Control Co-Design Concepts and Outcomes for Energy Systems

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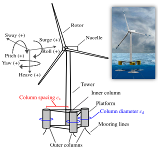
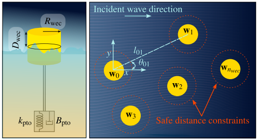
Outline

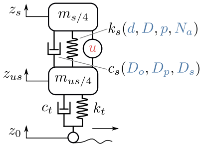
1

Control Co-Design (CCD) Introduction

→ Many Engineering Decisions for Dynamic Systems







- Floating offshore wind turbines with decisions[1] for [blade pitch, generator torque] and [platform column spacing, column diameter]
- Wave energy converter farms with decisions[2] for [power take-off gains] and [device geometry, device locations]
- Hybrid generator/storage systems with decisions[3] for [storage charging/discharging, generator power level] and [storage size]
- Active automotive suspension[4] with decisions for [open-loop actuator force] and [spring and damper geometry] decisions

[1] Sundarrajan, Lee, et al. 2024; Sundarrajan and Herber 2025; Sundarrajan 2025 [2] Azad, Herber, et al. 2025; Azad, Khanal, et al. 2024 [3] Azad, Gulumjanli, and Herber 2024 [4] Sundarrajan and Herber 2021

→ An Engineering Game Changer

#Control Co-Design (CCD) studies combining [control] and [plant] decisions

- Recently called an engineering game changer[1]

  "New products and systems are typically developed following a sequential approach, starting, for instance, with some mechanical engineering designs, continuing with aerodynamics/fluid discussions, then electrical/electronic solutions and at some point finishing up with the design and implementation of some control algorithms. An independent and sequential design process that develops the control system at a very late stage."

  *"This approach is a **game changer** for the control engineer[2], who will be not only the designer of advanced control algorithms but also the natural leader of the design of new products and systems."*

[1] Garcia-Sanz 2019 [2] For other engineers as well!

→ Optimization Model-Based CCD (1)

- There are many theories for model-based control engineering
  - Similarly, many CCD approaches leverage model-based CCD
- Often this is further structured as *optimization model-based CCD*:

$$\begin{aligned} \text{changing:} \quad & \text{[control decisions], [plant decisions]} && \text{(1a)} \\ \text{minimize:} \quad & \text{[objective]} && \text{(1b)} \\ \text{subject to:} \quad & \text{[what is possible]} && \text{(1c)} \end{aligned}$$

- Let's add some more structure to the CCD problem:

$$\begin{aligned} \text{changing:} \quad & \boldsymbol{p}_c, \boldsymbol{p}_p && \text{(2a)} \\ \text{minimize:} \quad & J(\boldsymbol{p}_c, \boldsymbol{p}_p) && \text{(2b)} \\ \text{subject to:} \quad & \boldsymbol{g}_c(\boldsymbol{p}_c) \leq \boldsymbol{0} \quad \text{(control design-only constraints)} && \text{(2c)} \\ & \boldsymbol{g}_p(\boldsymbol{p}_p) \leq \boldsymbol{0} \quad \text{(plant design-only constraints)} && \text{(2d)} \\ & \boldsymbol{g}_s(\boldsymbol{p}_c, \boldsymbol{p}_p) \leq \boldsymbol{0} \quad \text{(coupled system constraints)} && \text{(2e)} \end{aligned}$$

noting that the equality constraints are also included

→ Optimization Model-Based CCD (2)

- Equation (2) is not really representative of the dynamic nature of these systems — missing the states and dynamic constraint!

$$\begin{aligned}
\text{changing:}\quad & \boldsymbol{x}(t), \boldsymbol{p}_c, \boldsymbol{p}_p && \text{(3a)}\\
\text{minimize:}\quad & J(\boldsymbol{x}, \boldsymbol{p}_c, \boldsymbol{p}_p) && \text{(3b)}\\
\text{subject to:}\quad & \boldsymbol{g}_c(\boldsymbol{p}_c) \leq \boldsymbol{0} \quad \text{(control design-only constraints)} && \text{(3c)}\\
& \boldsymbol{g}_p(\boldsymbol{p}_p) \leq \boldsymbol{0} \quad \text{(plant design-only constraints)} && \text{(3d)}\\
& \boldsymbol{g}_s(\boldsymbol{x}, \boldsymbol{p}_c, \boldsymbol{p}_p) \leq \boldsymbol{0} \quad \text{(coupled system constraints)} && \text{(3e)}\\
& \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{p}_c, \boldsymbol{p}_p) - \dot{\boldsymbol{x}} = \boldsymbol{0} && \text{(3f)}
\end{aligned}$$

which generally is another coupled system constraint

Remark: Generally, $\boldsymbol{p}_c$ could be scalar parameters (e.g., gains), OL controls $\boldsymbol{u}(t)$, or some mixture.

Remark: Equation (3f) represents a time-domain perspective, but frequency-domain (i.e., transfer function) refinements of Eq. (2) are also possible.

→ Optimization Model-Based CCD (3): Illustrative Problem

- As an illustrative example, consider the following:

$$\text{changing:} \quad \boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}_p \qquad \text{(states, controls, plant design)} \tag{4a}$$

$$\text{minimize:} \quad \int_0^\infty [\boldsymbol{x}^\mathsf{T}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^\mathsf{T}\boldsymbol{R}\boldsymbol{u}]\mathrm{d}t + J_p(\boldsymbol{p}_p) \tag{4b}$$

$$\text{subject to:} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{p}_p)\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{p}_p)\boldsymbol{u}(t) \qquad \text{(dynamic constraints)} \tag{4c}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_0 \qquad \text{(initial conditions)} \tag{4d}$$

$$\boldsymbol{g}_p(\boldsymbol{p}_p) \leq \boldsymbol{0} \qquad \text{(plant design-only constraints)} \tag{4e}$$

- For a given feasible value of $\boldsymbol{p}_p$, Eq. (4) is equivalent to the classical infinite-horizon LQR problem where:

$$\boldsymbol{u}_*(t) = -\boldsymbol{K}(\boldsymbol{p}_p)\boldsymbol{x}_*(t) \tag{5}$$

is the the standard result (under certain conditions)

