Relay Selection from a Battery Energy Efficiency Perspective

Wenshu Zhang, Dongliang Duan, and Liuqing Yang

***Abstract*—The battery nonlinearity has never been considered for energy analysis in relay networks. In this letter, we adopt the realistic nonlinear battery model, apply the battery energy consumption results in [1] to general relay networks, investigate the optimum and suboptimum energy allocation solutions for relaying transmission, and establish the relay selection criterion from the battery energy efficiency perspective. Our analyses and comparisons show that relaying does not always increase the system energy efficiency. We further establish closed-form conditions that can be easily checked to determine whether the relay transmission is preferable to the direct transmission. Numerical examples are also presented to verify these results.**

***Index Terms*—Relay selection, energy efficiency, wireless sensor networks.**

I. Introduction

BATTERY energy efficiency is a critical factor in wireless sensor networks, since sensor nodes are typically driven by nonrenewable batteries [2]. There are several works studying the approaches to improve battery energy efficiency for sensor networks (see e.g. [3], [4]). It is well known that when relaying is utilized to split the direct transmission from the source to destination into two or more hops, the total battery energy consumption is expected to be greatly reduced, since the transceiver distances are smaller than that of the direct link and the path loss is thus significantly reduced [5]. However, the analyses in [6] and [7] show that it is not always the case due to the extra transceiver circuit energy consumed by the relay nodes. This gives rise to an intriguing relay selection problem.

Although this problem was introduced in [6] and [7], it was only considered under some very limiting assumptions, such as a linear relay setup, identical and fixed transmit power at the source and relay nodes, and a linear battery model. In this letter, we will consider the relay selection problem without invoking those assumptions. Specifically, we consider the general relay networks where the relay can be arbitrarily located, we look for an optimum power allocation rather than the identical and fixed assignment, and particularly, we employ the realistic nonlinear battery model. Our study of the battery energy efficiency issue in relay networks is inspired by [1] where a closed-form nonlinear battery energy consumption formula is established. We will first apply the nonlinear battery consumption formula to the relay network, and then derive the optimum and suboptimum energy distribution for the relay link. Subject to the average bit error rate (BER) constraint, we establish the relay selection criterion which dictates under what condition the relaying transmission is preferable to the direct transmission. This condition, in terms of the critical distance of the direct link, can be expressed in closed form. Moreover, we show that the *Amplify and Forward* (AF) and *Decode and Forward* (DF) relaying protocols are equivalent for relay selection over Rayleigh fading channels at high SNR.

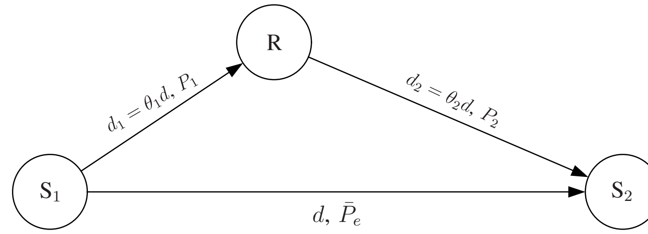

Fig. 1. General two-hop relay scenario.

II. System Model

Consider the general two-hop relay scenario with source node S_1, destination node S_2, and a relay node R, as shown in Fig. 1. We do not require the relay node R to stay on the straight line connecting the source and destination nodes as in [7]. Instead, we allow it to be anywhere in the space. There are two candidate links connecting S_1 and S_2, namely the *direct link* S_1–S_2 and the *two-hop relay link* S_1–R–S_2. Denote the distance of S_1–S_2 as d, the distance of S_1–R as $d_1 = \theta_1 d$, and R–S_2 as $d_2 = \theta_2 d$. Clearly, θ_1 and θ_2 should satisfy conditions $\theta_1, \theta_2 > 0$ and $\theta_1 + \theta_2 \geq 1$. The linear relay setup in [7] can be regarded as a special case of our setup with $\theta_1 + \theta_2 = 1$.

