Bridging the Gap between Open-Loop and Closed-Loop Control in Co-Design: A Framework for Complete Optimal Plant and Control Architecture Design*

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Abstract—Here we propose a novel framework for the combined plant and controller architecture design based on a set of systematic studies that culminate in an optimal plant architecture and associated realizable control law. This framework bridges the inherent gap between open-loop optimal trajectories provided by particular co-design studies and practically implementable control laws. This is accomplished through a step-by-step process where a series of optimization problems are solved that provide important system insights such as maximum system performance limits, controller architecture, actuator selection, etc. at appropriate design phases. Each optimization problem thus informs subsequent formulations. This methodology is applied to semi-active suspension design.

I. INTRODUCTION

Optimal design of complex dynamic systems is typically an iterative process involving plant and associated controller design. The strong interdependence between plant and control design must be addressed to obtain system-optimal solutions. This can be achieved through integrated plant and control design (co-design) methods, which can lead to significant performance improvements over traditional sequential methods [1], [2]. Moreover, recent advances in co-design can handle nonlinear dynamic, path, control, and plant constraints efficiently [1]–[4] using open-loop control (OLC) where no assumptions are made on control structure. Co-design has been used traditionally in early-stage design studies where optimal plant design and optimal control input trajectories are sought for a specified physical-system architecture. OLC trajectories can provide great insights into system performance limits, but may not be easily translated into practical control laws. Furthermore, optimal OLC trajectories can be highly sensitive to plant or other uncertainties. Current co-design formulations address only a small part of complete dynamic system development. To address this shortcoming we propose an extended framework for dynamic system design, supported by co-design, that lays out a full process from system architecture to implementable control laws. Our long-term objective is to bring integrated design methods into practice to support new levels of system performance and reduce costly design iterations.

In our proposed framework, we consider the complete architecture design of a dynamic system that is defined by the selection of the specific components and their connections. For example, candidate physical architectures for automotive hybrid powertrains include series, parallel, power-split, and other configurations. Many system architecture design problems have a large number of possible components and connections, making enumeration of architecture candidates impractical as a design strategy. OLC can instead replace some components or interfaces with optimal trajectories, reducing the design problem size while still providing an optimal solution [3]. In addition, OLC-based studies are particularly valuable at early design stages for gaining insights into upper system performance limits and into dynamic behaviors and interactions that lead to system-optimal performance [1], [3].

OLC techniques used in co-design include both indirect methods (dynamic programming [5], Pontryagin’s maximum principle [6]) and direct methods (shooting [1], direct transcription [7]–[9]). In OLC, optimal control trajectories are sought without assuming control architecture, whereas closed-loop control (CLC) design requires specification of a control structure (e.g., state/output feedback) that may implicitly limit performance or ability to satisfy system constraints. While OLC provides important insights at early design stages, CLC is normally required for actual implementation to provide stability, robustness, disturbance rejection, and other desirable properties of feedback systems (explaining the vast range of CLC methods, including LQR/LQG [10], adaptive control [11], and $H_\infty$ control [12]). CLC can produce optimal solutions if specific (but potentially restrictive) conditions are met such as with LQR/LQG (linear dynamics, quadratic objective, no path or plant constraints). The trade-offs between OLC and CLC has motivated a few techniques such as model predictive control (MPC) [13] and feedforward control [14].

Useful design insights can be extracted from optimal OLC solutions when employing co-design. Since the resulting solutions have the optimal dynamics, the desired natural dynamics of the system can emerge [3], [15]–[20]. Furthermore, the resulting trajectories can serve as a basis for developing implementable feedback control systems and physical-system/architecture design if the OLC was used in early-stage design [3], [15]–[18]. If the significant assumption of uni-directional plant and control coupling is made, control proxy functions may be used in the plant design formulation to yield a sequential design process that approximates (or in limited cases equals) the co-design solution [21].

Finally and perhaps most critically, OLC supports solution
of sophisticated co-design optimization problems that involve nonlinear dynamics, nonlinear plant constraints, path constraints, hybrid dynamics, and other challenging elements of realistic system design problems. OLC often is successful at capturing complex control solutions beyond what is possible with traditional CLC with assumed control architectures. Co-design based on CLC limits creative plant design exploration at early design stages. While OLC is an important tool for early studies, a significant gap exists between OLC and implementable CLC. An ideal design framework will support identification of system-optimal plant designs while supporting the development of CLC systems that yield approximately optimal system performance based on an understanding of how a system should behave dynamically that is gained through early-stage OLC co-design studies.