Design Coupling and Synergy Mechanisms

- Of key interest is the purple coupled parts — they enable the investigation of design coupling and synergy mechanisms
- *Design coupling* — how design decisions in one domain influence the ideal design decisions in other domains
  - For example, plant decisions might impact controller gains, or control decisions modify the states that force the plant decisions to change
  - ? Is it strong or significant? Is it captured (correctly) by the optimization model?
- *Synergy mechanism* — a specific underlying design mechanism that facilitates overall system performance improvements when two or more design elements are varied synergistically[1]
  - In wind energy, CCD enables the synergistic reduction in tower size with better-controlled maintenance of the optimal tip speed ratio, structural deflections, and stress[2]

[1] Allison, Herber, and Deshmukh 2015 [2] Deshmukh and Allison 2015

→ Control Co-Design Objective Discussion

- What determines the system's value that we are driving at through a CCD perspective?
- Are we looking to understand trade-offs (multi-objective perspectives)?
- Sometimes the result of these questions leads to separable "control"-focused goals J_c and "plant"-focused goals $J_p \rightarrow J = J_c + J_p$
- In other CCD applications, such a distinction might be unnatural or unnecessary
  - In energy systems, we might appropriately choose the levelized cost of energy (LCOE) that captures the impact of both plant and control decisions[1]
  - Other areas are cost-driven (minimize cost within prescribed specifications)
- Still, limited or simplified consideration of the dynamics and controls occurs
  - A counterbalanced robot as a proxy for minimizing energy consumption[2]
- Overall, we might consider appropriately "balanced" CCD approaches
  - Ones that identify the key system-level goals without undue influence of *either* area

[1] Garcia-Sanz 2020; Sundarrajan, Lee, et al. 2024 [2] Allison and Herber 2014

2

CCD Solution Strategies and Optimization Theory

→ Control Co-Design Solution Strategies (1): Introduction

- Consider the abstraction above for many of the (optimization-based) control approaches, which results in some kind of decided control solution (e.g., gains or OL $\boldsymbol{u}(t)$)
- We could be equally concerned about determining the decided plant solution

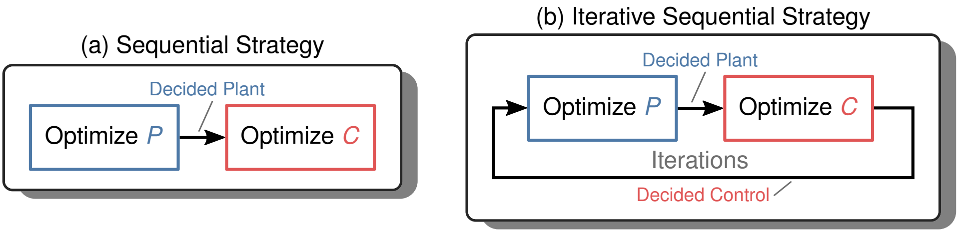
Remark

> This perspective is different from the classical "control-focused" perspective, where a plant is given to you that must be controlled.

- As there are two major decision sets here, we can consider finding solutions in several different ways
  - A key framing is through multidisciplinary analysis and optimization (MDAO)[1]

[1] Allison and Herber 2014; Allison, Guo, and Han 2014

→ Control Co-Design Solution Strategies (2): Sequential Perspective



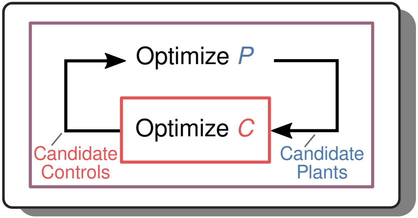
- *#sequential strategy* — determine the plant first, control second

> **Remark** This strategy is often the normal practice but has several critical deficiencies.

- *#iterative sequential strategy* — now we pass control design results back for plant changes or redesign and iterate
  - What is communicated back? — might be a fixed controller and/or insights into changes related to the plant domain
  - This approach can suffer from slow convergence and well-posedness issues

→ Control Co-Design Solution Strategies (3): Nested CCD Strategy

(c) Nested CCD Strategy



- *#nested CCD strategy* — the essence is continually asking and answering the question: if I made this plant, what would the best controller be?
    - Embedded inner-loop optimization problem (control subproblem) within the outer loop

→ Control Co-Design Solution Strategies (4): Simultaneous CCD Strategy

(d) Simultaneous CCD Strategy

Optimize P and C

- *#simultaneous CCD strategy* — consider both decision sets at the same time in one problem as we already saw in Eq. (2)
  - Could follow many paths toward the system-level optimum, including intermediate infeasible points

Example of CCD with Scalar Plant and Scalar Control (1)

- Let's consider a simple infinite-horizon problem[1] based on Eq. (4):

$$\text{changing:} \quad x(t), u(t), a \tag{6a}$$

$$\text{minimize:} \quad w_c \int_0^\infty \left[q x^2 + r u^2 \right] \mathrm{d}t - w_p a \tag{6b}$$

$$\text{subject to:} \quad \dot{x} = a x + u \tag{6c}$$

$$x(0) = x_0 \tag{6d}$$

$$a \leq 0 \tag{6e}$$

where a is a kind of nonpositive plant decision with some linear cost $-w_p a$

- For any feasible value of a, we can show that $u_*(t)$ is:

$$u_*(t) = -\left[a + \sqrt{a^2 + \frac{q}{r}} \right] x_*(t) \tag{7a}$$

$$= -k(a, q, r) x_*(t) \tag{7b}$$

[1] See §5.1 Test Problem 1: Scalar Plant, Scalar Control in Herber and Allison 2018

Example of CCD with Scalar Plant and Scalar Control (2)

