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The Behavior of Negatively Buoyant Stack Gases

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Abstract

The body of information presented in this paper is directed to engineers engaged in the design of stacks, relief valves and other devices which exhaust dense, cold or otherwise negatively buoyant gaseous emissions into the atmosphere. This paper summarizes mathematical expressions for dense gas plume dynamics and concentration behavior developed in detail in a previous report by the authors. Plume dynamic behavior was observed and measured in wind tunnel tests by use of titanium tetrachloride smoke. Concentrations were measured by the use of Krypton-85 as a tracer gas. Dense gas emissions were formed by mixing air and freon in varying proportions to form gas of the desired specific gravities. Experimental measurements were made of plume rise height for vertical plumes in a quiescent medium and for bent-over plumes in a laminar crosswind. The horizontal "touchdown" distance was also measured for the bent-over plumes. Concentration measurements were made for bent-over plumes in a laminar crosswind and for dense gas ground sources in a turbulent boundary layer under neutral and inversion stratifications. Experimental results were correlated with parameters developed in the theoretical analysis where appropriate. Expressions were developed for plume rise height for vertical and bent-over plumes and for "touchdown" distance and concentration behavior for bent-over plumes. The effect of specific gravity on dense ground sources is noted from experimental data.

Introduction

Negatively buoyant emissions into the atmosphere have been reported by several observers. Scorer[15] reports the case of two power plants emitting wet washed plumes with apparently insufficient elimination of free water, in which the subsequent evaporation of the free water cools the plume and causes it to sink. According to the report the plant itself is obscured from view on many occasions. Chesler and Jesser[6] and Bodurtha[1] have observed and discussed the descent of dense gases from stacks and relief valves.

The consequences of such behavior are potentially drastic. When the plume sinks rapidly to the ground, very little dilution will occur in comparison with normal emissions and high ground level concentrations of gases which may be toxic or explosive could result. In this report experimental results of the dynamic behavior of negatively buoyant emissions and the resulting concentration distributions are reported.

Summary of Mathematical Expressions

Plume Rise Equations. In a previous report[10], the authors developed an expression for the time of rise of a negatively buoyant plume. In this report the equations of conservation of mass, volume and buoyancy together with the entrainment hypothesis of Morton, Taylor and Turner[12] and the assumption of constant (potential) density yield a solution for dense plume rise height in a neutral atmosphere. This solution relates dimensionless rise height to a function of the Froude Number multiplied by an integral which can be evaluated as the sum of a Beta function and an infinite series. However, it is shown that for most applicable Froude Numbers the expression can be approximated by:

$$H/D_o = \frac{0.515F_R}{\sqrt{\alpha}} \qquad (1)$$

where

$$F_R = \frac{\sqrt{\rho_o}\;U_o}{\sqrt{(\rho_o-\rho_A)gD_o}}$$

ρ_A = Ambient Density

U_o = Exit Velocity

ρ_o = Exit Density

D_o = Exit Diameter

α = Dimensionless Entrainment Constant

The foregoing expression was obtained with the assumption of constant density equal to the exit density. If density is assumed equal to the ambient the expression becomes:

$$H/D_o = \frac{0.515SG^{1/4}F_R}{\sqrt{\alpha}} \quad (2)$$

where $SG = \rho_o/\rho_A$

An expression for α which is expected to be valid in the jet region close to the stack and permit evaluation from smoke pictures was developed and is:

$$\alpha = \frac{1}{Z}\left[\frac{b-b_o}{2} + \frac{b_o(SG-1)}{4\sqrt{SG}} \ln\left\{\frac{\sqrt{[b_o{}^2U_o(SG-1)]^2 + 4SGb_o{}^2U_o{}^2b^2} + 2\sqrt{SG}\, b_oU_ob}{\sqrt{[b_o{}^2U_o(SG-1)]^2 + 4SGb_o{}^4U_o{}^2} + 2\sqrt{SG}\, b_o{}^2U_o}\right\}\right] \quad (3)$$

where

b_o = Stack Radius

Z = Height Above Stack

b = Plume Radius at Height Z

Plumes in a laminar crosswind are assumed to approximate behavior at low wind speeds when plume generated turbulence will govern rise and concentration behavior. For this case the expression for time of rise is still valid. The equations of conservation of mass, momentum and buoyancy are closed with the Hoult, Fay and Forney[11] model for the entrainment of outside air into a bent-over plume where entrainment due to parallel and perpendicular velocity differences are assumed to superimpose. Using these equations, it is shown that for the asymptotic cases of near vertical trajectory close to the stack and for near horizontal trajectory far from the stack, the dimensionless mass flux is a function of R, the ratio of exit velocity to wind speed and Z/D_o, where Z is the height of trajectory centerline. This permits separation of time and distance variables and a generalized expression for plume rise results:

$$H/D_o = \phi(R, (SG)(F_R^2)) \tag{4}$$

If the plume is considered to be in a near horizontal configuration during the entire rise, this expression becomes:

$$H/D_o = \left[\frac{3R(SG)F_R^2}{8\beta^2}\right]^{1/3} \tag{5}$$

where β is a dimensionless entrainment constant for velocity differences perpendicular to the plume axis. The preceding equation predicts values of H/D_o which vary a maximum of ≈10% in the range of specific gravities from 1.25 to 5 if the exit and crosswind velocities are held constant. The minimum predicted rise is at a specific gravity of two. This agrees qualitatively with the observations of Bodurtha[1] who stated that the rise of plumes he observed with roughly this range of specific gravities was independent of specific gravity.