Here we propose a novel process for dynamic system design that integrates OLC and CLC studies (illustrated in Fig. 1a). Stage 1 finds the optimal plant architecture \( a_\star \), employing established co-design and OLC methods to compare architecture candidates. Stage 2 finds the optimal controller architecture, i.e. control laws, \( c_\star \) and optimal plant design \( x_{p,\star} \), again employing co-design but with CLC. Stage 3 realizes the digital controller design. Stage 1 will be typically handled by mechanical engineers while Stages 2 and 3 will be solved by control engineers. Iterations between the stages should be performed as necessary, including modification of problem formulations as insights are gained. Stages 1 and 2 will be further expanded upon in the following sections.

II. FORMULATION CONSIDERATIONS FOR CO-DESIGN AND SYSTEM ARCHITECTURE DESIGN

Effective implementation of the design process outlined in Fig. 1a requires consideration of several optimization problem formulations. The outer-loop nonlinear programming (NLP) formulation for optimal physical architecture design in Stage 1 is:

\[
\begin{align*}
\min_{a} & \quad \Phi_\star (a) \\
\text{where:} & \quad a \in \mathcal{F}_a
\end{align*}
\]

where \( a \) represents plant architecture design (e.g., an adjacency matrix). Architecture design is a discrete design problem. Fair comparison between architecture candidates in the set of possible architectures \( \mathcal{F}_a \) requires evaluation of the best possible performance for each candidate. This requires optimization with respect to continuous design variables (plant design variables \( x_p \) and control trajectories \( u(t) \)) for each candidate architecture, through solution of the following inner-loop problem (co-design with OLC) to obtain \( \Phi_\star (\cdot) \):

\[
\begin{align*}
\min_{\xi \in \mathcal{X}_p, \xi(t), u(t)} & \quad \Phi^i (t, \xi^i(t), u^i(t), x_{p}^i) \\
\text{subject to:} & \quad \dot{\xi}^i - f_\xi^i(t, \xi^i(t), u^i(t), x_{p}^i) = 0 \quad (2b) \\
& \quad C^i(t, \xi^i(t), u^i(t), x_{p}^i) \leq 0 \quad (2c) \\
& \quad \phi^i(t, \xi^i(t), u^i(t), x_{p}^i) \leq 0 \quad (2d)
\end{align*}
\]

where:

\[
\{\xi(t), u(t), x_{p}^i, f_\xi^i(t), C^i(t), \phi^i(t)\} \in a^i
\]

Figure 1b illustrates the Stage 1 problem in more detail. Linking the outer- and inner-loop formulation is an important aspect that needs to be done efficiently to facilitate a thorough exploration of the physical architecture design space. While a human designer could formulate and evaluate the optimized performance of each candidate architecture, this task would be quite time consuming and limit the number of candidate architectures that could be evaluated. Automating this process is an ongoing task. An automated interface between the outer- and inner-loops requires a number of tasks to be performed in an automated fashion including model creation, optimization problem generation, and model parameterization. Developing such an interface with the proper inputs to Stage 1 would allow a truly diverse set of candidate architectures to be explored efficiently.

Some existing approaches for physical-system architecture design (other than the enumeration of possible architectures) include agent-based synthesis for mechatronics systems [22], graph grammar for general engineering systems [23], and generative representations (for robotics [24] and static trusses
In these approaches, the feasible set of architectures are implicitly ensured through the underlying algorithms and rules as suggested in Eqn. (1b), rather than through traditional constraints.

Physical architecture design traditionally has involved simplified dynamic models. The above design strategy accounts fully for system dynamics, including passive and active elements, through the solution of the inner-loop co-design problem. Co-design with OLC will be utilized primarily in Stage 1 as an infinite-dimensional problem where we seek optimal control trajectories, $u(t)$ [1], [3]. In Stage 2 we seek a finite set of optimal control variables (e.g., feedback gains) for the CLC system, $x_c$ [26].

With the formulation in place, additional solution considerations will be discussed.

A. Classifications of Information Horizons

OLC typically has been used on complete horizon systems where the environment is completely known for the time horizon. This, however, is a strong assumption. Considering smaller horizons is more realistic.

1) Complete Horizon: The control (either OLC or CLC) problem is solved with complete information about environment (e.g., exogenous inputs, physical model). Examples include robotic shuttle reentry trajectory design [7]. These problems are typically offline optimization problems.

2) Instantaneous: This class of problems deal with instantaneous control problem, where the environment information is available only at the current instant of time. CLC for tracking and regulation are well-known examples. With instantaneous information, providing an acceptable level of performance over the entire horizon (such as satisfying path constraints) can be challenging.

3) Limited Horizon: This control problem is solved for only a small portion of the horizon. This solution is repeated at regular intervals during operation. Some examples include look ahead control of wind turbines [27] and fuel optimal hauling trucks or railways [28]. MPC is an important solution strategy since it can provide comparable performance to complete horizon solutions and the desirable properties of instantaneous ones [13]. These problems can be offline or online optimization problems. Limited horizon is a middle ground between complete and instantaneous.

