarity theory, the comparison of diffusion characteristics is limited to those theoretical approaches that utilize this similarity to formulate the diffusion problem.

2. THEORETICAL BACKGROUND

Any formulation of the diffusion problem requires knowledge of wind characteristics in the lower layers of the atmosphere. Monin–Obukhov's similarity arguments have formed the basis of much of the present understanding of turbulent flow in the diabatic surface layer. According to the similarity theory, the stationary turbulent regime is completely determined by three parameters viz. shear stress τ and vertical heat flux H which do not vary with height, and the ratio g/T, where T is absolute average temperature of the layer under consideration. The only scale of velocity is $u_* = \sqrt{\tau/\rho}$ and the only scale of length is

$$L = -\,u_*^3 \Big/ \left[\kappa \frac{g}{T}\frac{H}{\rho C_p}\right]. \tag{1}$$

All dimensionless variables can be functions of dimensionless height ζ $(=z/L)$ only. Thus, the Monin–Obukhov theory predicts for the non-dimensional wind shear,

$$S = \frac{\kappa z}{u_*}\frac{\mathrm{d}u}{\mathrm{d}z} = \phi(\zeta). \tag{2}$$

Under neutral conditions, S is unity and the velocity profile is logarithmic. Monin and Obukhov approximated $\phi(\zeta)$ for small ζ as

$$\phi(\zeta) = 1 + \beta\zeta, \tag{3}$$

which on integration gives

$$u = \frac{u_*}{\kappa}\left[\ln\frac{z}{z_0} + \beta(\zeta - \zeta_0)\right]. \tag{4}$$

This log-linear law of velocity distribution is shown to be valid over a wide range in stable conditions by WEBB (1970) in the atmosphere, and by ARYA and PLATE (1969) in the wind tunnel.

Since the basis of modeling stratified atmospheric flows is the similarity theory of Monin and Obukhov, we are interested in those solutions of the point source diffusion problem which incorporate the results of this similarity. There are only two such solutions that one finds in the literature viz. Lagrangian similarity theory, and a solution of the three-dimensional diffusion equation by YAMAMOTO and SHIMANUKI (1964).

The lagrangian similarity theory was originally proposed by BATCHELOR (1959) to investigate turbulent diffusion in neutral conditions. This approach is based on an hypothesis and an assumption. The Lagrangian similarity hypothesis states that the rate of increase of average vertical displacement of particles $\bar{Z}$ is uniquely determined by u_* within the constant stress region of the atmosphere. Hence, on dimensional grounds

$$\frac{\mathrm{d}\bar{Z}}{\mathrm{d}t} = bu_*, \tag{5}$$

where b is a universal constant. The assumption is that the rate of increase of mean displacement $\bar{X}$ downwind of the source is equal to average wind velocity at $\bar{Z}$, i.e.

$$\frac{\mathrm{d}\bar{X}}{\mathrm{d}t} = \bar{u}(\bar{Z}). \tag{6}$$

Combination of equations (5 and 6, in the logarithmic form) gives, upon integration,

$$\bar{X} = \frac{\bar{Z}}{b\kappa}\left(\mathrm{ln}\,\frac{\bar{Z}}{Z_0} - 1\right). \tag{7}$$

On the basis of further dimensional arguments, BATCHELOR (1959) and ELLISON (1959) expressed the axial ground level concentration from a point source, $C_{\max}$, as

$$\frac{C_{\max}}{Q} \propto \frac{1}{\bar{Z}^2 \dfrac{\mathrm{d}\bar{X}}{\mathrm{d}t}} \simeq \frac{1}{\bar{Z}^2\,\bar{u}(\bar{Z})}. \tag{8}$$

Extension of the hypothesis of the Lagrangian similarity to diffusion in thermally stratified flow has been considered in the light of Monin–Obukhov similarity theory by GIFFORD (1962); CERMAK (1963); YAGLOM (1965); PANOFSKY and PRASAD (1965); PASQUILL (1966); KLUG (1968) and CHAUDHRY (1969). According to Gifford, $\mathrm{d}\bar{Z}/\mathrm{d}t$, must, on dimensional grounds, be proportional to u_* times some universal dimensionless function involving L. Therefore,

$$\frac{\mathrm{d}\bar{Z}}{\mathrm{d}t} \equiv bu_*\,\phi_1(\bar{\zeta}) \tag{9}$$

where $\bar{\zeta} = \bar{Z}/L$.