If a near horizontal trajectory is assumed during descent an expression results for the dimensionless horizontal distance from plume high point to "touchdown":

$$\frac{X_D-\overline{X}}{D_o} = \left[\frac{8\beta^2}{3}\left(\frac{H}{D_o}\right)^3 \left\{\left(2 + \frac{h_s}{H}\right)^3 - 1\right\}\right]^{1/2} \frac{F_{RH}}{\sqrt{R}} \tag{6}$$

where F_{RH} = the "horizontal" Froude Number = $\dfrac{\sqrt{\rho_A}\,V}{\sqrt{(\rho_o-\rho_A)gD_o}}$

X_D = the horizontal "touchdown" distance

$\overline{X}$ = horizontal distance to plume high point

h_s = stack height

V = crosswind velocity

The horizontal distance from the stack to the plume high point can be estimated by assuming the plume to be convected horizontally with the crosswind velocity V:

$$\frac{\bar{X}}{D_o} \approx \frac{F_R^{\ 2}}{R} \qquad (7)$$

Concentration Determinations. Rouse, Yih and Humphries[14] and Morton et al.[12] obtained theoretical expressions and experimental verification for the decay of maximum concentration in vertical buoyant plumes in a quiescent medium, using mixing length and proportional entrainment hypotheses respectively. Measurements indicated Gaussian cross-sectional concentration distributions. In this case mixing length and entrainment hypotheses are consistent, both predicting "straight-sided" plumes. For negatively buoyant plumes this is not the case and since the entrainment prediction comes closer to the "mushroom" cloud observed, it was used. The approximate prediction for the maximum concentration at the high point of a vertical plume is:

$$\frac{\chi_m D_o^{\ 2} U_o}{Q} \approx \left[\frac{0.36}{(\alpha)^{3/4}}\right]\left(H/D_o\right)^{-1} \qquad (8)$$

where

χ_m = Maximum (centerline) concentration at plume high point

Q = Effluent flow rate = $\frac{\pi}{4}\chi_o U_o D_o^{\ 2}$

χ_o = Concentration of effluent at stack exit

For plumes in a laminar crosswind, the assumptions of Gaussian cross-sectional distribution, characteristic mixing length and similarity in the form of mass and momentum diffusivities are consistent with entrainment theory if a near horizontal trajectory is assumed over the plume path. This predicts a concentration relationship at plume "touchdown" point of:

$$\frac{\chi V D_o^{\ 2}}{Q} \sim \left[\frac{2H + h_s}{D_o}\right]^{-2} \qquad (9)$$

Wind Tunnel Simulation of Turbulent Diffusion. Cermak, et al.[4], point out that for small scale turbulent diffusion in which there is no variation in the mean wind direction, and non-neutral stratification dynamic similarity of flows with geometrically similar boundary conditions is determined by the Reynolds Number, Prandtl Number and Richardson Number. For wind tunnels utilizing air in shear flows of high Reynolds Number, Prandtl Number similarity is assured and experience has shown that if the Reynolds Number is high enough, the diffusion behavior is essentially independent of Reynolds Number. Thus the significant parameter determining diffusion is the Richardson Number:

$$Ri = \frac{g}{T_o} \frac{(\frac{dT}{dZ})}{(\frac{dU}{dZ})^2} \qquad (10)$$

The reliability of the use of wind tunnel shear layers for modeling atmospheric flows has been demonstrated by several investigators[4,5,13], even though the ratio in Reynolds Number between model and prototype of the order of 10^3 may exist. The effects of density on diffusion governed by atmospheric turbulence therefore can be estimated by wind tunnel simulation of ground sources in a turbulent boundary layer.

Experimental Measurements

Plume Rise

Experiments on negatively buoyant plumes were conducted to check previous and present theories for plume rise characteristics and to obtain reliable data on the nature of the descending plume. Vertical plume rise experiments with no crosswind were conducted in the Industrial Aerodyanmics wind tunnel in the Fluid Dynamics and Diffusion Laboratory at Colorado State University. The 6 x 6 ft area of the test section (which was closed at each end for this experiment to limit convective currents) provided a very still environment for the experiment. The experiments with bent-over plumes were performed in the thermal tunnel in the Fluid Dynamics and Diffusion Laboratory. The tunnel has a 24 x 24 inch cross section. Turbulent plumes were injected into both laminar and turbulent crosswinds.

The pertinent parameters regarding plume rise appeared in the equations in the Introduction. Values of these parameters applicable to actual conditions can be duplicated in the wind tunnel. Implicit in the assumptions was the existence of a Reynolds Number sufficiently large to justify the assumptions of turbulent entrainment. The Reynolds Numbers based on pipe flow calculations did not meet this criteria for all cases studied so that the turbulence was artifically generated. A short-edged orifice was placed in the stacks 8 diameters upstream from the exit. Hewett[8] has shown that plume rise is independent of Reynolds Number if the plume is turbulent at exit.

The experiments were conducted as follows. Tunnel and stack flow rates were set. The specific gravities were obtained by mixing Freon 12 with Air in appropriate proportions. Freon 12 has a molecular weight of 121. The effluent was bubbled through T_iCL_4 to produce smoke. Extended time exposure photographs were taken against a blackboard divided into marked horizontal and vertical increments. Measurements were then corrected for the parallax resulting from the fact that this plume was in the center of the tunnel and the blackboard at the tunnel wall.

