Control Co-design Direct Transcription Solution Strategies: Overview and Challenges

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Outline

1. Control Co-design

2. Direct Transcription

Control Co-design as a Dynamic Optimization Problem

One way to represent a control co-design (CCD) problem is in the time[1] domain using a dynamic optimization (DO) formulation[2]:

$$\min_{\boldsymbol{x}_c, \boldsymbol{x}_p} \quad \Psi(\boldsymbol{x}_c, \boldsymbol{x}_p) = \int_{t_0}^{t_f} \mathcal{L}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right) dt + \mathcal{M}\left(\boldsymbol{\xi}(t_0), \boldsymbol{\xi}(t_f), \boldsymbol{x}_c, \boldsymbol{x}_p\right) \tag{1a}$$

$$\text{subject to:} \quad \dot{\boldsymbol{\xi}} = \boldsymbol{f}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right) \tag{1b}$$

$$\boldsymbol{C}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right) \leq \boldsymbol{0} \tag{1c}$$

$$\boldsymbol{\phi}\left(\boldsymbol{\xi}(t_0), \boldsymbol{\xi}(t_f), \boldsymbol{x}_c, \boldsymbol{x}_p\right) \leq \boldsymbol{0} \tag{1d}$$

- $t \in [t_0, t_f]$: time defined in the time horizon between t_0 and t_f
- $\boldsymbol{\xi}(t)$: states
- $\boldsymbol{x}_c$: control design variables
- $\boldsymbol{x}_p$: plant design variables

[1] Herber and Allison 2018 [2] Note that for simplicity of presentation, this is a fixed-horizon, single-phase problem

→ Control Co-design as a DO Problem (continued)

One way to represent a control co-design (CCD) problem is in the time[1] domain using a dynamic optimization (DO) formulation[2]:

$$\min_{\boldsymbol{x}_c, \boldsymbol{x}_p} \quad \Psi(\boldsymbol{x}_c, \boldsymbol{x}_p) = \int_{t_0}^{t_f} \mathcal{L}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right) dt + \mathcal{M}\left(\boldsymbol{\xi}(t_0), \boldsymbol{\xi}(t_f), \boldsymbol{x}_c, \boldsymbol{x}_p\right) \tag{1a}$$

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$$\boldsymbol{\phi}\left(\boldsymbol{\xi}(t_0), \boldsymbol{\xi}(t_f), \boldsymbol{x}_c, \boldsymbol{x}_p\right) \leq \boldsymbol{0} \tag{1d}$$

- $\mathcal{L}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right)$: Lagrange or running cost term (*time dependent*)
- $\mathcal{M}\left(\boldsymbol{\xi}(t_0), \boldsymbol{\xi}(t_f), \boldsymbol{x}_c, \boldsymbol{x}_p\right)$: Mayer or terminal cost term
- $\boldsymbol{f}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right)$: state derivative function (*time dependent*)
- $\boldsymbol{C}\left(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p\right)$: path constraints (*time dependent*)
- $\boldsymbol{\phi}\left(\boldsymbol{\xi}(t_0), \boldsymbol{\xi}(t_f), \boldsymbol{x}_c, \boldsymbol{x}_p\right)$: boundary constraints

[1] Herber and Allison 2018 [2] Note that for simplicity of presentation, this is a fixed-horizon, single-phase problem

→ Basic CCD Solution Strategies

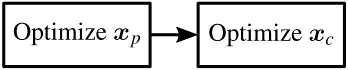

Figure: Sequential design.

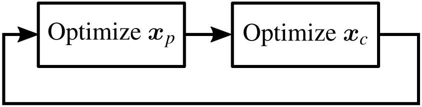

Figure: Iterated sequential design.

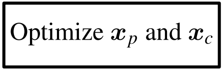

Figure: Simultaneous design.

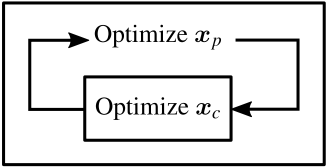

Figure: Nested design.

Optimal Open-Loop Control in CCD

- There often is a choice in control design variables whether it be the gains in a particular control architecture or open-loop trajectories ($\boldsymbol{u}$)
- Many recent CCD studies have utilized optimal open-loop control (OOLC) in early-stage design[1]
- Closed-loop control (CLC) design requires specification of control structure (e.g., state/output feedback) that may implicitly limit performance or the ability to satisfy system constraints
  - But there are also certain advantages…
- With OOLC, optimal control trajectories are sought without assuming a control architecture[2]
- CDD using OOLC results in physical systems with natural dynamics that interact with an active control system in a way that yields maximal system performance[3]
- Can provide important insights at early design stages

[1] Allison, Guo, and Han 2014 [2] Including the cases where the particular problem has a known feedback structure that is equivalent to the OOLC [3] Deshmukh, Herber, and Allison 2015

→ Some Limitations Found in Earlier CCD Research

- Some studies investigated the specific case when separate plant and control objectives were well defined[1]:

$$\Psi(\boldsymbol{x}_c, \boldsymbol{x}_p) = w_p \Psi_p(\boldsymbol{x}_p) + w_c \Psi_c(t, \boldsymbol{\xi}, \boldsymbol{x}_c, \boldsymbol{x}_p)$$