- The objective function can then be shown using k in place of $u(t)$ to be:

$$J(k, a) = \frac{w_c x_0^2 [r k^2 + q]}{2[k - a]} - w_p a \tag{8}$$

- We can then solve this problem resulting in the following optimal values[1]:

$$k_* = \sqrt{\frac{w_p q}{2 r^2 w_c x_0^2 - w_p r}} \tag{9a}$$

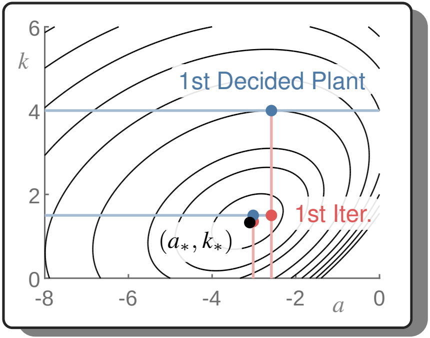
$$a_* = [w_p - r w_c x_0^2] \sqrt{\frac{q}{2 r^2 w_c x_0^2 w_p - w_p^2 r}} \tag{9b}$$

  - w_c smaller $\implies k_*$ larger, a_* smaller
  - $w_p \to 0 \implies k_* \to 0,\ a_* \to -\infty$
  - q smaller $\implies k_*$ smaller, a_* smaller

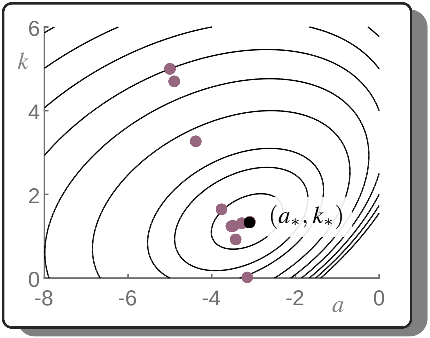
[1] Also, there are some conditions on the parameters such that these are real-valued solutions

Example of CCD with Scalar Plant and Scalar Control (3)

(b) Iterative Sequential Strategy



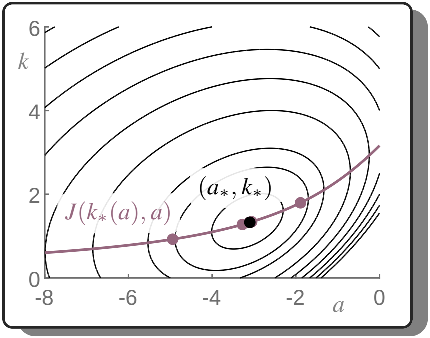
(d) Simultaneous CCD Strategy

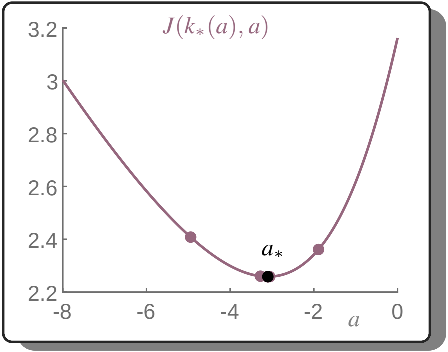


[1] Parameters values here are $x_0 = 1$, $q = 10$, $r = 1$, $w_c = 1$, and $w_p = 0.3$

Example of CCD with Scalar Plant and Scalar Control (4)

(c) Nested CCD Strategy





- Using the nested CCD strategy, every candidate plant results in a candidate control that is on the purple curve

→ CCD Strategies Discussion (1)

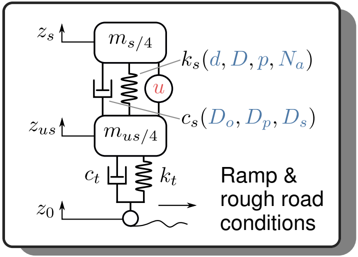
1. **X** is faster and more scalable than **Y** — *it depends!*[1]
   - A poorly implemented **Y** is worse than a well implemented **X**
2. **X** is easier to implement than **Y** — *it depends!*
   - Sometimes it is easier to create one problem with simultaneous CCD
   - Sometimes, it is easier to partition based on an existing control design technique for the inner-loop subproblem
   - While nested CCD has very similar complexity to the sequential strategy, sequential strategies can be more straightforward in practice
3. **X** is more robust and accurate than **Y** — *it depends!*
   - Simultaneous CCD has more flexibility to explore since infeasibility is allowed while iterating/solving
   - Nested CCD can support hybrid approaches with focused exploration (often the plant parameters) but might fail to converge if the inner loop does not always have a solution

[1] Sundarrajan and Herber 2021; Herber and Allison 2018

➞ CCD Strategies Discussion (2)

4. **X** will result in the same solution than **Y** — *it depends!*
   - Many CCD problems do not readily support “nice” formulations
   - In certain CCD problems, concerns regarding local optima are valid
5. **X** will result in a better understanding and performance for my dynamic system than **Y** — *it depends!*
   - Improvements to existing solutions are better than no change
   - Often, more than a single CCD problem needs to be solved to understand what the best solution is and why
   - Some strategies better support analysis techniques like parameter sweeps, sensitivity calculations, and infeasibility insights
   - Should seek to identify design coupling strength and synergy mechanisms

→ Numerical Investigations (1): Active Suspension Example



- Automotive suspension with [open-loop actuator force] and [spring and damper geometry] decisions
- Plant design dependent dynamics:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{p}_p)\boldsymbol{x}(t) + \boldsymbol{B}u(t) + \boldsymbol{E}\dot{z}_0(t) \tag{10a}$$

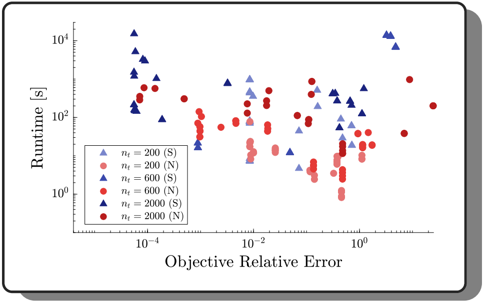
$$\boldsymbol{A}(\boldsymbol{p}_p) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_t(\boldsymbol{p}_p)}{m_{us}/4} & \frac{-[c_s(\boldsymbol{p}_p)+c_t]}{m_{us}/4} & \frac{k_s(\boldsymbol{p}_p)}{m_{us}/4} & \frac{c_s(\boldsymbol{p}_p)}{m_{us}/4} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s(\boldsymbol{p}_p)}{m_s/4} & \frac{-k_s(\boldsymbol{p}_p)}{m_s/4} & \frac{-c_s(\boldsymbol{p}_p)}{m_s/4} \end{bmatrix} \tag{10b}$$