The function ϕ_1 was assumed to have the form

$$\phi_1(\zeta) = \left(1 - \frac{\zeta}{\phi(\zeta)}\right)^{\frac{1}{4}}. \tag{10}$$

as suggested by MONIN (1959).

The relation between $\bar{X}$ and $\bar{Z}$ for the stably stratified case, as a consequence of equations 4, 6, 9 and 10, is given by

$$\frac{b\kappa\bar{X}}{L} = \int_{\zeta_0}^{\zeta} [\mathrm{ln}\,\zeta/\zeta_0 + \beta(\zeta - \zeta_0)]\,\phi_1{}^{-1}(\zeta)\,\mathrm{d}\zeta. \tag{11}$$

GIFFORD (1962) also showed that the relationship for maximum ground level concentration $C_{\max}$ due to a point source in the thermally stratified case is similar to that in equation (8), i.e.

$$\frac{u_* L^2 C_{\max}}{\kappa Q} = G \frac{1}{\bar{\zeta}^2 \left[1n\bar{\zeta}/\zeta_0 + \beta(\bar{\zeta} - \zeta_0) \right]}^{\dagger} \tag{12}$$

where G is the constant of proportionality.

Gifford evaluated equations (11 and 12) for $G = 1$ and presented curves for the dimensionless average vertical displacement $\bar{Z}/L$ and the dimensionless concentration $u_* L^2 \; C_{\max}/\kappa Q$ as a function of downwind distance $b\kappa\bar{X}/L$.

KLUG (1968) compared a relationship for $C_{\max}$ similar to that in equation (12) to the Project "Prairie Grass" data to evaluate the constant of proportionality G. He found that though G was independent of stability, it did, however, depend upon σ_v/u_* where σ_v is the r.m.s. of fluctuations of lateral velocity. A comparison of the wind tunnel concentration data will be made in section 4 with the similarity theory prediction in equation (12) using Klug's estimates of G.

YAMAMOTO and SHIMANUKI (1964) also used the results of similarity theory to solve the diffusion equation

$$u \frac{\partial C}{\partial x} = \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial C}{\partial y} \right) \tag{13}$$

under the following conditions:

$$K_z = \kappa u_* z \; \phi^{-1}(\zeta'), \tag{14}$$

and

$$K_y = \kappa u_* z \; \alpha(\zeta'_0), \tag{15}$$

where ϕ defined in equation (2) is given by

$$\phi^4 + \zeta' \phi^3 - 1 = 0. \tag{16}$$

Also, $\zeta' = -\gamma z/L$ and $\zeta'_0 = -\gamma z_0/L$,

where γ is a constant. The parameter $\alpha(\zeta'_0)$ is determined by comparison of the calculated and observed flow patterns. Whereas the expression for K_z follows from the similarity theory under the assumption that $K_z = K_M$, there is no basis presented for the assumed form for K_y. A comparison of their calculated distribution of concentration with wind tunnel experimental data should aid in evaluating this assumption.

3. EXPERIMENTAL TECHNIQUE

The experimental work was carried out in the closed circuit micro-meterological wind tunnel at the Fluid Dynamics and Diffusion Laboratory of Colorado State University. This wind tunnel has a 1.8 m square test section which is 24 m long and permits control of stability and humidity conditions. For the present study, the temperature of the floor was held at about 4°C and that of the air in the free stream at 50°C. The variations in flow characteristics over the last 5 m length of the test section are negligible making the wind tunnel shear layer a good analog to the stably stratified

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† Equation (12) as given above is the correct form of Giffords formulation. The Constant G in this equation is indeed the same as G' suggested by Hosker and was calculated as a product of G_{Klug} and g_{Klug}. It should, however, be pointed out that KLUG (1968) found g not to be a function of ζ but that of σ_v/u_*. This is the reason why we included the values of the parameter σ_v/u_* in TABLE 1.

atmospheric boundary layer. A reference should be made to the detailed analysis of the wind tunnel flow by ARYA (1968) who used a temperature differential similar to the one described above. Measured mean velocity profiles are compared in non-dimensional form with Monin and Obukhov's similarity theory in FIG. 1.