Concentration Measurements

Concentration measurements at the maximum rise height were made of 18 plumes injected into the laminar crosswind. Concentrations at the points where these plumes touched the floor was also taken.

Detailed cross-sectional concentration measurements taken in roughly equal longitudinal increments between the point of maximum rise height and plume touchdown were made for six plumes of specific gravities 2 and 3. The cross-sectional measurements were made so that the plane of measurement was at right angles to the plume motion as determined from the smoke pictures.

Concentration measurements were taken in a turbulent boundary layer of negatively buoyant ground sources. Measurements were made in neutral and inversion stratifications. Velocity of discharge from the source was set equal to tunnel velocity at the height of the source centerline. The turbulent boundary layer was artificially generated in the Colorado State University thermal tunnel by the use of vortex generators after the method of Counihan[7]. An approximate 1/7 law velocity profile was obtained. Using the thermal stratification capabilities of the tunnel, an approximate 1/7 law temperature profile was also generated.

Apparatus and Instrumentation

A complete description of the wind tunnel facilities utilized and the laboratory procedures followed may be found in Reference 9. The thermal and tunnel utilized a bank of electrical heaters to establish the initial thermal gradients. The extremely low velocities required for atmospheric simulation were measured by an eddy shedding hot wire technique (also see Hewett[8]). Concentration measurements were made utilizing a Krypton-85 tracer gas as described by Chaudhry and Meroney[5].

Results

Plume Rise

The vertical plumes emitted into a quiescent atmosphere were observed to rise initially in a jet with almost linear growth of radius with vertical distance. This jet region appeared to encompass from 1/4 to 1/3 the total rise height. Measurements of the point of maximum rise of the top of the plume indicated better correlation with Froude Number than with Froude Number multiplied by the one-fourth power of specific gravity. Actual least squares correlation indicated a proportionality to $SG^{.08}F_R$. This indicates the assumption $\rho \simeq \rho_o$ produces better results than $\rho \simeq \rho_A$. This at first seems completely unreasonable, particularly since it is later shown that for bent-over plumes, $\rho \simeq \rho_A$ produces good results. There are, however, important physical differences between two situations. Measurements indicate that the entrainment constant perpendicular to the flow direction is greater than the one for the parallel flow by a factor of from six to eight; thus the presence of crosswind greatly increases the entrainment. The negatively buoyant vertical plume may also re-entrain some of the falling dense fluid so that the flux of negative buoyancy increases with distance, rather than being constant. This would have the effect of reducing rise time. Since correlation with specific gravity yields such a small power, the correlation was made with Froude Number. The least squares value of the proportionality constant was determined to be 2.96. So that (See Figure 1)

$$H/D_o = 2.96 \; F_R \qquad (11)$$

The value of the entrainment constant, α, was determined for the "top hat" region immediately downstream from the stack exit. Matching radial and height measurements from the smoke photographs to Equation (3) suggests that:

$$\alpha \approx 0.045.$$

When this value of α is used in Equation (1) to calculate plume rise one predicts:

$$H/D_o = 2.43 \; F_R$$

This relation somewhat underestimates rise height. Apparently, in the upper regions of the plume where velocities are low, the proportional entrainment theory over-predicts entrainment.

Plumes in a Laminar Crosswind. Bodurtha noted that over a given range in specific gravity the plume rise is independent of specific gravity. This was also found in the present experiments for plume rise in a laminar cross-wind. Rise height was obtained by measuring the maximum centerline plume height from the smoke pictures. Froude Number dependence was determined by comparing rise heights from 1/4 inch and 1/8 inch diameter stacks of equal specific gravity and velocity ratio. Velocity ratio dependence was determined by comparing rise heights for equal Froude Numbers and specific gravities but differing velocity ratios. The relationships appeared to follow those suggested by Equation (5). The constant of proportionality determined by least squares correlation is 1.32 so that (See Figure 2):

$$H/D_o = 1.32 R^{1/3} SG^{1/3} F_R^{\;2/3} \qquad (12)$$

The correlation with this equation is good except for the lowest Froude Numbers, where the rise is moderately less than predicted. Apparently the rise time is low enough that the net entrainment is rather low; hence the Bossinesq approximation is not valid over a sufficient range of the height.

Measurements also indicate that Equation (7) gives a good estimate for the horizontal position of the point of maximum rise.

For plumes such that the point where the lower edge of the plume touched the floor could be determined with reasonable certainty (diffusion of the smoke downstream made some plumes visually indeterminate at larger distances) the results indicate that strong correlation of touchdown distance with the "horizontal" Froude Number occurs for plumes of equal diameter, velocity ratio and rise height, but differing specific gravities. Linear proportionality was noted. Correlation with Equation (6) was noted but with a fair amount of scatter. The correlation of variables of Equation (6) is plotted in Figure 3. A least squares value of the constant of proportionality of 0.56, was found, so that:

$$\frac{X_D - \overline{X}}{D_o} = 0.56 \left\{ \left(\frac{H}{D_o}\right)^3 \left[\left(2 + \frac{h_s}{H}\right)^3 - 1 \right] \right\}^{1/2} \frac{F_{RH}}{\sqrt{R}} \tag{13}$$