- 16 inequality constraints — fatigue, buckling, manufacturability, etc.
- Single objective balancing handling, comfort, and control

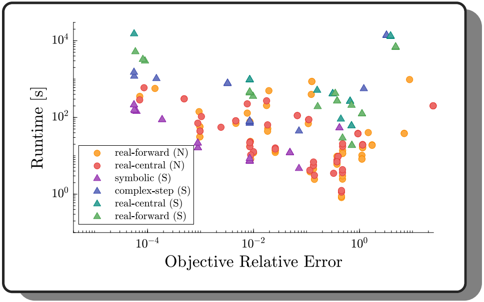
[1] Sundarrajan and Herber 2021 and github.com/danielrherber/dt-qp-project/examples/ACC2021

→ Numerical Investigations (2): Results

- Solved using direct transcription (discretized dynamics and control), similar to model predictive control (MPC) methods

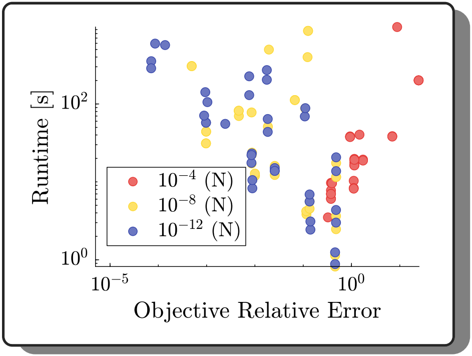


Depends on problem discretization



Depends on derivative method

→ Numerical Investigations (3): Results

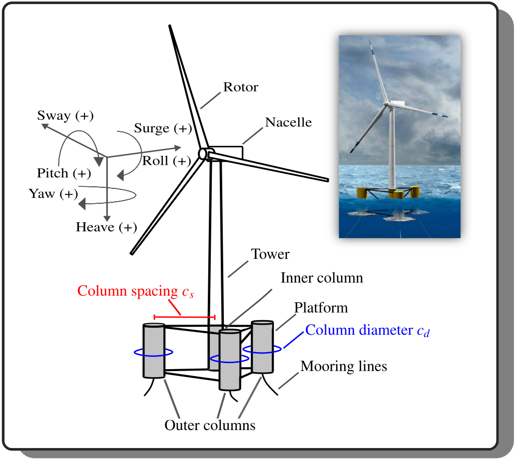


Depends on inner-loop precision

3

Select CCD Applications

➞ Floating Offshore Wind Turbines (1): Decisions



- Decisions[1] for [blade pitch, generator torque] and [platform column spacing, column diameter]
- The aero-servo-elastic simulations to predict FOWT dynamic behavior can be expensive
- A fixed plant and single design load case (DLC) might take 21 minutes

[1] Sundarrajan, Lee, et al. 2024; Sundarrajan and Herber 2025; Sundarrajan 2025

→ Floating Offshore Wind Turbines (2): Surrogate Models

- Surrogate models (data-driven approximations) can be used to reduce computational costs
- A derivative-function surrogate model (DFSM) approximating $\dot{\boldsymbol{x}}$ and $\boldsymbol{y}$ supports controller optimization and/or CCD activities

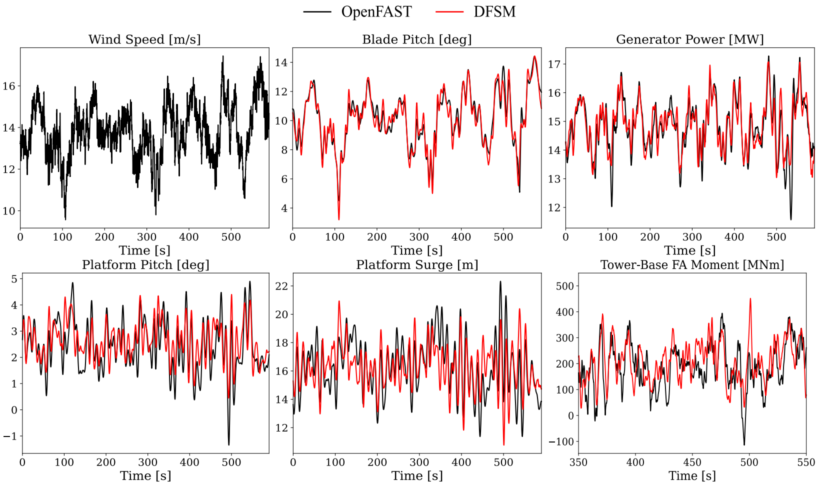
$$\dot{\boldsymbol{x}} = \boldsymbol{A}(w, \boldsymbol{p}_p)\boldsymbol{x} + \boldsymbol{B}(w, \boldsymbol{p}_p)u \tag{11a}$$

$$\boldsymbol{y} = \boldsymbol{C}(w, \boldsymbol{p}_p)\boldsymbol{x} + \boldsymbol{D}(w, \boldsymbol{p}_p)u + \underbrace{\boldsymbol{e}(w, \boldsymbol{p}_p, \boldsymbol{x}, u)}_{\text{LSTM}} \tag{11b}$$

where w is the wind speed and LSTM is long short-term memory approach with recurrent neural network

- Currently, 21 minutes $\rightarrow$ 45 seconds (28$\times$) computational cost improvements with trade-offs in accuracy

