Control Co-design Direct Transcription
Solution Strategies: Overview and Challenges

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Outline

1. Control Co-design

2. Direct Transcription
Control Co-design as a Dynamic Optimization Problem

One way to represent a control co-design (CCD) problem is in the time domain using a dynamic optimization (DO) formulation:

\[
\begin{align*}
\min_{x_c, x_p} & \quad \Psi(x_c, x_p) = \int_{t_0}^{t_f} \mathcal{L}(t, \xi, x_c, x_p) \, dt + \mathcal{M}(\xi(t_0), \xi(t_f), x_c, x_p) \\
\text{subject to:} & \quad \dot{\xi} = f(t, \xi, x_c, x_p) \\
& \quad C(t, \xi, x_c, x_p) \leq 0 \\
& \quad \phi(\xi(t_0), \xi(t_f), x_c, x_p) \leq 0
\end{align*}
\]

- \( t \in [t_0, t_f] \): time defined in the time horizon between \( t_0 \) and \( t_f \)
- \( \xi(t) \): states
- \( x_c \): control design variables
- \( x_p \): plant design variables

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1 Herber and Allison 2018  
2 Note that for simplicity of presentation, this is a fixed-horizon, single-phase problem
Control Co-design as a DO Problem (continued)

One way to represent a control co-design (CCD) problem is in the time\(^1\) domain using a dynamic optimization (DO) formulation\(^2\):

\[
\begin{align*}
\min_{x_c, x_p} & \quad \Psi(x_c, x_p) = \int_{t_0}^{t_f} \mathcal{L}(t, \xi, x_c, x_p) \, dt + \mathcal{M}(\xi(t_0), \xi(t_f), x_c, x_p) \\
\text{subject to:} & \quad \dot{\xi} = f(t, \xi, x_c, x_p) \\
& \quad C(t, \xi, x_c, x_p) \leq 0 \\
& \quad \phi(\xi(t_0), \xi(t_f), x_c, x_p) \leq 0
\end{align*}
\]

- \(\mathcal{L}(t, \xi, x_c, x_p)\): Lagrange or running cost term (\textit{time dependent})
- \(\mathcal{M}(\xi(t_0), \xi(t_f), x_c, x_p)\): Mayer or terminal cost term
- \(f(t, \xi, x_c, x_p)\): state derivative function (\textit{time dependent})
- \(C(t, \xi, x_c, x_p)\): path constraints (\textit{time dependent})
- \(\phi(\xi(t_0), \xi(t_f), x_c, x_p)\): boundary constraints

\(^1\) Herber and Allison 2018  \(^2\) Note that for simplicity of presentation, this is a fixed-horizon, single-phase problem
Basic CCD Solution Strategies

- **Sequential design:** Optimize $x_p$ → Optimize $x_c$
  
  *Figure: Sequential design.*

- **Simultaneous design:** Optimize $x_p$ and $x_c$
  
  *Figure: Simultaneous design.*

- **Iterated sequential design:** Optimize $x_p$ → Optimize $x_c$
  
  *Figure: Iterated sequential design.*

- **Nested design:**
  
  *Figure: Nested design.*
Optimal Open-Loop Control in CCD

- There often is a choice in control design variables whether it be the gains in a particular control architecture or open-loop trajectories ($u$).
- Many recent CCD studies have utilized optimal open-loop control (OOLC) in early-stage design.
- Closed-loop control (CLC) design requires specification of control structure (e.g., state/output feedback) that may implicitly limit performance or the ability to satisfy system constraints.
  - But there are also certain advantages.
- With OOLC, optimal control trajectories are sought without assuming a control architecture.
- CDD using OOLC results in physical systems with natural dynamics that interact with an active control system in a way that yields maximal system performance.
- Can provide important insights at early design stages.

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1 Allison, Guo, and Han 2014
2 Including the cases where the particular problem has a known feedback structure that is equivalent to the OOLC
3 Deshmukh, Herber, and Allison 2015
Some Limitations Found in Earlier CCD Research

- Some studies investigated the specific case when separate plant and control objectives were well defined\(^1\):

\[ \Psi(x_c, x_p) = w_p \Psi_p(x_p) + w_c \Psi_c(t, \xi, x_c, x_p) \]

- Some studies used the assumption of unidirectional coupling where a plant design objective and constraints did not depend on \(x_c\)\(^2\)
  - Realistic treatment of plant design requires the inclusion of constraints that contain both \(x_p\) and \(\xi\) such as fatigue
  - There may not exist a feasible control/state solution for a fixed plant design with bidirectional coupling
  - This is design coupling between the physical-system and control-system
- Many early approaches for solving time-domain CCD problems had other potentially restrictive assumptions
  - For example, infinite-horizon, linear dynamics, and no path constraints so there is a linear–quadratic regulator (LQR) subproblem in nested CCD\(^3\)
- Frequency domain approaches can address some challenges but not readily nonlinear dynamics and path constraints