We adopt BER as the performance metric, which requires the two links, S_1–S_2 and S_1–R–S_2, to achieve the same target average BER $\bar{P}_e$. The channel for each link is modeled as Rayleigh fading with Kth-power path loss. With BPSK modulation, the average BER for the Rayleigh fading channel with coherent detection can be approximated at high SNR as [9, Ch.3]

$$P_e = \frac{N_0}{4\mathcal{E}_r} \tag{1}$$

where $\mathcal{E}_r$ is the average received bit energy and N_0 is the single-sided power spectrum density (PSD) of the additive white Gaussian noise (AWGN).[1] The bit energy at the transmitter $\mathcal{E}_t$ and the bit energy at the receiver $\mathcal{E}_r$ follow the

[1] All analyses in this letter can be readily extended to other modulations by modifying (1) accordingly.

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relationship:

$$\mathcal{E}_t/\mathcal{E}_r = M_l G_1 d^K \tag{2}$$

where M_l is the link margin, G_1 is the gain factor at $d=1$, and $K \geq 2$ is the pathloss exponent.

III. Total Battery Energy Consumption

In this section, we apply the nonlinear battery model derived in [1] for direct transmission to relay networks, and investigate the optimum and suboptimum energy allocation schemes for the relaying transmission.

A. Total Battery Energy Consumption of Direct Transmission

In [1], a realistic nonlinear model is adopted to describe the battery discharge process. It is shown that the total battery energy consumption formula can be expressed as a quadratic function of the receive pulse energy $\mathcal{E}_{pr}$ for the direct transmission [1, Theorem 1]:

$$\begin{aligned}\mathcal{E} &= \frac{M_l^2 G_1^2 \omega \gamma_p (1+\alpha)^2}{V\eta^2}\mathcal{E}_{pr}^2 d^{2K} + \frac{M_l G_1 (1+\alpha)}{\eta}\mathcal{E}_{pr} d^K \\ &\quad + \frac{\mathcal{P}_{ct}T_P + \mathcal{P}_{cr}T_d}{\eta} \\ &\triangleq C_2 \mathcal{E}_{pr}^2 d^{2K} + C_1 \mathcal{E}_{pr} d^K + C_0 \end{aligned} \tag{3}$$

where $\gamma_p \triangleq \int_0^{T_p} (p_0(t))^2 dt$ is a parameter determined by the normalized transmit pulse shape $p_0(t) = p(t)/\int_0^{T_p} |p(t)|dt$, ω is the battery efficiency factor, V is the battery voltage, η is the transfer efficiency of the DC/DC converter, α is the extra power loss factor of the power amplifier (PA), $\mathcal{P}_{ct}$ and $\mathcal{P}_{cr}$ are the transmitter and receiver circuit power consumption respectively, T_p is the pulse duration and T_d is the pulse duty-time.

As detailed in [1], the total battery energy consumption $\mathcal{E}$ consists of three parts corresponding to the three respective terms in (3): the excess energy loss due to the nonlinear battery discharge, the energy carried by the transmitted signal, and the transceiver circuit energy consumption. It is not surprising that the first two terms both depend on the transmission distance d, whereas the third term is a constant independent of d.

Considering BPSK modulation over the Rayleigh fading channel, we substitute (1) into (3). Then the total battery energy consumption $\mathcal{E}_{\mathrm{d}}$ of the direct transmission S_1–S_2 over Rayleigh fading channel can be expressed as an explicit function of d and $\bar{P}_e$:

$$\mathcal{E}_{\mathrm{d}} = L_2 \frac{d^{2K}}{\bar{P}_e^2} + L_1 \frac{d^K}{\bar{P}_e} + L_0 \tag{4}$$

where $L_2 \triangleq C_2 N_0^2/16$, $L_1 \triangleq C_1 N_0/4$, and $L_0 \triangleq C_0$. Notice that $\mathcal{E}_{\mathrm{d}}$ can be regarded either as a quadratic function of $\bar{P}_e^{-1}$ for a given S_1-S_2 distance d, or as a quadratic function of d^K with some desired $\bar{P}_e$.