Concentrations

The plumes injected into a laminar crosswind exhibited a somewhat skewed Gaussian distribution of concentration during the rising portion of the trajectory and a more symmetric distribution during the falling portion. The predictions of mixing length theory for maximum concentrations at the plume high point and "touchdown" are approximated with the determined constants of proportionality as follows:

$$\frac{\chi V D_o^2}{Q} = 2.15 (H/D_o)^{-1.85} \quad \text{at point of maximum rise} \tag{14}$$

$$\frac{\chi V D_o^2}{Q} = 3.10 \left(\frac{2H + h_s}{D_o}\right)^{-1.95} \quad \text{at a point of plume "touchdown"} \tag{15}$$

The plumes injected into the turbulent boundary layer exhibited an extremely large lateral spreading immediately after touchdown. The concentration profiles in the lateral direction were quite flat. The initial decay rate is approximately proportional to $X^{-0.65}$ (See Figure 4), which is similar to the behavior noted by Holly and Grace with salt water plumes in an open channel. The decay rate appears to approach ground source behavior as all plumes approach a -1.7 power law decay rate. (Values of velocity in the $\overline{\chi}UZ_o^2/Q$ are free stream velocity). Such behavior is expected to occur only in those plumes where as a result of high densities and/or low exit velocities and crosswinds, the plume touches down a short distance from the stack.

Dense Ground Source in a Turbulent Boundary Layer. Density differences were observed to have significant effect on the downstream diffusion pattern of a ground source. This effect was primarily multiplicative, however, other than a change in the power law of decay with downstream distance as is normally observed with an inversion stratification. Figures 5 and 6 show the rate of decay of maximum values of the quantity of $\overline{\chi}U/Q$ with downstream distances from the source. U in this case is taken as the velocity at the source centerline. In this case U was taken as 3.5 ft per second. Decay rates of maximum concentration with downstream distance for a specific gravity if one were observed to be proportional to power laws of -1.68 and -1.45 for the neutral and inversion cases respectively, which is in good agreement with previously observed values. The maximum concentrations of the denser gases decay at a slightly greater rate when the concentrations slowly approach those of air since the density effects are attenuated by diffusion. Over the range of downstream distances examined, a specific gravity of two, for example, increases maximum ground concentrations by a factor of approximately 30 percent in both neutral and inversion stratifications.

Conclusions

1. The rise height of a negatively buoyant plume is increased by increasing the discharge velocity. For a given flow rate, this can be accomplishd by decreasing the stack or relief valve diameter. For a constant flow rate, rise height is proportional to $D_o^{-3/2}$ for vertical plumes in a quiescent atmosphere and to $D_o^{-4/3}$ for plumes in a crosswind. The horizontal position of plume descent to the ground will also be increased by decreasing the stack diameter for a given flow rate and stack height. This horizontal distance is proportional to D_o^{-1}, and is of course increased with increased stack height being approximately proportional to $(2+h_s/H)^{3/2}$ for significant values of h_s/H.

2. For plumes of relatively high density exhausted into light winds, such that the density difference and resulting vertical motion dominates diffusion, the ground concentration will be approximately proportional to $(2H+h_s)^{-2}$. It is recommended that the least value of concentration as computed from vertical motion and atmospheric diffusion from Pasquill-Gifford Theory be employed at plume touchdown.

3. The effect of negative buoyancy on the behavior of ground source is primarily multiplicative as the decay relationship is not changed in form. Large specific gravities produce only moderate percentage increases in downstream concentration values rather than order of magnitude changes. Negative buoyance causes larger lateral and smaller vertical plume dimensions that are observed in cases of neutral buoyancy.