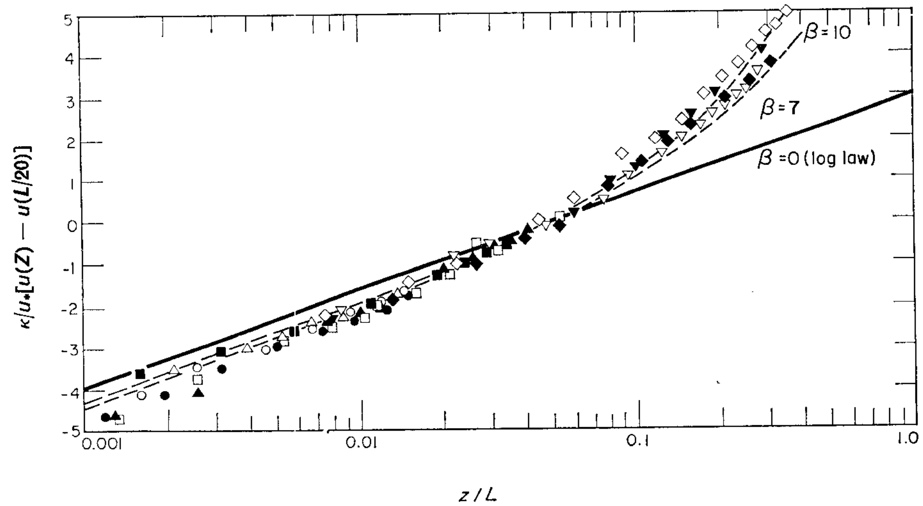

FIG. 1. Comparison of mean velocity distribution in the wall layer, with Monin and Obukhov's similarity theory (after ARYA, 1968).

Krypton-85 was used as a tracer for obtaining concentration distributions downwind of continuous point sources. This beta emitting radioactive noble gas has many advantages over other tracers. As it is considerably diluted before use, its properties are similar to those of air. The technique is simple and economical. The sampling

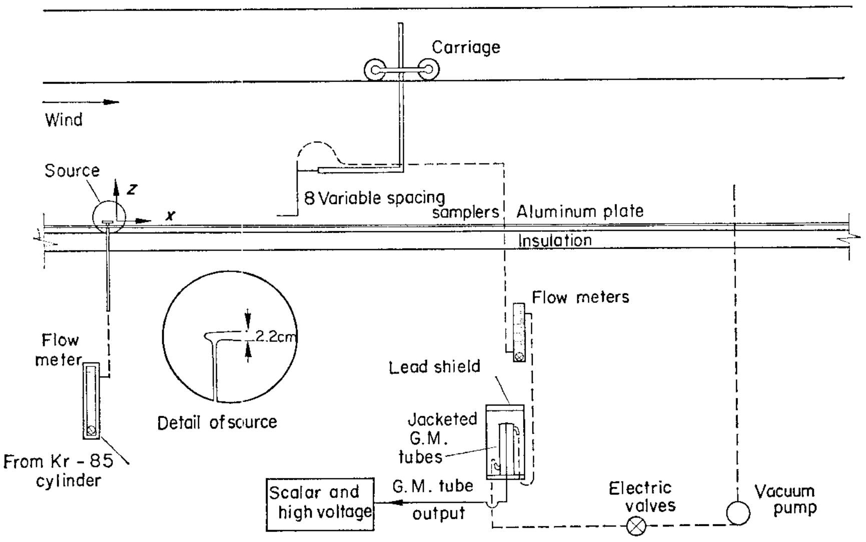

FIG. 2. Tracer feed and sampling system.

system is sketched in FIG. 2. A sampling rake of eight 1.6 mm dia. probes was mounted in an adjustable array on a remote control traversing carriage. Samples were aspirated through jacketed Geiger–Mueller tubes for a fixed interval of time and then enclosed by closing inlet and outlet valves. Each sample was subsequently counted on a scaler. An elaborate description of the tracer technique is given by CHAUDHRY (1969). Free stream velocities of 6 and 2 m s^{-1} were used for these experiments.