B. Total Battery Energy Consumption of Relaying Transmission

When the relay link is deployed, either the DF or AF relaying protocol can be employed at the relay node R. For simplicity, we first consider the DF relaying protocol. The AF and DF equivalence will be established in Section V.

Using the DF protocol, the relay node R first demodulates the signal from the source and then forwards the remodulated signal to the destination. Therefore, one can essentially separate the relay link S_1–R–S_2 into two decoupled links S_1–R_D and R_F–S_2, each having received pulse energy $\mathcal{E}_{pri}$ and transmission distance $d_i = \theta_i d$, for $i = 1, 2$. As a result, the total battery energy consumption for the S_1–R–S_2 link can be simply expressed as the superposition of the energy consumption of the two decoupled links [c.f. (3)]:

$$\begin{aligned}\mathcal{E}_{\mathrm{r}}(\mathcal{E}_{pr1}, \mathcal{E}_{pr2}) = C_2 \left(d_1^{2K}\mathcal{E}_{pr1}^2 + d_2^{2K}\mathcal{E}_{pr2}^2\right) \\ + C_1 \left(d_1^K \mathcal{E}_{pr1} + d_2^K \mathcal{E}_{pr2}\right) + 2C_0 \,. \end{aligned} \tag{5}$$

Equation (5) leads to two important implications: i) Although utilization of the relay node can reduce the pathloss effect, it will double the distance-independent circuit energy consumption ($2C_0$ in (5)), in comparison with the direct link case (C_0 in (3)); and ii) For a given relay location specified by d_1 and d_2, $\mathcal{E}_{\mathrm{r}}(\mathcal{E}_{pr1}, \mathcal{E}_{pr2})$ is a quadratic function of two variables $\mathcal{E}_{pr1}$ and $\mathcal{E}_{pr2}$. For a fair comparison between the battery energy efficiency of the direct and relaying transmission, the energy consumption levels $\mathcal{E}_{pr1}$ and $\mathcal{E}_{pr2}$ should be optimized to minimize the overall energy of the relay link.

Due to the inversely proportional relationship between the received pulse energy and the average BER given in (1), the energy distribution among $\mathcal{E}_{pr1}$ and $\mathcal{E}_{pr2}$ are exclusively determined by the average BER of the two decoupled links P_1 and P_2. The overall average BER for the relay link can also be upper bounded using P_1 and P_2 as $\bar{P}_e \leq 1-(1-P_1)(1-P_2) = P_1 + P_2 - P_1P_2$. In practice, the desired BER is usually small ($\bar{P}_e \leq 10^{-3}$), which means $P_1 + P_2 \gg P_1P_2$ and therefore $\bar{P}_e \approx P_1 + P_2$. The latter will be considered as the error performance constraint for the relaying transmission in the sequel, and the total battery energy consumption in (5) can be rewritten in terms of P_1 as

$$\begin{aligned}\mathcal{E}_{\mathrm{r}}(P_1) &= L_2 \left(\frac{d_1^{2K}}{P_1^2} + \frac{d_2^{2K}}{(\bar{P}_e - P_1)^2}\right) \\ &\quad + L_1 \left(\frac{d_1^K}{P_1} + \frac{d_2^K}{\bar{P}_e - P_1}\right) + 2L_0 \end{aligned} \tag{6}$$

with constraint $0 < P_1 < \bar{P}_e$. Now we are ready to formulate the equivalent energy optimization problem: Given the relay location d, θ_1 and θ_2, determine the optimum P_1 which minimizes the total battery energy consumption $\mathcal{E}_{\mathrm{r}}(P_1)$; that is,

$$\mathcal{E}_{\mathrm{r}} = \min_{P_1} \; \mathcal{E}_{\mathrm{r}}(P_1), \text{ subject to } 0 < P_1 < \bar{P}_e \tag{7}$$

where $\mathcal{E}_{\mathrm{r}}(P_1)$ is given by (6).