Acknowledgments

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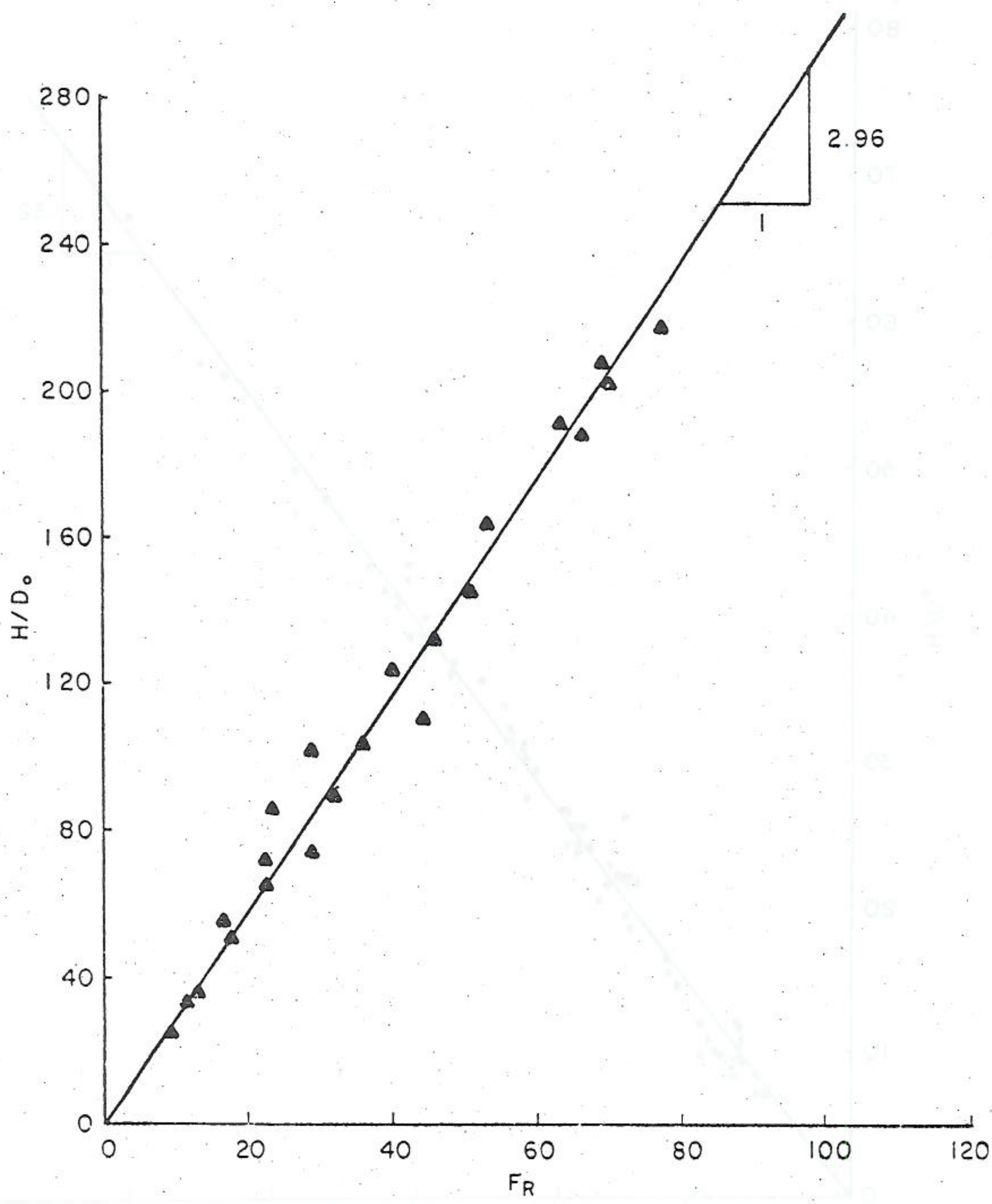