B. Early-Stage Design Considerations

Multiple special formulations in Stage 1 can be used to make better informed plant and control design decisions. Utilizing co-design with OLC maintains an unrestricted (and potentially system optimal) formulation while performing these studies. Conventional Stage 1 design strategies involve physical-system design without complete consideration of the interaction between physical- and control-system design, leading to suboptimal results.

Unstructured OLC can be used replace select components or interfaces in a system to simplify early design studies. Electric subsystems may be approximated as OLC trajectories in mechatronic systems [19], particularly if fast electrical system dynamics support a direct mapping from OLC to CLC with tracking. Co-design with OLC temporarily forgoes some architecture design decisions for this problem, such as electric machine or gearbox selection. Another example is deciding between semi-active/active control strategies [29]. At a fundamental level, OLC formulations can model idealized versions of these components (e.g., constraints on power: $P \leq 0$ for semi-active, $P \in \mathbb{R}$ for active) and guide the selection process based insights extracted from OLC results (e.g., Does the performance benefit of an active system outweigh the cost penalty compared to a semi-active one?).

Another recent example from the literature involves power-take off (PTO) design for a wave energy converter (WEC) that extracts (or inputs) power through a force [3]. Several WEC PTO types have been investigated, including linear generators, rotary electric machines, and hydraulics systems, each of which have widely different dynamics. If the PTO type is ignored at early design stages, and the force trajectory provided by the PTO is optimized using OLC, important insights about how the system should be operated can be extracted before some architecture decisions are made. These insights can then inform both physical and control architecture design decisions. Distinct behaviors, such as latching (holding the WEC in place for a short time), emerge via OLC studies. Physical- and control-system architecture decisions can then be made to achieve the type of behavior exhibited in the OLC co-design results [3]. This strategy helps engineers develop implementable systems that approach the fundamental maximum system performance limit, and offers a path toward integrated mechatronic system design processes than can be adopted in engineering practice.

Similarly, for the transition from computational models to fabricated plants, structured studies addressing the common issues associated with reconfigurability of the plant such as cost estimation and selection can be performed in a complete way [30]. Co-design with OLC enables the proper study of reconfigurability of controlled dynamic systems.

III. Extracting Implementable Control Solutions from Open-Loop Studies

The objective of the framework presented here is to support the development of integrated, implementable actively controlled systems. Recent advances in co-design based on optimal OLC methods make possible the design of systems that account fully for the interaction between physical- and control-system design, including detailed and realistic physical-system design considerations. These methods are highly effective for early-stage design, generating physical systems with natural dynamics that interact with an active control system in a way that yields maximal system performance. The associated optimal control trajectories can lead to new insights, and help engineers discover what the true performance limits are on the system without constraints imposed by control architecture assumptions. These optimal
control trajectories, however, cannot be used directly in a control-system implementation.

Elements of optimal OLC may be used in realizable control systems. For example, open-loop optimization can be repeated online based on feedback of measured/estimated variables [9], [31], [32]. Robustness may be improved using open-loop multi-objective optimization that aims to improve nominal performance and reduce variance [33]. Alternatively, classical feedback methods may be used in well-modeled regimes, but is complemented through the use of open-loop trajectories in poorly modeled regions [34], [35].

While the above strategies enhance the utility of OLC in practice, in many cases a feedback control architecture is required. There is a significant gap that exists between the output of OLC co-design methods that are appropriate for early-stage design, and implementable control-system design. This article presents a first effort to formalize this gap in the context of co-design and integrated system development, and presents a first approach for addressing this gap. Several approaches may be used to extract CLC designs from OLC co-design results with varying levels of rigor. Optimal OLC trajectories may be analyzed for patterns, spectral properties, or other characteristics that can guide control architecture development. Given sufficient data from optimal control trajectories, system identification [36] or trajectory matching strategies [37] might be used to determine a CLC system design that approximates OLC performance.

Bridging the gap between OLC and CLC in integrated dynamic system design is an opportunity to make possible new levels of design integration. The existing co-design literature offers significant advancements in design integration between physical- and control-system design, capitalizing on synergy to improve system performance, but does not by itself offer a means for integrated design in a realistic system development process. Connecting co-design to CLC architecture design is an important step toward incorporating co-design into design practice, and toward a more comprehensive integrated design framework for designing systems with new levels of performance in shorter time periods.