Optimum Energy Allocation: Since $\mathcal{E}_r(P_1)$ is a convex function, it is straightforward to solve the minimization problem in (7). Taking first-order derivative of $\mathcal{E}_{\mathrm{r}}(P_1)$ with respect to P_1 and setting it to zero, we obtain:

$$\begin{aligned}\frac{\partial \mathcal{E}_{\mathrm{r}}(P_1)}{\partial P_1} &= -2L_2 d_1^{2K} P_1^{-3} + 2L_2 d_2^{2K} (\bar{P}_e - P_1)^{-3} \\ &\quad - L_1 d_1^K P_1^{-2} + L_1 d_2^K (\bar{P}_e - P_1)^{-2} \\ &= 0 \end{aligned} \tag{8}$$

which can be reorganized into a fourth-order (quartic) equation of variable P_1 by multiplying $P_1^3(P_e - P_1)^3$ to both sides.

The root $P_1^{\rm o}$ of the quartic equation satisfying the constraint $0 < P_1^{\rm o} < \bar{P}_e$ will give rise to the minimum $\mathcal{E}_{\rm r}(P_1)$, i.e., the minimum total relay energy consumption $\mathcal{E}_{\rm r}$. The corresponding optimum energy allocation will be $\mathcal{E}_{pr1} = N_0/\left(4P_1^{\rm o}\right)$ and $\mathcal{E}_{pr2} = N_0/\left(4(\bar{P}_e - P_1^{\rm o})\right)$.

Although the quartic equation is exactly solvable [10, Ch.1], its four root expressions are too complicated for further analyses. The computational complexity motivates us to look for a suboptimum method to simplify the energy allocation process.

Suboptimum Energy Allocation: It turns out that the first-order term in the battery energy consumption formula in (3) and the corresponding first-order term in the total battery energy consumption $\mathcal{E}_{\rm r}(P_1)$ formula for the relay link in (6) dominate the total energy consumption within the BER range of $0 < P_1 < \bar{P}_e$. Therefore, those second-order terms can be temporarily discarded when finding the best P_1 with negligible effect on the energy allocation optimality.

After removal of the second-order term, the first-order total battery energy consumption becomes

$$\mathcal{E}_{\rm r}^1(P_1) = L_1\left(\frac{d_1^K}{P_1} + \frac{d_2^K}{\bar{P}_e - P_1}\right) + 2L_0\,. \tag{9}$$

Likewise, we take the derivative of $\mathcal{E}_{\rm r}^1(P_1)$ with respect to P_1 and set it to zero to obtain:

$$\left(d_2^K - d_1^K\right)P_1^2 + 2d_1^K\bar{P}_eP_1 - d_1^K\bar{P}_e^2 = 0 \tag{10}$$

which is a quadratic equation in terms of P_1. Note that the root of this equation only depends on the distance ratio $d_2/d_1 = \theta_2/\theta_1$ and the overall target BER $\bar{P}_e$. It has nothing to do with the battery consumption formula coefficients L_1 and L_0. The quadratic equation has well-known simple root expressions. Within the range of $0 < P_1 < \bar{P}_e$, (10) always has a single positive root, and we can obtain the suboptimal solution

$$\begin{aligned} P_1^{\rm so} &= \frac{1}{(\theta_2/\theta_1)^{\frac{K}{2}} + 1}\bar{P}_e\,, \\ \text{and}\quad P_2^{\rm so} &= \bar{P}_e - P_1^{\rm so} = \frac{(\theta_2/\theta_1)^{\frac{K}{2}}}{(\theta_2/\theta_1)^{\frac{K}{2}} + 1}\bar{P}_e\,. \end{aligned}$$