Figure 1. Dimensionless Rise Height vs. Froude Number-
Vertical Plumes

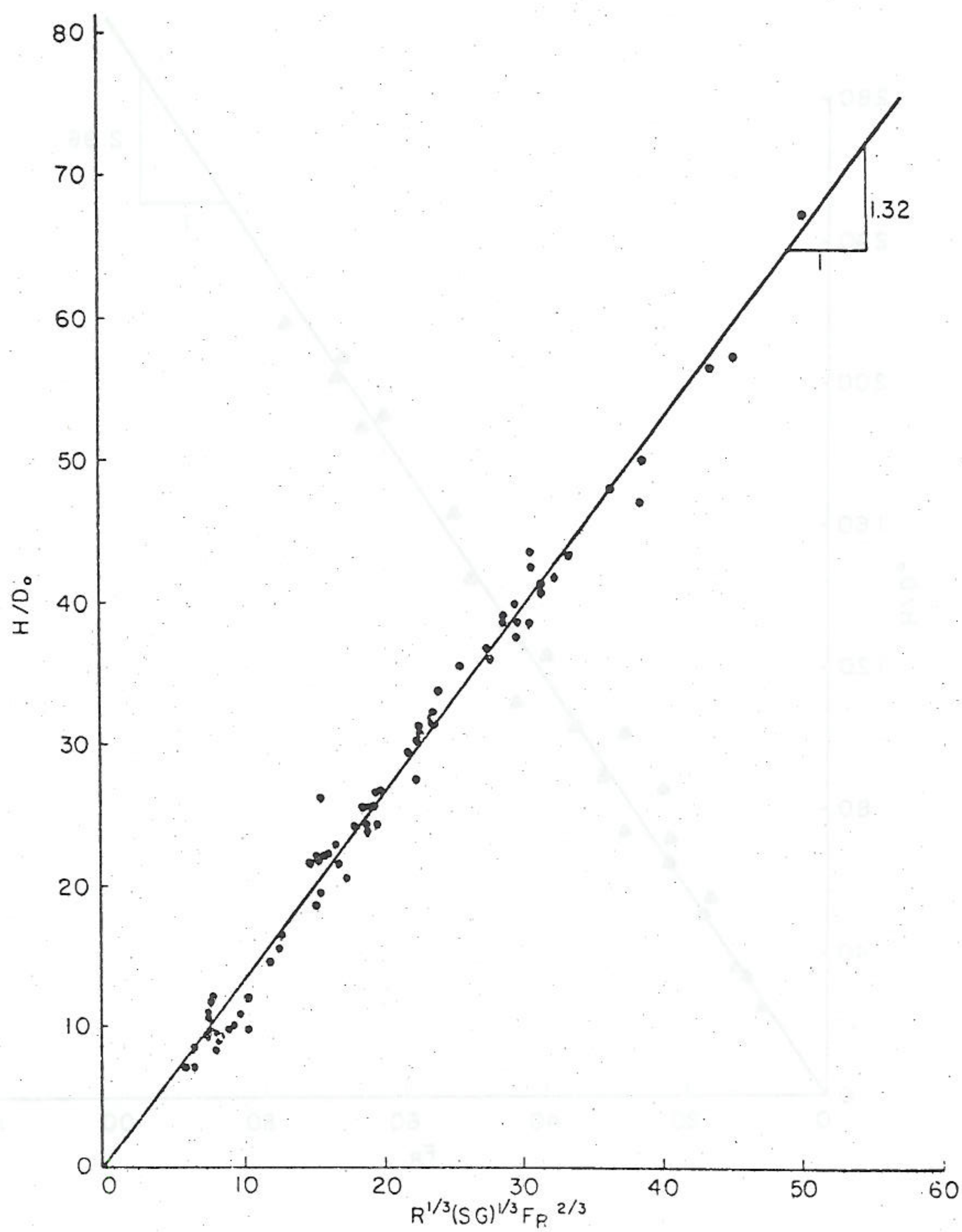

Figure 2. Dimensionless Rise Height vs. Rise Height Parameter

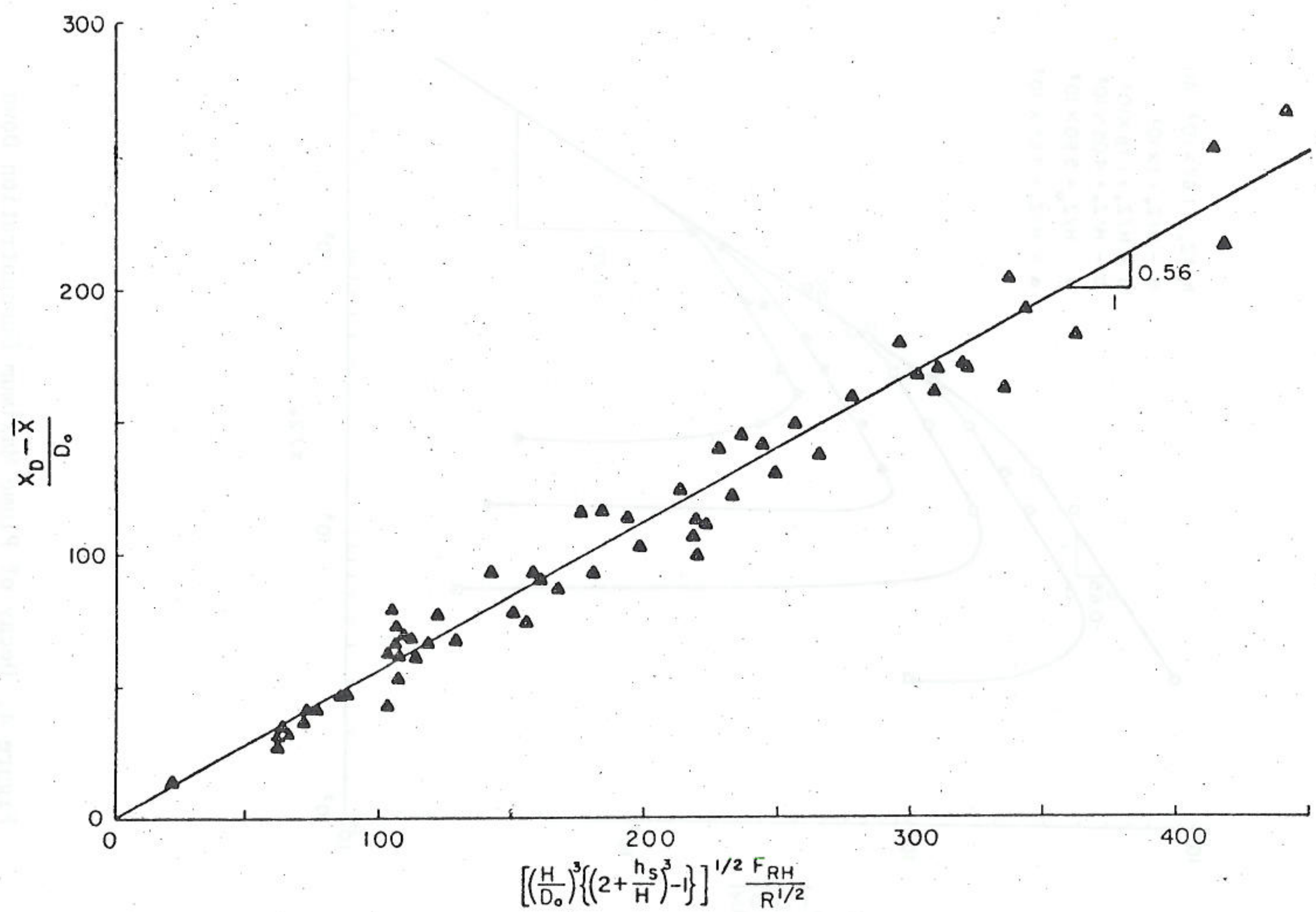

Figure 3. Dimensionless "Touchdown" Distance vs. "Touchdown Parameter"