4. RESULTS

Concentration distributions were measured at distances of 0.3, 0.61, 0.92, 1.52, 2.14, 2.75, 3.36, 3.97 and 4.58 m from the source under various stability conditions. A consistency check of the data was performed by integrating the concentration profiles to calculate

$$Q(x) = \int_{0}^{\infty} \int_{-\infty}^{\infty} u(z)\, C(x, y, z)\, \mathrm{d}y\, \mathrm{d}z \tag{17}$$

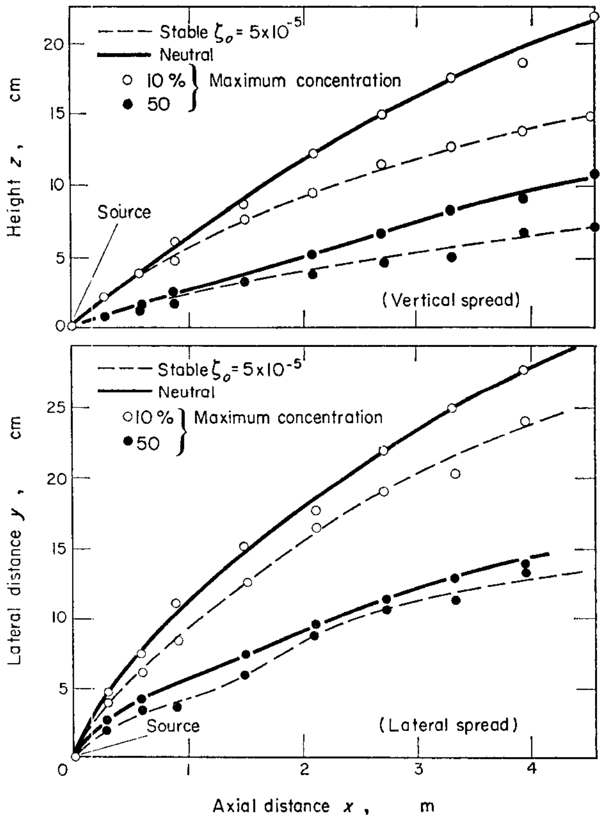

FIG. 3. Vertical and lateral spreads of the ground source plume under neutral and stable conditions.

for all x positions. The Q values obtained in this manner and the value based on the source concentration and flow rate (kept constant in all the experiments) checked within 10 per cent over the range of x from 0.3 to 4.58 m.

One of the objectives of this study was to investigate the effect of stable stratification on the plume geometry. The vertical and lateral spread of a ground source plume for the same free velocity but under different stability conditions are shown in FIG. 3. Lines of constant concentration (one-tenth and one-half of the plume's axial concentration) show consistently that the stable stratification limits the horizontal as well as vertical spread of the plume. However, while the vertical spread is suppressed due to moderately stable stratification by about 30 per cent, the lateral spread is decreased by only about 10 per cent. The effect of stratification is further illustrated in FIG. 4 which contains typical plume cross sections under both stable and neutral conditions of flow.

TABLE 1. WIND TUNNEL FLOW PARAMETER

Run	Free stream velocity m s^{-1}	u_* cm s^{-1}	L m	σ_v/u_*	ζ_0	G	β
Neutral	3	12.4	∞		0		—
Slightly stable	6	17.7	2.44	1.66	1×10^{-5}	0.131	10
Moderately stable	3	7.34	4.87	1.37	5×10^{-5}	0.143	10

The experimental data on concentration are in good agreement with the results of the Lagrangian similarity theory as given by GIFFORD (1962). The values of various flow and similarity parameters are given in TABLE 1. The variation of $\bar{Z}$ with axial distance, calculated from the concentration profiles measured at various $x = \bar{X}$ positions, is compared with theory in FIG. 5. The agreement between experimental data, taken for two different velocities, and the theory is fairly good. The maximum ground concentrations for two stabilities are compared with predictions of similarity theory in FIG. 6 showing excellent agreement as to the magnitude and rate of decrease of concentration with axial distance. The constant G, appearing in the expression for non-dimensional concentration and given in TABLE 1, was determined according to KLUG's (1968) method of evaluation at adiabatic conditions. It correlates the wind tunnel data with similarity theory remarkably well. The fact that both field and wind tunnel data are correlated to similarity theory by the same parameter is significant to the similarity between the two flows. It is customary to express the variation of concentration with axial distance in terms of a power index in the empirical formula $C_{\max} \alpha x^{-m}$. The value of this index for the wind tunnel is 1.7 which may reflect the effect of stability as m is found in the field for neutral conditions to be about 1.8 (SUTTON, 1953).