Floating Offshore Wind Turbines (3): Validation Results



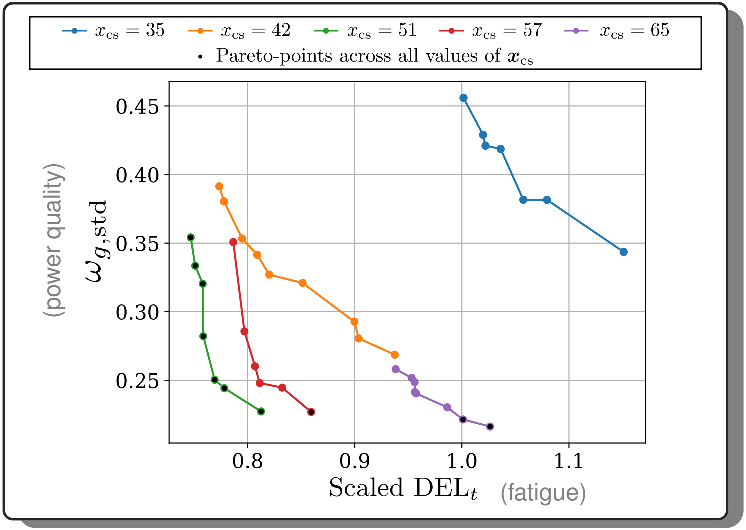
→ Floating Offshore Wind Turbines (4): Multi-Fidelity Results

- Now, we employ a trust-region-based multi-fidelity optimization scheme using the DFSM for the low-fidelity (LF) component:

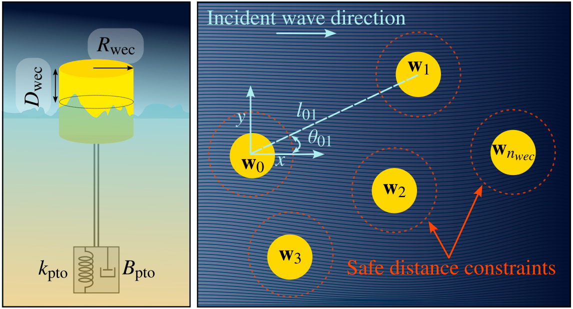
$$f_{\text{HF}}(\boldsymbol{p}_c) \approx f_{\text{CORR}}(\boldsymbol{p}_c) = f_{\text{LF}}(\boldsymbol{p}_c) + f_{\text{SUR}}(\boldsymbol{p}_c) \tag{12}$$

Fidelity	LF calls	HF calls	$\boldsymbol{x}_c$	$\text{DEL}_{t,\text{HF}}$ at $\boldsymbol{p}_c$
Low-fidelity	24	–	$[0.14, 1.65, -07.69, 0.36]^T$	1.62×10^5
Multi-fidelity	468	11	$[0.15, 2.12, -15.37, 0.38]^T$	1.56×10^5
High-fidelity	–	38	$[0.10, 2.92, -14.42, 0.36]^T$	1.55×10^5

➞ Floating Offshore Wind Turbines (5): Multi-Objective CCD Trade-Offs



→ Wave Energy Converter Farms (1): Decisions



- Decisions[1] for [power take-off gains] and [device geometry, device locations]

[1] Azad, Herber, et al. 2025; Azad, Khanal, et al. 2024 [2] WEC for wave energy converter

→ Wave Energy Converter Farms (2): Dynamics and Surrogate Modeling

- Motions of all WECs for a regular wave of frequency ω and unit amplitude can now be described as a transfer function matrix:

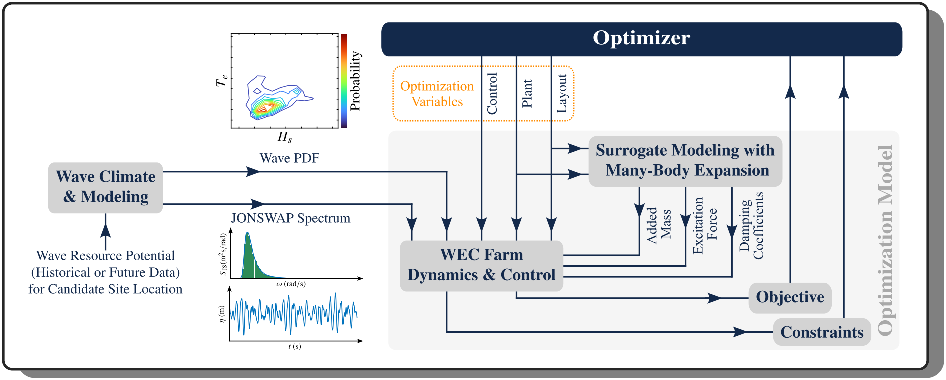
$$\hat{\boldsymbol{\xi}}(\omega) = \mathbf{H}(\omega)\hat{\mathbf{F}}_{\mathsf{e}}(\omega) \tag{13}$$

$$\mathbf{H}(\omega) = \left[\left[\omega^2(\mathbf{M} + \mathbf{A}(\omega)) + G + \mathbf{K}_{\mathsf{pto}}\right] + i\omega(\mathbf{B}(\omega) + \mathbf{B}_{\mathsf{pto}})\right]^{-1} \tag{14}$$

- We can estimation of hydrodynamic coefficients through numerical methods such as multiple scattering (MS) — but expensive!
- We construct data-driven surrogate models using artificial neural networks (ANNs) in combination with concepts from many-body expansion (MBE)
- Rough process outline for added mass matrix:

$$(R_{\mathsf{wec}}, D_{\mathsf{wec}}, l_{pq}, \theta_{pq}, \omega) \xrightarrow[\text{device}]{\text{ANN}} (\tilde{a}, \tilde{a}_{11}) \xrightarrow[\text{device}]{\text{scaling}} (a, a_{11}) \xrightarrow[\text{farm}]{\text{MBE}} \longrightarrow \mathbf{A}(\omega) \tag{15}$$

→ Wave Energy Converter Farms (3)