\(^1\) Peters, Papalambros, and Ulsoy 2009; Peters, Papalambros, and Ulsoy 2013; Fathy et al. 2001
\(^2\) Peters, Papalambros, and Ulsoy 2013; Allison, Guo, and Han 2014
\(^3\) Herber and Allison 2018; Fathy et al. 2001
Some Needs in a General CCD Solution Strategy

1. Inequality constraints
   • Many realistic CCD problems have inequality constraints to represent different failure modes such as stress or fatigue or even simple bounds on states and controls\(^1\)

2. Bidirectional coupling

3. Comprehensive plant design representations including independent design variables and nonlinear dynamics

4. Identification of optimal dynamic and control behaviors
   • The desirable control architecture might be unknown in early-stage design (so support OOLC)

5. Computationally efficient and robust

Direct transcription (DT) methods have been shown to be effective at addressing these needs

\(^1\) Allison and Herber 2014; Allison, Guo, and Han 2014; Herber and Allison 2018
Direct Transcription Overview

- In DT, the time horizon is discretized into a number of segments.
- The values of the states $\xi$ and controls $u$ at the boundaries of these segments (discrete time points) are included directly as optimization variables.
  - Discretization of the time-varying quantities.
- The dynamic constraints are included as a set of equality constraints (known as defect constraints).
  - Many potential methods such as the basic trapezoidal rule, pseudospectral methods, or zero-order hold (only for linear dynamic systems).
- The Lagrange term is evaluated using numerical quadrature.
- Path constraints are directly included as finite-dimensional constraints through their evaluation only at the discrete time points.
- Therefore, a DT method creates a (potentially large) nonlinear program (NLP).
- Many good resources available

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1 Biegler 2010; Biegler 2007; Betts 2010; Herber 2015; Patterson and Rao 2014; Divya 2011
Direct Transcription Overview (continued)

- CDD using DT and OOLC results in physical systems with natural dynamics that interact with an active control system in a way that yields maximal system performance\(^1\)
- This NLP has a specific structure and sparsity pattern that can be exploited in solvers to reduce total computational effort
  - Certain classes of dynamic optimization problems can be solved with convex optimization or quadratic programming\(^2\)
- DT has been shown to have good convergence properties, be parallelizable, handle unstable DAEs, and have specific advantages for singular control problems and high-index path constraints
- It is a direct method
  - Versus an indirect method such as the use of Pontryagin’s minimum principle to derive optimality conditions
- It is simultaneous or all-at-once approach because the optimization algorithm handles all design and analysis tasks
  - Analysis equations are embedded as optimization equality constraints
- Analogous ideas are used in (nonlinear) model predictive control (MPC)

\(^1\) Deshmukh, Herber, and Allison 2015  \(^2\) Usually with nested CCD solution strategy
Limitations and Potential Directions for CCD with DT

• Uncertainty
  • Address certain uncertainties using robust and reliability-based optimization principles¹
  • Merge nested CCD with experimental data²
  • Utilize recent developments in robust trajectory optimization such as polynomial chaos (PC) theory and DT³

• Implementable controllers
  • How can we bridge the “gap”⁴ between optimal open-loop control CCD studies and implementable control systems?
  • Determine how to synergize with feedback control architectures or model predictive control methods
  • Overall, understand how we can extract generalizable design knowledge from appropriate CCD problems and solutions (decision support tool)

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¹ Azad and Alexander-Ramos 2019; Cui, Allison, and P. Wang 2019  
² Deese and Vermillion 2018  
³ F. Wang et al. 2019  
⁴ Deshmukh, Herber, and Allison 2015
Limitations and Potential Directions for CCD with DT

- Efficient optimization methods for complex and large CCD problems
  - Provide better guidance on nested vs. simultaneous CCD strategies\(^1\)
  - Developments in decomposition-based optimization methods for CCD with DT\(^2\)
  - Leverage surrogate models, global optimization, and mixed discrete-continuous programming

- Inclusion of design-appropriate models
  - Better use of independent plant-design variables rather than dependent quantities (requirements) (e.g., instead using spring stiffness, we use the spring geometry as a design variable)\(^3\)
  - While bidirectional coupling can be challenging to model, it is needed for CCD to accurately represent real system design problems\(^4\)

\(^1\) Herber and Allison 2018 \quad \(^2\) Behtash and Alexander-Ramos 2020; Liu, Azarm, and Chopra 2020 \quad \(^3\) Allison, Guo, and Han 2014; Allison and Herber 2014 \quad \(^4\) Allison, Guo, and Han 2014; Allison and Herber 2014
## References

References (continued)


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