- Some studies used the assumption of unidirectional coupling where a plant design objective and constraints did not depend on $\boldsymbol{x}_c$[2]
  - Realistic treatment of plant design requires the inclusion of constraints that contain both $\boldsymbol{x}_p$ and $\boldsymbol{\xi}$ such as fatigue
  - There may not exist a feasible control/state solution for a fixed plant design with bidirectional coupling
  - This is design coupling between the physical-system and control-system
- Many early approaches for solving time-domain CCD problems had other potentially restrictive assumptions
  - For example, infinite-horizon, linear dynamics, and no path constraints so there is a linear–quadratic regulator (LQR) subproblem in nested CCD[3]
- Frequency domain approaches can address some challenges but not readily nonlinear dynamics and path constraints

[1] Peters, Papalambros, and Ulsoy 2009; Peters, Papalambros, and Ulsoy 2013; Fathy et al. 2001
[2] Peters, Papalambros, and Ulsoy 2013; Allison, Guo, and Han 2014 [3] Herber and Allison 2018; Fathy et al. 2001

→ Some Needs in a General CCD Solution Strategy

1. Inequality constraints
   - Many realistic CCD problems have inequality constraints to represent different failure modes such as stress or fatigue or even simple bounds on states and controls[1]
2. Bidirectional coupling
3. Comprehensive plant design representations including independent design variables and nonlinear dynamics
4. Identification of optimal dynamic and control behaviors
   - The desirable control architecture might be unknown in early-stage design (so support OOLC)
5. Computationally efficient and robust

Direct transcription (DT) methods have been shown to be effective at addressing these needs

[1] Allison and Herber 2014; Allison, Guo, and Han 2014; Herber and Allison 2018

→ Direct Transcription Overview

- In DT, the time horizon is discretized into a number of segments
- The values of the states $\boldsymbol{\xi}$ and controls $\boldsymbol{u}$ at the boundaries of these segments (discrete time points) are included directly as optimization variables
    - Discretization of the time-varying quantities
- The dynamic constraints are included as a set of equality constraints (known as defect constraints)
    - Many potential methods such as the basic trapezoidal rule, pseudospectral methods, or zero-order hold (only for linear dynamic systems)
- The Lagrange term is evaluated using numerical quadrature
- Path constraints are directly included as finite-dimensional constraints through their evaluation only at the discrete time points
- Therefore, a DT method creates a (potentially large) nonlinear program (NLP)
- Many good resources available[1]

[1] Biegler 2010; Biegler 2007; Betts 2010; Herber 2015; Patterson and Rao 2014; Divya 2011

→ Direct Transcription Overview (continued)

- CDD using DT and OOLC results in physical systems with natural dynamics that interact with an active control system in a way that yields maximal system performance[1]
- This NLP has a specific structure and sparsity pattern that can be exploited in solvers to reduce total computational effort
  - Certain classes of dynamic optimization problems can be solved with convex optimization or quadratic programming[2]
- DT has been shown to have good convergence properties, be parallelizable, handle unstable DAEs, and have specific advantages for singular control problems and high-index path constraints
- It is a direct method
  - Versus an indirect method such as the use of Pontryagin's minimum principle to derive optimality conditions
- It is simultaneous or all-at-once approach because the optimization algorithm handles all design and analysis tasks
  - Analysis equations are embedded as optimization equality constraints
- Analogous ideas are used in (nonlinear) model predictive control (MPC)

[1] Deshmukh, Herber, and Allison 2015 [2] Usually with nested CCD solution strategy

Limitations and Potential Directions for CCD with DT

- Uncertainty
  - Address certain uncertainties using robust and reliability-based optimization principles[1]
  - Merge nested CCD with experimental data[2]
  - Utilize recent developments in robust trajectory optimization such as polynomial chaos (PC) theory and DT[3]
- Implementable controllers
  - How can we bridge the "gap"[4] between optimal open-loop control CCD studies and implementable control systems?
  - Determine how to synergize with feedback control architectures or model predictive control methods
  - Overall, understand how we can extract generalizable design knowledge from appropriate CCD problems and solutions (decision support tool)

[1] Azad and Alexander-Ramos 2019; Cui, Allison, and P. Wang 2019 [2] Deese and Vermillion 2018 [3] F. Wang et al. 2019 [4] Deshmukh, Herber, and Allison 2015

→ Limitations and Potential Directions for CCD with DT

- Efficient optimization methods for complex and large CCD problems
  - Provide better guidance on nested vs. simultaneous CCD strategies[1]
  - Developments in decomposition-based optimization methods for CCD with DT[2]
  - Leverage surrogate models, global optimization, and mixed discrete-continuous programming
- Inclusion of design-appropriate models
  - Better use of independent plant-design variables rather than dependent quantities (requirements) (e.g., instead using spring stiffness, we use the spring geometry as a design variable)[3]
  - While bidirectional coupling can be challenging to model, it is needed for CCD to accurately represent real system design problems[4]

[1] Herber and Allison 2018 [2] Behtash and Alexander-Ramos 2020; Liu, Azarm, and Chopra 2020 [3] Allison, Guo, and Han 2014; Allison and Herber 2014 [4] Allison, Guo, and Han 2014; Allison and Herber 2014

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