→ Wave Energy Converter Farms (4): Results

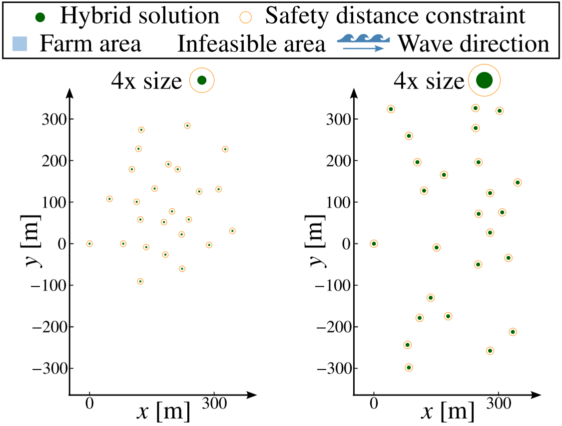
Surrogate model effectiveness

Method	Power	Time [hr]
MS-GA	25.51	639.32
SM-GA	25.27	0.90
Hybrid	25.51	6.02

Site dependence for optimal [plant] and [control]

Site	**Plant**		**Control**		Power
	R_{wec}	RD_{wec}	$\mathbf{B}_{\text{pto}}$	$\mathbf{K}_{\text{pto}}$	
NAWC24	2.79	5.29	0.92	-226.01	9.7×10^7
NAEC8	2.11	4.22	1.40	-116.00	4.2×10^1
PI14	4.06	6.80	13.84	-419.47	2.9×10^6
N46229	3.99	6.79	3.62	-434.66	5.2×10^7

→ Wave Energy Converter Farms (5): Results



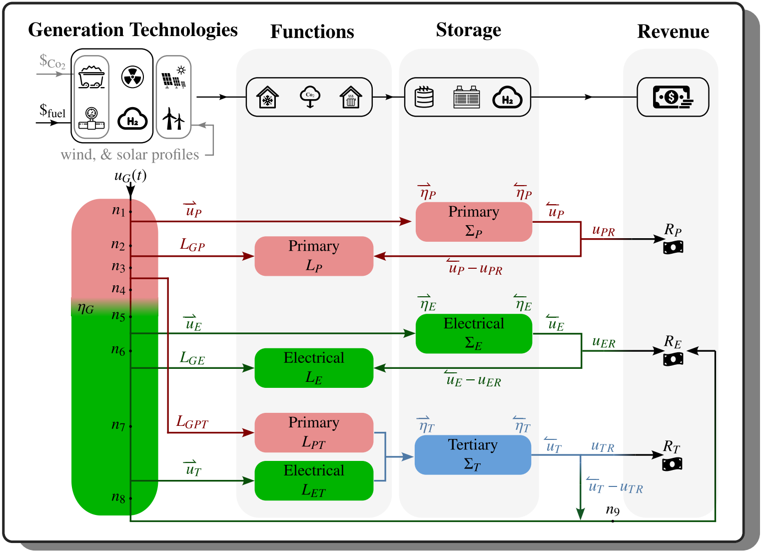
(a) Alaska Coast (NAWC24). (b) East Coast (NAEC8).

Hybrid Generator/Storage Systems (1)

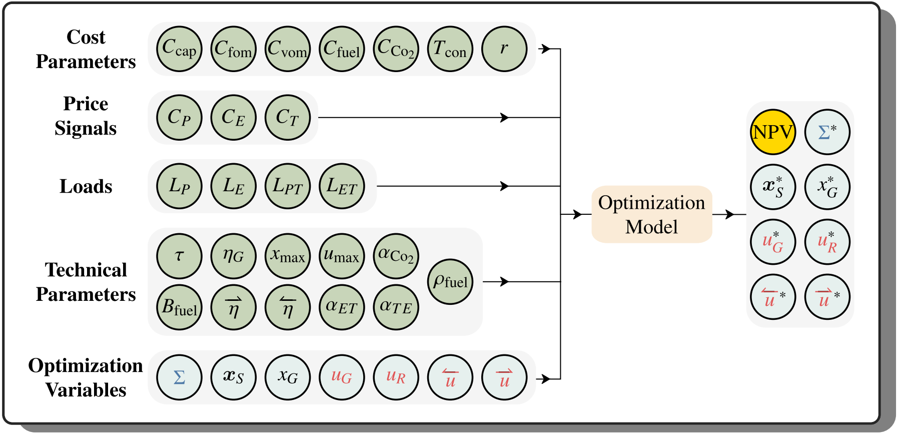
- Integrating storage into energy grids is of increasing interest due to the expanding variability of energy-generating systems
  - Wind turbines with batteries
  - Natural gas power using carbon capture combined with hot thermal storage
  - Nuclear power combined with hydrogen production
- Decisions[1] for [storage charging/discharging, generator power level] and [storage size]

[1] Azad, Gulumjanli, and Herber 2024

→ Hybrid Generator/Storage Systems (2): Framework

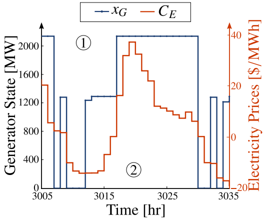


➞ Hybrid Generator/Storage Systems (3): Inputs and Outputs

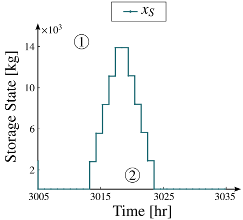


Hybrid Generator/Storage Systems (4): Nuclear with Hydrogen Results

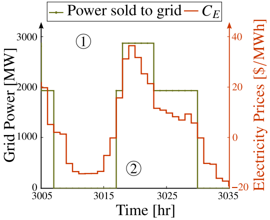
- Solved as a linear program (simultaneous CCD) with hourly decisions made over 30 years of operation
- Optimal storage capacity was found to be 195930 [kg]



(a) Optimal generator state.



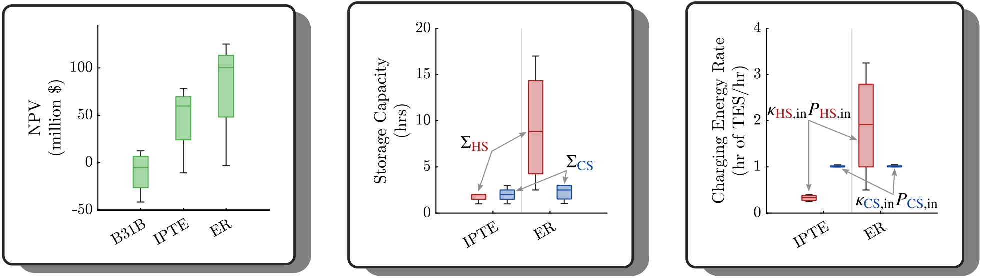
(b) Optimal storage state.