By integrating the point source data in the lateral direction, equivalent line source data are obtained. The boundary concentrations C_B for such a line source are also compared with the predictions of similarity theory obtained from

$$\frac{C_B}{Q_1} \propto \frac{1}{Z\dfrac{d\bar{X}}{dt}} \tag{18}$$

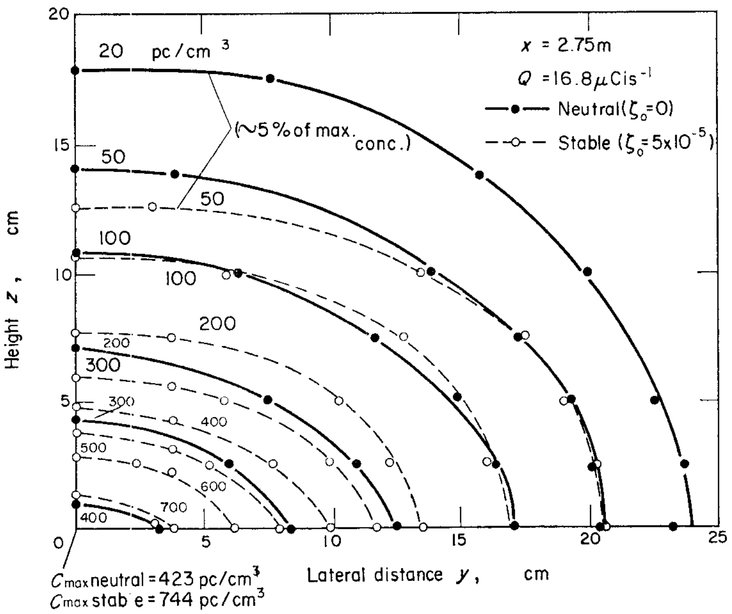

Fig. 4. Effect of stable stratification on the crosswind-distribution of concentration.

which is analogous to the proportionality in equation (8). No estimate of the constant of proportionality for line sources, similar to the one for a point source given by Klug, is available in the literature. In the point source problem, such a constant expresses the relationship between the lateral and the vertical spread of the plume as well as the relationship between $\bar{Z}$ and the vertical standard deviation σ_z (Panofsky and Prasad, 1965). As the line source problem is two-dimensional, the lateral characteristics are not relevant and the constant of proportionality B, say, expresses only the

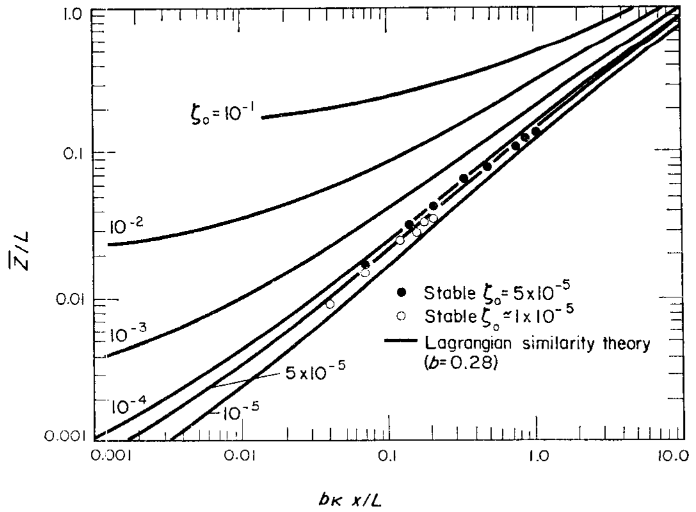

Fig. 5. Comparison of average non-dimensional plume rise data with similarity theory.

latter relationship. Its value should thus be much smaller than the value of G suggested by Klug and may lie between 1 and 2. Substitution of the assumption expressed in equation (6) into the proportionality (18) gives

$$\frac{u_* L\, C_B}{\kappa Q_1} = B \frac{1}{\zeta[\ln\zeta/\zeta_0 + \beta(\zeta - \zeta_0)]} \tag{19}$$

FIGURE 7 displays the comparison of experimental data with the estimates from equation (19). The values of constant B for slightly and moderately stable runs, respectively, are found to be 1.8 and 1.5. There is total agreement between these cases in the rate of decrease of concentration. The exponent m expressing this variation ($C_B \propto X^{-m}$) is about 0.99.

