Comments on "Boundary-Layer Turbulence Measurements with Mass Addition and Combustion"

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IN Ref. 1 an investigation was made of the turbulent velocity fluctuation field in an isothermal boundary layer with homogeneous injection. The authors measured the shear stress distribution across the boundary layer and displayed the results in Fig. 5 of their paper as normalized shear stress as a function of velocity ratio ϕ.

In Ref. 2 Meroney developed an analytical expression for the velocity profile across the turbulent transpired boundary layer together with an approximate technique to determine the normalized shear stress distribution. He proposed for a velocity profile near the wall

$$u^+ = y^+ + \tfrac{1}{2}(v_w^+)y^{+2} + (1/3!)(v_w^{+2})y^{+3} + U_4^+y^{+4} \quad (1)$$

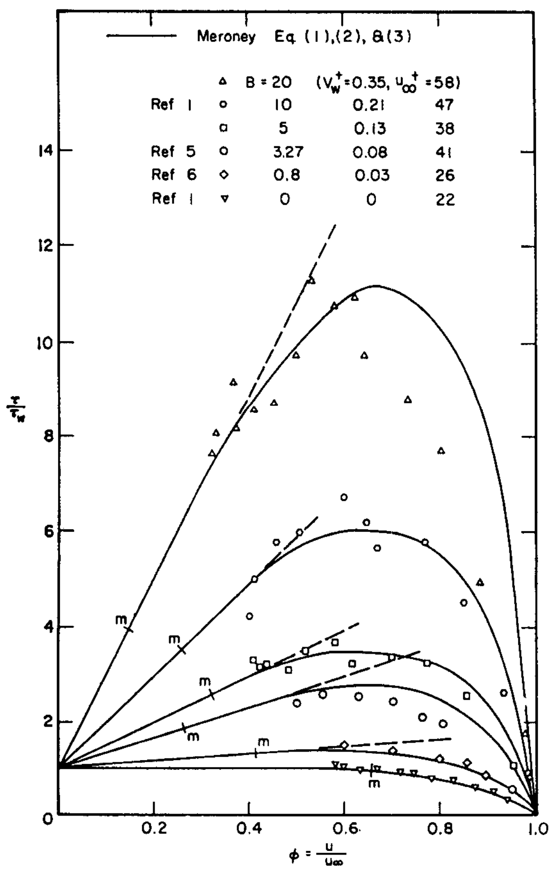

Fig. 1 Shear stress distribution across the isothermal turbulent boundary.

and farther from the wall

$$u_\infty^+ - u^+ = \frac{v_w^+}{4k^2}\ln^2\frac{y^+}{\delta^+} - \frac{1}{k}(1 + v_w^+u_\infty^+)^{1/2}\ln\frac{y^+}{\delta^+} + \frac{\pi}{k}(1 + v_w^+u_\infty^+)^{1/2}\left[2 - \omega\left(\frac{y^+}{\delta^+}\right)\right] \quad (2)$$

where the nondimensional nomenclature have their conventional definitions, and the matching conditions have been computed to be

$$y_m^+ = 15.67 - 860(v_w^+)^{1/2}/u_\infty^{+2} - 11.4v_w^{+0.45}$$

$$U_v^+ = -5.4 \times 10^{-4} - 1.3(v_w^+)^{1.7}/u_\infty^{+2} - 1.6 \times 10^{-2}v_w^{+1.7}$$

The shear profile may be determined from the velocity profiles from

$$\frac{\tau}{\tau_w} \simeq (1 + v_w^+u^+) - (1 - v_w^+u_\infty^+) \times \frac{2\int_0^{y/\delta}(u^+)^2d\xi - u^+\int_0^{y/\delta}u^+d\xi}{2\int_0^1(u^+)^2d\xi - u_\infty^+\int_0^1u^+d\xi} \quad (3)$$

Near the boundary Eq. (3) reduces to

$$\tau/\tau_w \simeq 1 + v_w^+y^+ + (v_w^{+2}/2!)y^{+3} + v_w^+U_4^+y^{+4} \quad (4)$$

Figure 1 compares the calculated distributions of shear stress from Eqs. (1–3) with the data displayed in Fig. 5 of Ref. 1. The comparison is good except at very high blowing rates. Figure 2 compares measured values of the normalized maximum shear stress with computed values. Equation (3) seems to provide better agreement with this variation than a previously suggested correlation by Tennekes in Ref. 3.

The rather large divergence of theory and data at high blowing rates in Fig. 1 may represent separation of the main flow from the transpired wall. If the flow parameters are compared with the somewhat limited criteria for attachment suggested by Hacker[4], this premise is strengthened. Hacker suggested an empirical blow-off criteria of $[(\rho v)_w/(\rho u)_\infty]Re^{1/5} = 0.08$ for air into air transpiration at low Mach numbers.

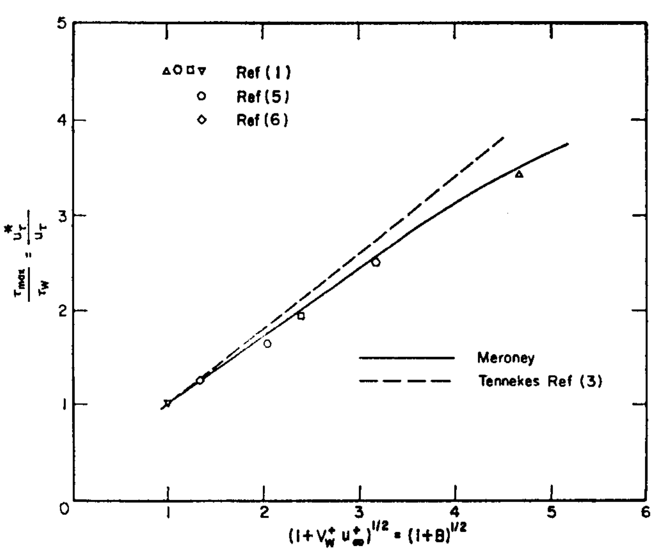

Fig. 2 Variation of pseudofriction velocity with mass transfer number.

Received January 2, 1968.
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Reprinted from AIAA JOURNAL, Vol. 6, No. 6, June 1968, pp. 1213–1214 CEP67-68-27

When the mass transfer number B equals 20 for the data in Ref. 1, this expression equals approximately 0.11. In addition, the kink observed in the velocity profiles displayed as Fig. 2 of Ref. 1 for $B = 20$ suggests the presence of separation.

References

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