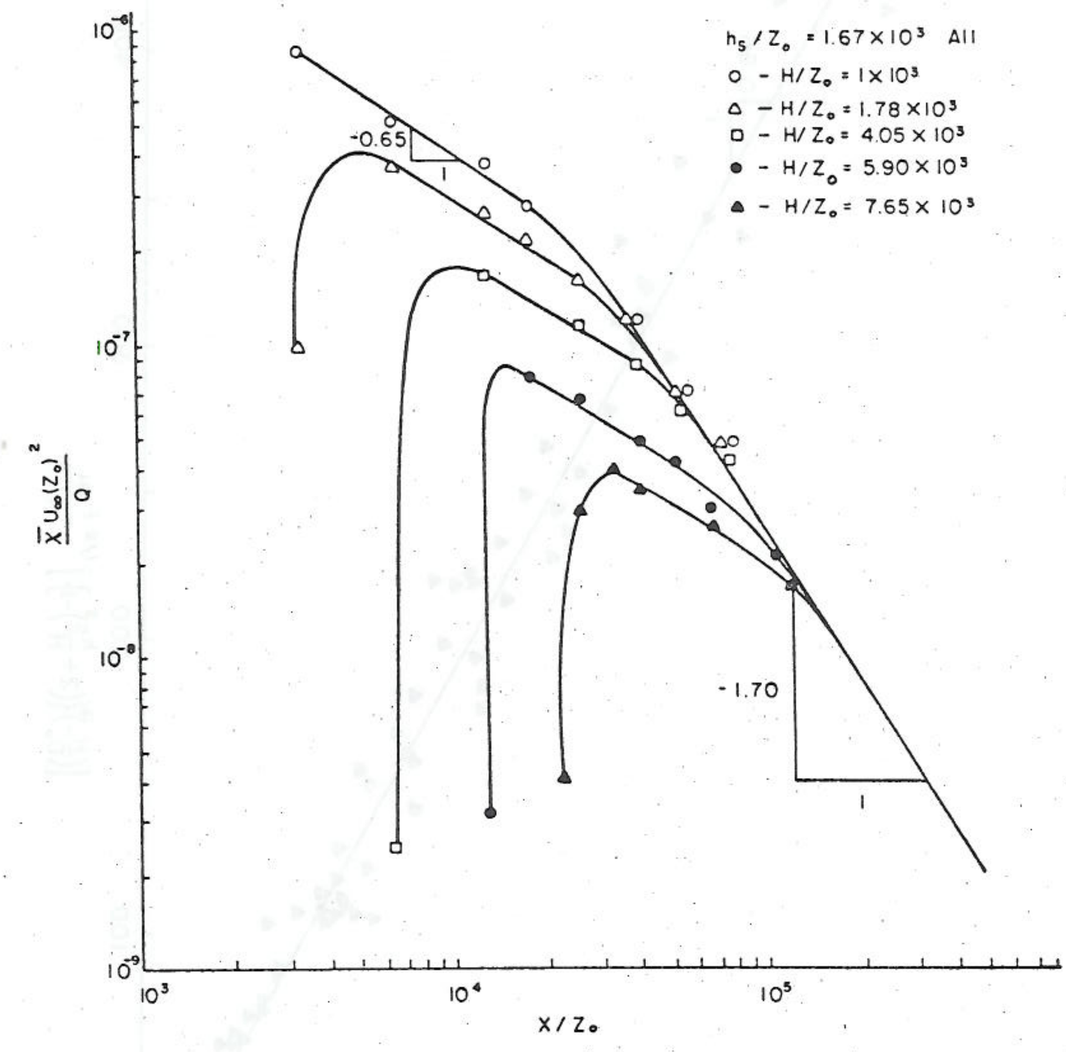

Figure 4. Decay of Plume Maximum Concentration Downstream from Touchdown Point

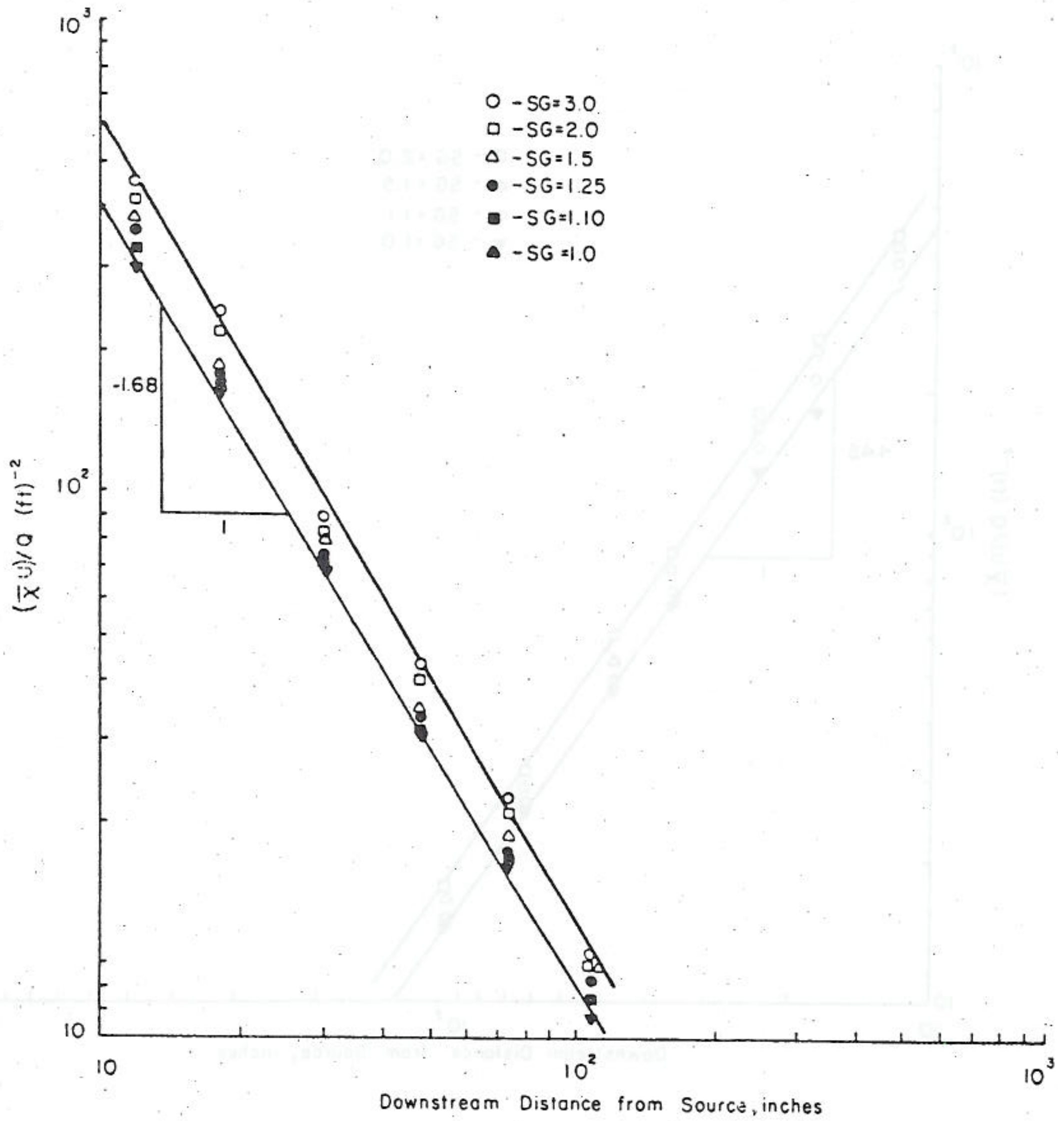

Figure 5. Maximum Concentrations vs. Downstream Distance, Negatively Buoyant Ground Source, Neutral Stratification.