Substituting $P_1^{\rm so}$ back into (6), one can obtain the suboptimum $\mathcal{E}_{\rm r}$ for the relay link S_1–R–S_2:

$$\begin{aligned} \mathcal{E}_{\rm r} &= \frac{L_2}{\bar{P}_e^2}\left(\theta_1^{\frac{K}{2}} + \theta_2^{\frac{K}{2}}\right)^2\left(\theta_1^K + \theta_2^K\right)d^{2K} \\ &\quad + \frac{L_1}{\bar{P}_e}\left(\theta_1^{\frac{K}{2}} + \theta_2^{\frac{K}{2}}\right)^2 d^K + 2L_0\,. \end{aligned} \tag{11}$$

In Fig. 2, both the optimum and suboptimum allocation solutions $P_1^{\rm o}$ and $P_1^{\rm so}$ are plotted with respect to the relay location d_1 with fixed $d_2 = 200$m. The two curves nearly coincide with each other, validating that the second-order terms have negligible effect on the optimum power allocation.

With the optimum relaying transmission for S_1–R–S_2, we will next address this question: Whether or not, and under what conditions, should one choose the relay link S_1–R–S_2 rather than the direct link S_1–S_2 in order to achieve higher energy efficiency.

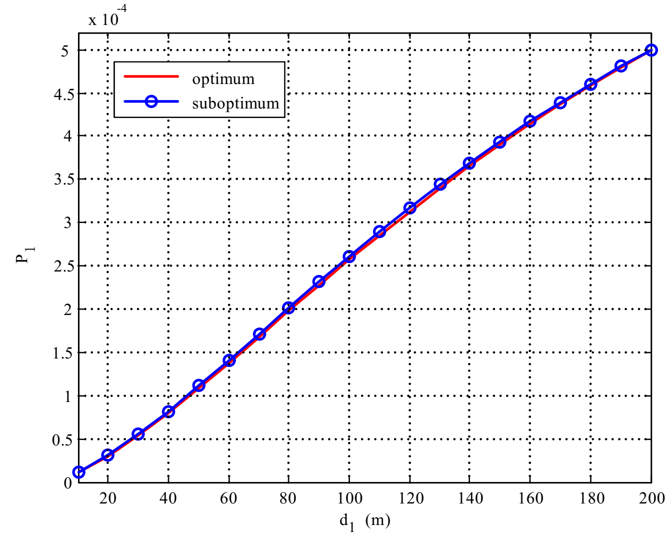

Fig. 2. Optimum and suboptimum allocation comparison.

IV. Relay Selection Criterion

To compare the direct and relaying transmissions in terms of the battery energy efficiency, we will evaluate the difference of the total battery energy consumptions, denoted as $\Delta\mathcal{E}$, between the direct and the optimal relaying transmissions. The sign of the difference will reveal the more efficient one: direct link outperforms the relay link if the difference is negative, and vice versa.

$\Delta\mathcal{E}$ can be readily obtained by taking the difference of (4) and (11):

$$\begin{aligned} \Delta\mathcal{E} &= \mathcal{E}_{\rm d} - \mathcal{E}_{\rm r} \\ &= \frac{L_2}{\bar{P}_e^2}\left(1-\left(\theta_1^{\frac{K}{2}}+\theta_2^{\frac{K}{2}}\right)^2\left(\theta_1^K+\theta_2^K\right)\right)\left(d^K\right)^2 \\ &\quad + \frac{L_1}{\bar{P}_e}\left(1-\left(\theta_1^{\frac{K}{2}}+\theta_2^{\frac{K}{2}}\right)^2\right)d^K - L_0 \end{aligned} \tag{12}$$

as a quadratic function of d^K. The constant term is negative, which is resulted from the fact that the relaying transmission consumes more distance-independent transceiver circuit energy than the direct transmission. Based on the sign of $\Delta\mathcal{E}$, we establish the relay selection criterion as follows.