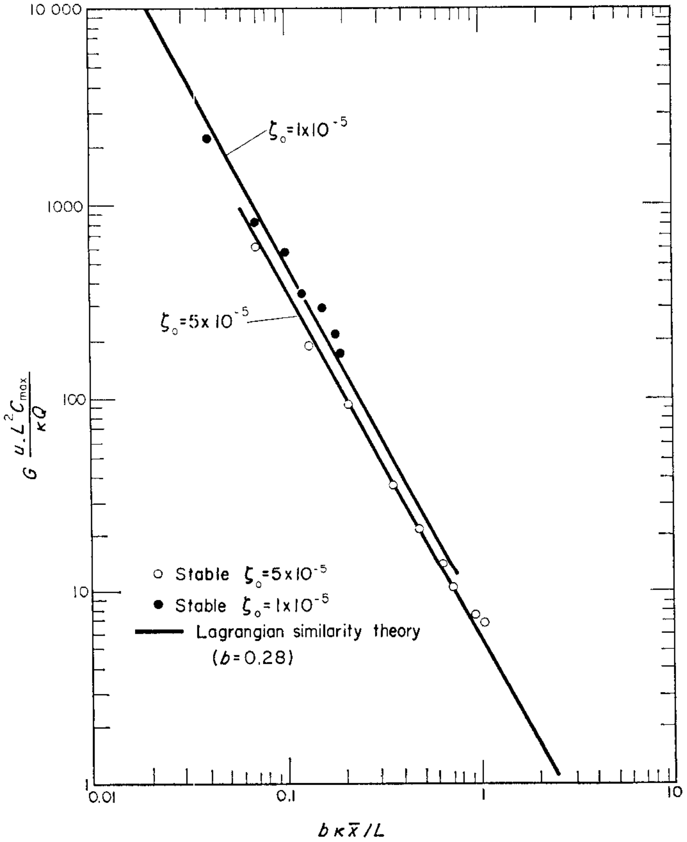

FIG. 6. Comparison of maximum ground level concentrations due to ground level point sources with predictions of similarity theory.

The detailed concentration measurements are next compared with the YAMAMOTO and SHIMANUKI (Y and S) numerical solution of the point source problem briefly reviewed earlier. The whole form of the Y and S solution depends on α. Yamamoto and Shimanuki have evaluated α as a function of $\zeta'_0 = -\gamma z_0/L$ using field data which,

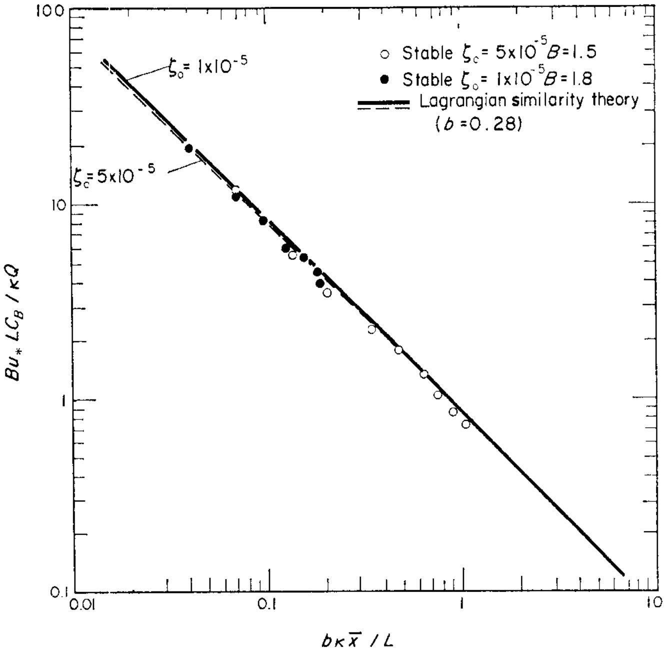

FIG. 7. Comparison of boundary concentration due to equivalent line source with similarity theory.

for our moderately stable run ($\zeta'_0 \simeq -0.001$), is 7.2. A value of 18 is assumed for γ in this comparison. FIGURES 8 and 9 show the observed isopleths of concentration and those predicted by Y and S solution at two different distances from the source, namely, $x/z_0 = 62{,}500$ and 137,000. Theoretical curves have been included for various α values, including the one suggested by Y and S, to facilitate selection of an α value for best fit. The parameter α controls not only the shape of an isopleth but its magnitude also. Agreement of measured and calculated values is reasonable only if α is allowed to change according to distance from centreline. This suggests that α is not merely a function of ζ'_0 but also depends on the position relative to the source. In other words, the Y and S formulation for K_y is inadequate. Unfortunately, no alternative relationship is analytically apparent. It is suggested that the wind tunnel data may be used in non-dimensional form in field applications if the relationship for K_y is established by co-ordinated laboratory and field tests.