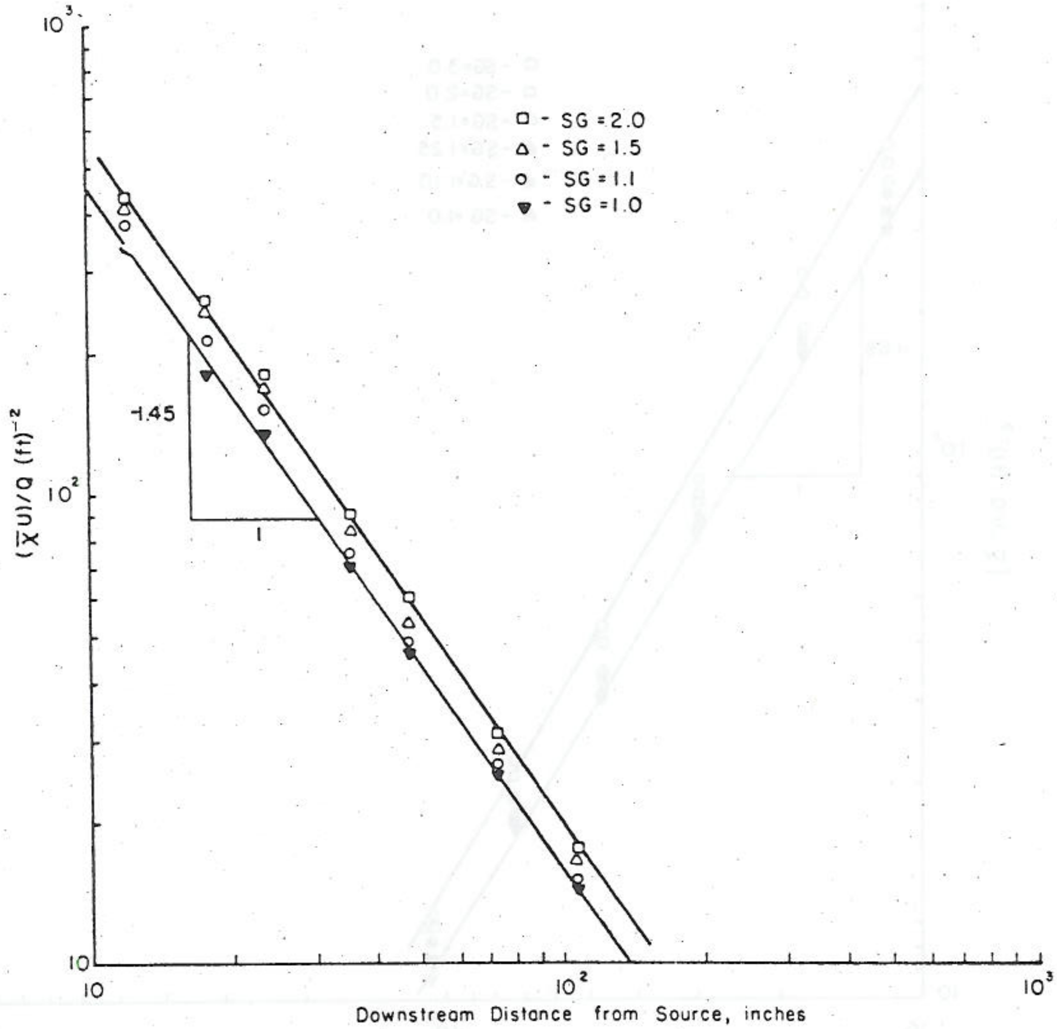

Figure 6. Maximum Concentrations vs. Downstream Distance, Negatively Buoyant Ground Source, Inversion Stratification

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