(d) Optimal power to grid.

→ Hybrid Generator/Storage Systems (5): NGCC with TES Results

- CCD studies can help *fairly* compare different technology options, including current solutions[1]
- Variability is due to different scenarios (previous and future prices, locations due to temperature dependence)

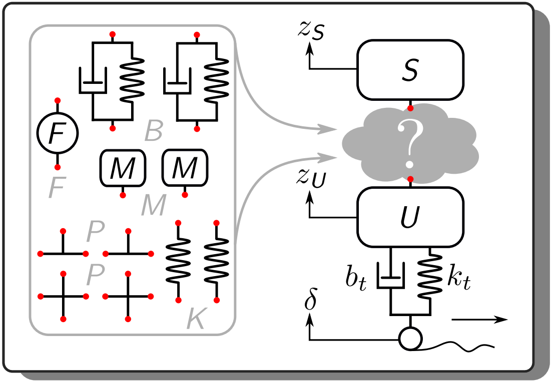


[1] Vercellino et al. 2022

➞ System Architecture and Discrete Decisions (1)

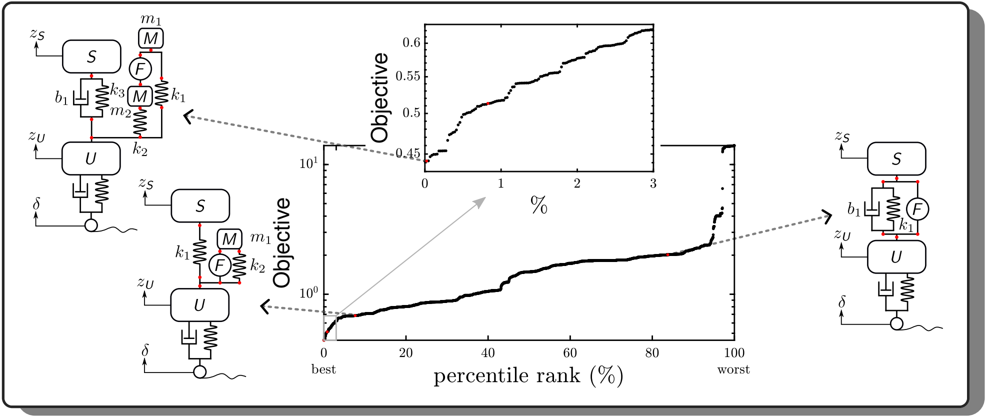
- We may also want to consider architecture-level decisions, such as for the previous automotive suspension[1]

Architecture-dependent [plant] and [control] decisions



[1] Herber and Allison 2019

→ System Architecture and Discrete Decisions (2): Results



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- Special thanks to my many collaborators on the various works presented in this talk, including James Allison (PhD advisor), Saeed Azad (postdoc), Athul Sundarrajan (PhD student), among many others

- Many thanks to ARPA-E and NSF for supporting this research — please see the cited papers for specific project acknowledgements

Questions?

Control Co-Design Concepts and Outcomes for Energy Systems

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Please reach out if you are interested in further discussion and materials on CCD, controls, and optimization, or any of the shown applications!

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➞ Plant Modeling and Decisions for Effective and Balanced CCD (1)

What should $\boldsymbol{p}_p$ be then?

- Lots of ways to model and represent change to our plants or physical systems

Abstract		Concrete
Metamodel	Lumped	Direct

- *Metamodel* — includes abstract parameters with limited direct physical or engineering interpretation
  - Coefficients in a state-space model or transfer function, eigenvalues, or transfer function perturbations
- *Lumped* — includes semi-abstract or physics-driven intermediate parameters
  - Spring constant k in the $a_{2,3}$ coefficient k/m of the state-space model
- *Direct* — includes independent decisions that are more closely connected to direct manufacturing specifications
  - Instead of k from before, we might consider the spring wire diameter and number of coils directly

Plant Modeling and Decisions for Effective and Balanced CCD (2)

- More abstract representations might be considered plant requirements or targets, but the issue comes when there is a disconnect between this CCD result and what is physically possible, especially when plant-design constraints are ignored[1]

Remark

> This isn't to say there isn't value in more abstract CCD problems — we should consider the realizability of the outcomes in relation to the questions we are trying to answer with a CCD perspective.

- Linear vs. nonlinear models, low vs. high fidelity models — ensure that system assessment is sufficiently close to reality and goals
  - Overly abstract or simplified plant models might not enable sufficient feasible exploration and exploitation of design coupling
  - Drivers are often failure theories, manufacturing limits, stakeholder preferences, or even engineering judgment
  - May need to include inequality constraints as fundamental limits

[1] Allison and Herber 2014

→ Practical Methods Discussion

- Very few CCD examples have closed-form solutions — general dynamic and design optimization problems are still hard to solve this way
- Early CCD research and methods focused on extending classical control methods, such as LQR, to include plant decisions[1]
  - However, there were many required assumptions and limited flexibility
- For more flexible solution methods for CCD general problems, direct methods have become quite popular
  - Both simulation-based (shooting) and direct transcription methods are used in many CCD strategies
- In most cases, finite-dimensional optimization theory and methods are utilized in a selected strategy (e.g., `fmincon` or `quadprog` in Matlab)

[1] Herber 2023

Example of CCD with Simple State Transfer (1): Formulation

- Second, let's consider the following finite-time CCD problem:

$$\text{changing:} \quad \boldsymbol{x}(t), u(t), k \tag{16a}$$

$$\text{minimize:} \quad \int_0^{t_f} u^2 \mathrm{d}t \tag{16b}$$

$$\text{subject to:} \quad \dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -k & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{16c}$$

$$\boldsymbol{x}(0) = \begin{bmatrix} x_0 & v_0 \end{bmatrix}^{\mathsf{T}} \tag{16d}$$

$$\boldsymbol{x}(t_f) = \boldsymbol{0} \tag{16e}$$

which seeks to move a second-order system from an arbitrary initial state to rest while minimizing control effort

Remark

> This problem is related to Minimum Energy Transfer[1].