Proposition 1 *In a general two-hop BPSK modulated relay network with a single DF relay node located at distance* $d_1 = \theta_1 d$ *to the source and* $d_2 = \theta_2 d$ *to the destination, where* d *is the source-destination distance,* $\theta_1, \theta_2 > 0$ *and* $\theta_1 + \theta_2 \geq 1$, *the direct versus relay link selection subject to the average BER* $\bar{P}_e$ *constraint should obey the following rules:*

R1. If $\left(\theta_1^{K/2} + \theta_2^{K/2}\right)^2\left(\theta_1^K + \theta_2^K\right) \geq 1$, *then the direct transmission is always more energy efficient than the relaying transmission;*

R2. If $\left(\theta_1^{K/2} + \theta_2^{K/2}\right)^2\left(\theta_1^K + \theta_2^K\right) < 1$, *then the direct link is preferable as long as* d *is less than the critical distance* d_c, *and vice versa, with* $d_c^K = (-B + \sqrt{B^2 - 4AC})/(2A)$, *where* $A \triangleq \left(1 - \left(\theta_1^{K/2} + \theta_2^{K/2}\right)^2\left(\theta_1^K + \theta_2^K\right)\right)L_2/\bar{P}_e^2$, $B \triangleq \left(1 - \left(\theta_1^{K/2} + \theta_2^{K/2}\right)^2\right)L_1/\bar{P}_e$ *and* $C \triangleq -L_0$ *are the coefficients of the quadratic function* $\Delta\mathcal{E}$ *in* (12).

Proof: The sign of $\Delta\mathcal{E}$ can be determined by checking the roots of the quadratic equation $\Delta\mathcal{E} = 0$. Since $C \triangleq -L_0 < 0$, the quadratic curve always crosses y-axis in the negative half.

R1. The first-order derivative evaluated at $d^K = 0$ is $(\partial\Delta\mathcal{E}/\partial d^K)|_{d^K=0} = (L_1/\bar{P}_e)\left(1-\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2\right)$. This must be negative, or $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2 > 1$, under the condition $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2\left(\theta_1^K+\theta_2^K\right) \geq 1$. One can validate the statement by contradiction:

Suppose that $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2 \leq 1$, then $\theta_1^K + \theta_2^K + 2\theta_1^{K/2}\theta_2^{K/2} \leq 1$. Since $\theta_1, \theta_2 > 0$ and correspondingly $\theta_1^{K/2}, \theta_2^{K/2} > 0$, we will have $\theta_1^K + \theta_2^K < 1$, giving rise to $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2\left(\theta_1^K+\theta_2^K\right) < 1$, which is contradictory to the condition $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2\left(\theta_1^K+\theta_2^K\right) \geq 1$. Therefore, it has to be $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2 > 1$.

Negative first-order derivative at $d^K = 0$, together with the condition $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2\left(\theta_1^K+\theta_2^K\right) \geq 1$ (i.e., $A \leq 0$) and $C < 0$, guarantee $\Delta\mathcal{E} < 0$, $\forall\ d^K > 0$. This means that the direct link is always energy efficient than the relay link.

R2. $\left(\theta_1^{K/2}+\theta_2^{K/2}\right)^2\left(\theta_1^K+\theta_2^K\right) < 1$ and $C < 0$ ensure that the quadratic equation $\Delta\mathcal{E} = 0$ has a single positive root, which is given by $d_c^K = (-B + \sqrt{B^2 - 4AC})/(2A)$. When the distance $d < d_c$, $\Delta\mathcal{E} < 0$ implies that the direct transmission is more energy efficient than the relaying transmission; otherwise, when $d > d_c$ the relay link is preferable. ■

Remarks: Proposition 1 provides the rules for determining whether the relay link should be used for the purpose of energy conservation. The rules are based on the realistic nonlinear battery model and are applicable to the general two-hop relay network topology. Intuitively, the relaying transmission will consume more energy than the direct link in R1 when the relay node is located so far away that the S_1–R_D and/or R_F–S_2 distance is even larger than the direct distance, or in R2 when the direct transmission is at a short range so that the transceiver circuit energy dominates the total battery energy consumption.

V. AF and DF Equivalence in Our Scenario

In preceding sections, we considered the DF protocol. For the AF protocol, the network setup in Fig. 1 is still valid, except that the relay amplifies what it receives from the source, including the noise, and forwards to the destination. With two links S_1–R and R–S_2, the AF relaying transmission follows the same total battery energy consumption formula as the DF case in (5).