5. CONCLUSIONS

The modeling of diffusion in a stably stratified atmospheric boundary layer is shown to be feasible through comparison of the laboratory data with the Lagrangian similarity theory. The length scale of Monin and Obukhov and the roughness height z_0 are found to be adequate scaling parameters for transferring wind tunnel data to the field. It appears that the detailed diffusion patterns obtained from wind tunnel experiments may be preferable to numerical solutions of the point source problem which require arbitrary specification of lateral diffusivity.

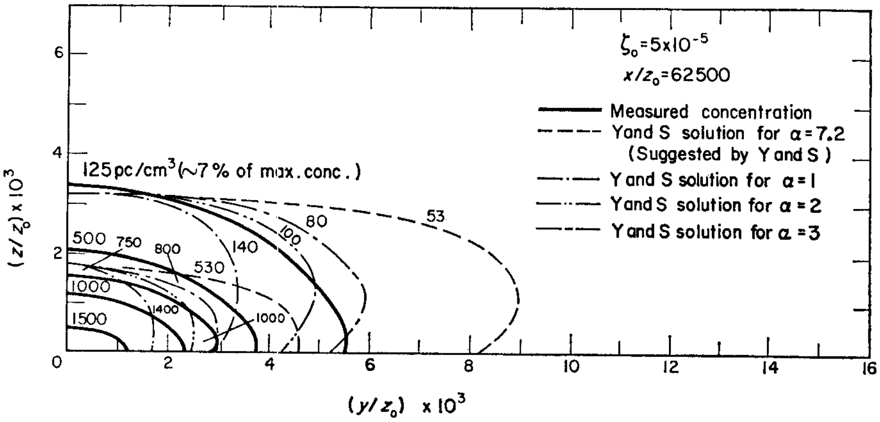

FIG. 8. Comparison of observed isopleths of concentrations with YAMAMOTO and SHIMANUKI (1964) solution at $x/z_0 = 62{,}500$.

The experimental data show plume characteristics which are in general agreement with field observations. Stability suppresses plume spread in the vertical direction more than in the horizontal. The index m describing a power law variation of ground concentration with distance matches well with reliable field estimates.

This paper reports diffusion results under relatively mild stability conditions. Further study of this phenomenon for a wide range of stabilities should prove most illuminating. This study is distinguished from others on the subject in that a better tracer was employed and that the boundary layer was naturally developed over a long test section.

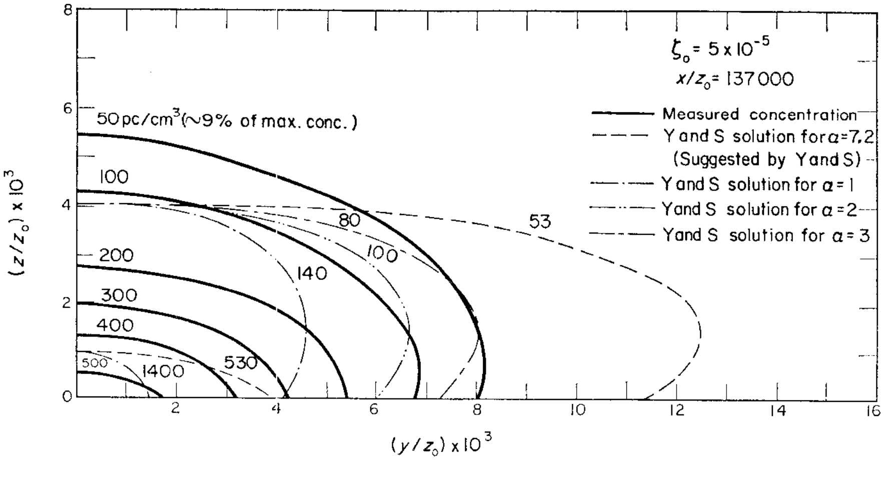

FIG. 9. Comparison of observed isopleths of concentration with YAMAMOTO and SHIMANUKI (1964) solution at $x/z_0 = 137{,}000$.

Acknowledgements—Financial support for this study was provided by U.S. Army Electronics Command, Fort Huachuca, under Contract No. DAAB07-68-C-0423.

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