[1] See §5.2 Test Problem 2: Co-Design Transfer in Herber and Allison 2018

→ Example of CCD with Simple State Transfer (2): Nested CCD Solution

- The nested CCD inner-loop solution for $k > 0$ can be shown to be:

$$u_*(t,k) = \frac{2k}{\sin^2(\sqrt{k}t_f) - kt_f^2}\left[c_1(t,k)x_0 + c_2(t,k)\frac{v_0}{\sqrt{k}}\right] \tag{17a}$$

$$c_1(t,k) = \sin(\sqrt{k}[t_f - t])\sin(\sqrt{k}t_f) - \sqrt{k}t_f \sin(\sqrt{k}t) \tag{17b}$$

$$c_2(t,k) = -\cos(\sqrt{k}[t_f - t])\sin(\sqrt{k}t_f) + \sqrt{k}t_f \cos(\sqrt{k}t) \tag{17c}$$

Remark

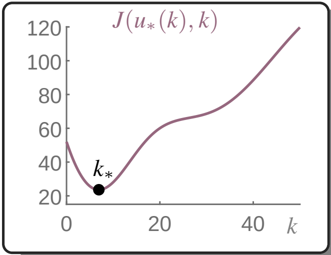
> This control solution demonstrates the complicated nature of the analytical solutions for even the simplest of CCD problems, similar to what we observed for optimal control in general.

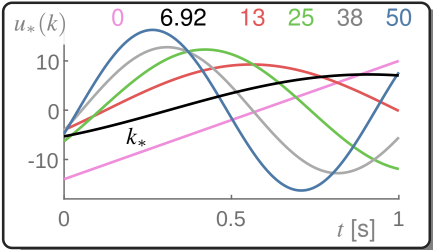
→ Example of CCD with Simple State Transfer (3): Single Global Minimum

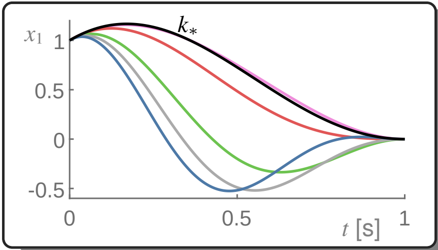
- Parameters values here are $t_f = 1$, $x_0 = 1$, and $v_0 = 2$

Remark

> There is a single global minimum.





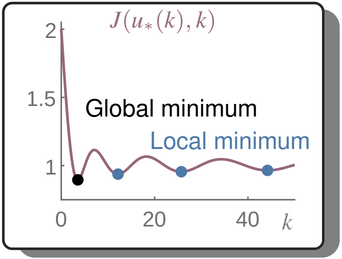


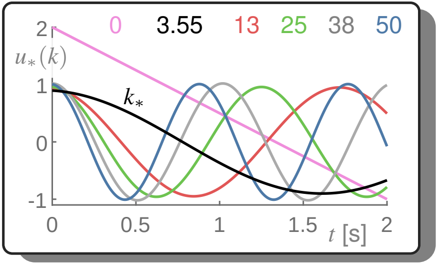
→ Example of CCD with Simple State Transfer (4): Mutliple Local Minimum

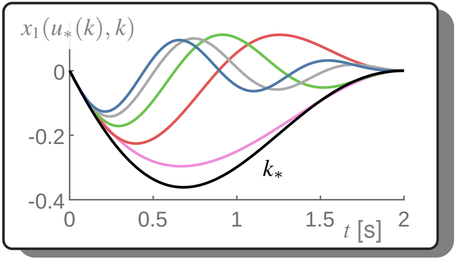
- Parameters values here are $t_f = 2$, $x_0 = 0$, and $v_0 = -1$

Remark

> There are many local solutions.





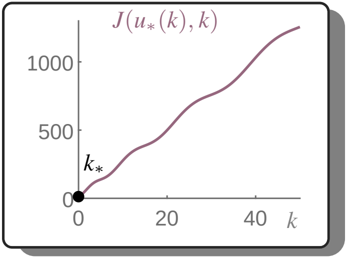


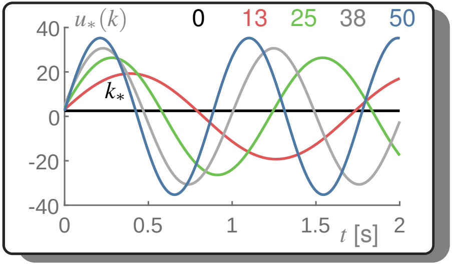
→ Example of CCD with Simple State Transfer (5): Degenerate Plant Solution

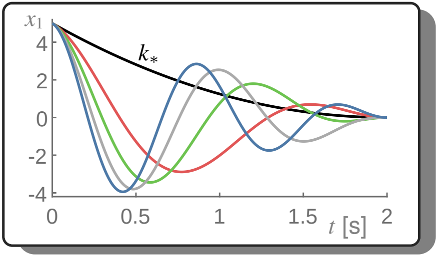
- Parameters values here are $t_f = 2$, $x_0 = 5$, and $v_0 = -5$

Remark

> Optimal plant solution is $k_* = 0$.







→ References for Control Co-Design

- Some references include:
  - Garcia-Sanz 2019
  - Herber and Allison 2018; Allison, Guo, and Han 2014; Allison and Herber 2014; Sundarrajan and Herber 2021; Peters, Papalambros, and Ulsoy 2015; Fathy et al. 2001
  - Azad and Herber 2023; Azad and Herber 2025
- Some applications include:
  - See Slide 3
  - Thermal systems in Nash, Pangborn, and Jain 2021
  - Wave energy in Coe et al. 2020
  - Marine hydrokinetics in Hasankhani et al. 2022
  - Airborne wind energy in Deese, Deodhar, and Vermillion 2017