Denote the SNR for the two links as $\rho_1 = \mathcal{E}_{pr1}/N_0$ and $\rho_2 = \mathcal{E}_{pr2}/N_0$. Using the AF protocol, the received signal at the destination essentially involves two parts: useful signal $\mathcal{E}_{pr2}\cdot\rho_1/(\rho_1+1)$ and noise $\mathcal{E}_{pr2}/(\rho_1+1)$ magnified at the relay node. Therefore, the overall SNR for the relay link S_1–R–S_2 is $\rho = (\mathcal{E}_{pr2}\cdot\rho_1/(\rho_1+1))/(N_0+\mathcal{E}_{pr2}/(\rho_1+1))$. Taking the reciprocal of ρ, we get $1/\rho = (1/\rho_2)\cdot((\rho_1+1)/\rho_1)+1/\rho_1 \approx 1/\rho_1 + 1/\rho_2$, where the approximation comes from the fact that the practical desired $\bar{P}_e \leq 10^{-3}$ is small and thus we only consider the medium to high SNR $\rho_1 \gg 1$. Again, due to the relationship in (1), we reach the following average BER constraint for the AF relaying transmission:

$$\bar{P}_e = P_1 + P_2 \tag{13}$$

which is identical to that in the DF case. Since the AF relaying transmission shares the same battery energy consumption formula (5) and BER performance constraint (13) with the DF transmission, the optimum energy allocation analyses and the relay selection criterion for DF also hold for AF. In other words, the two relaying protocols are equivalent in terms of the relay selection over Rayleigh fading channels at high SNR.

TABLE I
Simulation Parameters

$\mathcal{P}_{cr} = 52.5$mW	$V = 3.7$A	$G_1 = 27$dB
$\mathcal{P}_{ct} = 105.8$mW	$\alpha = 0.33$	$M_l = 40$dB
$N_0/2 = -171$dBm/Hz	$\omega = 0.05$	$K = 3$
$T_p = 1.33 \times 10^{-4}$s	$\eta = 0.8$	$p_0(t) = 1/T_p$

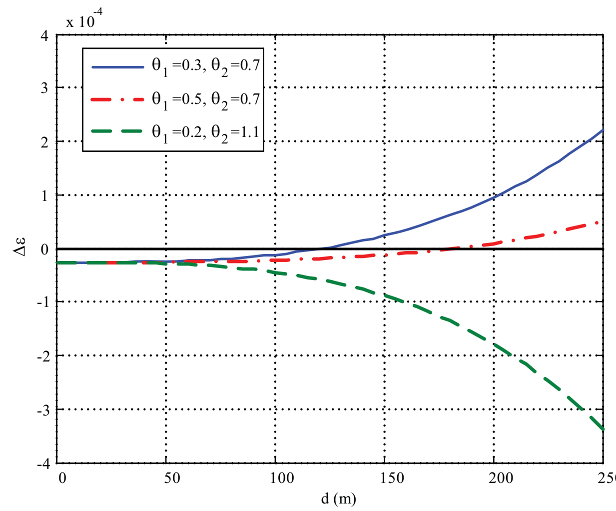

Fig. 3. Battery energy consumption difference between direct and relaying transmissions for various relay locations.

VI. Numerical Results

The system parameters for our numerical examples are listed in Table I. First, the energy difference $\Delta\mathcal{E}$ between the direct and relay links is plotted in Fig. 3 for various θ_1 and θ_2 pairs. The first two pairs, $\theta_1 = 0.3$ and $\theta_2 = 0.7$, $\theta_1 = 0.5$ and $\theta_2 = 0.7$, yield the increasing difference curves along with the distance d, and the curves cross the positive-half d-axis. This phenomenon confirms the existence of specific critical transmission distances for these relay setups as stated in R2 of Proposition 1. In contrary, one will never take advantage from the third pair $\theta_1 = 0.2$ and $\theta_2 = 1.1$ because the battery energy consumption difference is strictly negative, corresponding to R1 in Proposition 1.

Fig. 4 and Fig. 5 depict the critical distance d_c versus θ_1 while keeping the sum of θ_1 and θ_2 a constant. Those θ_1 values, which do not have d_c result, are always in favor of the

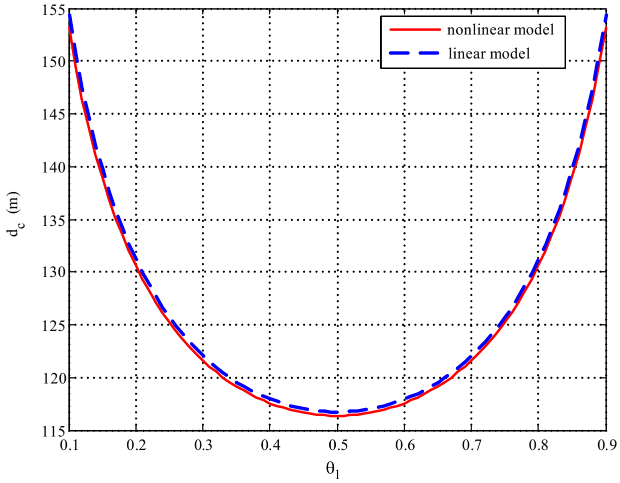

Fig. 4. Critical distances obtained from the nonlinear and linear models, $\theta_1 + \theta_2 = 1$.

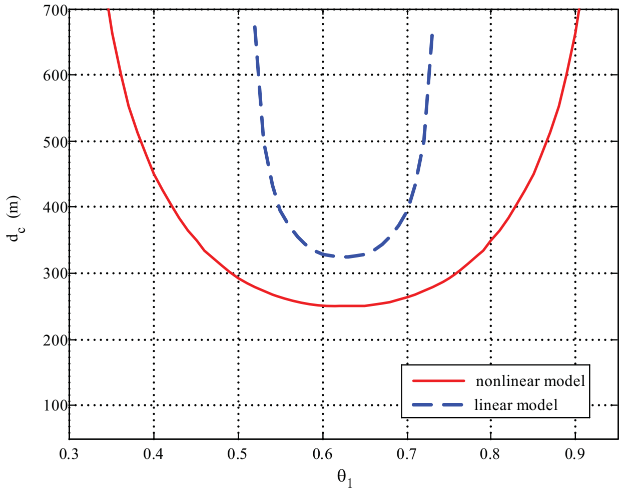

Fig. 5. Critical distances obtained from the nonlinear and linear models, $\theta_1 + \theta_2 = 1.25$.

direct transmission. The critical transmission distance curves obtained from the linear battery consumption model are also provided for comparison. Based on the linear battery model, the critical distance is simply $d_c = -C/B$ where B and C are defined in Proposition 1. In Fig. 4, $\theta_1 + \theta_2 = 1$ indicates the linear relay setup. We observe that the two curves obtained from the nonlinear and linear battery model agree well for the linear relay setup. However, Fig. 5, where $\theta_1 + \theta_2 = 1.25$, shows a different picture. Not only the decision range of θ_1 about whether to use the relaying transmission but also the critical distances are markedly distinct for the two battery models. As a result, for the relay topology which is not on the straight line, one cannot characterize its energy consumption accurately using the linear battery model. The nonlinear model provides a more accurate relay selection criterion to improve the battery energy efficiency.

VII. Conclusions

In a general two-hop relay network, we adopted the realistic nonlinear battery model, investigated the optimum and sub-optimum relaying transmission, developed the relay selection criterion from a battery energy efficiency perspective, and established the equivalence between the AF and DF protocols over Rayleigh fading channels at high SNR. Theoretical derivations and numerical results show that, due to the extra transceiver circuit consumption, relaying does not necessarily increase the system energy efficiency. We also provided the explicit conditions under which relaying is beneficial.

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