The study presented here begins with an early-stage co-design problem that makes no assumptions regarding actuator or control-system architecture. A sequence of problems is solved, each informed by the results of the previous problem, moving toward greater levels of system specificity, eventually resulting in an optimal system design based on a detailed CLC architecture. While adding detail brings us closer to an implementable design, it also constrains the design problem. It is important to use a multi-step approach where early-stage design problems entail few assumptions and support a broad design space search.

IV. CO-DESIGN STUDIES FOR A SEMI-ACTIVE SUSPENSION

The co-design and early-stage architecture design process specified in the previous sections is applied to a trailing-arm type suspension (shown in Fig. 2). The objectives are to 1) minimize the sprung mass acceleration, \( \ddot{x}_s \), (i.e., improve passenger comfort) and 2) minimize tire deflection, \( z_{us} - z_0 \), (improve road handling). The gray box in Fig. 2 indicates an unknown component in the system that we need to determine through optimal OLC studies. The performance can be improved further by optimizing the plant design variables: \( x_p = [x_{p1}, x_{p2}, x_{p3}, x_{p4}, K_{in}] \) where \( x_{p1} \) and \( \alpha \) are geometric variables, and \( K_{in} \) the stiffness of the physical spring that is assumed to be linear. The 1D simplification is made accurate by modeling geometric nonlinearities for both \( K_s(x_{p1}, \alpha) \) and the unknown component.

The process outlined in Fig. 1a is completed using four linked co-design problems that are solved using direct transcription with trapezoidal collocation [2], [3], [7], [8]. Solution accuracy was verified using high-order simulation. The insights from each problem inform the subsequent problem, with the final study culminating in a realizable feedback controller. The differences in each problem (additional structure on \( F(u) \) and control bounds) are shown in Table I. The underlying problem formulation is:

$$\min_{x_{p1}, u(t)} \Phi(x_p, u(t)) = \int_0^{t_F} \left( r_1 \dot{\xi}_1^2 + r_2 \dot{\xi}_2^2 \right) dt $$

subject to:

$$\dot{\xi} = A(x_p, \xi) \xi + B_1 q + B_2 F(u)$$

$$A_p x_p \leq 0$$

$$x_p \geq 0$$

where:

$$A = \begin{bmatrix} -\frac{K_s(x_{p1}, \alpha)}{k_s} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\omega^2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{K_s(x_{p1}, \alpha)}{k_s} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \xi = \begin{bmatrix} z_{us} - z_0 \\ \dot{z}_{us} \\ \ddot{z}_{us} \\ x_s - z_{us} \end{bmatrix}$$

$$A_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where the spring rate \( K_s \) is a nonlinear function of \( x_{p1} \) and states \( \xi, M_{us}, M_s, C_t \), and \( K_t \) are the unsprung mass, sprung mass, tire damping, and tire stiffness, respectively. The road disturbance and disturbance velocity are \( z_0 \) and...
TABLE I: Sequence of problem descriptions and optimal solutions for the semi-active suspension study.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Stage</th>
<th>Type</th>
<th>Control</th>
<th>(u)</th>
<th>(F(u))</th>
<th>Bounds</th>
<th>(\Phi_\alpha)</th>
<th>(K_{\text{lin}}) (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>–</td>
<td>Passive</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>(5.80 \times 10^{-4})</td>
<td>26810</td>
</tr>
<tr>
<td>P1</td>
<td>(1)</td>
<td>Active</td>
<td>OLC</td>
<td>(u)</td>
<td>(u)</td>
<td>(0 \leq u(t) \leq 2.06 \times 10^{-4})</td>
<td>20694</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>(1)</td>
<td>Semi-active</td>
<td>OLC</td>
<td>(u)</td>
<td>(-u \cdot (\dot{z}<em>s - \dot{z}</em>{us}))</td>
<td>(0 \leq u(t) \leq 3.72 \times 10^{-4})</td>
<td>27634</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>(1)</td>
<td>Semi-active</td>
<td>OLC</td>
<td>(u)</td>
<td>(-F(u, \dot{z}<em>s - \dot{z}</em>{us}))</td>
<td>(0 \leq u(t) \leq 1)</td>
<td>(9.58 \times 10^{-4})</td>
<td>26262</td>
</tr>
<tr>
<td>P4</td>
<td>(2)</td>
<td>Semi-active</td>
<td>CLC</td>
<td>(\Phi)</td>
<td>(-F(\Phi, \dot{z}<em>s - \dot{z}</em>{us}))</td>
<td>(0 \leq \Phi(t) \leq 2.00 \times 10^{-4})</td>
<td>12507</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3: Solutions for each of the problems outlined in Table I: (a)–(c) demonstrate the sprung mass displacement vs. road disturbance, (d)–(f) demonstrate the passenger comfort, and (g)–(i) demonstrate the resulting optimal forces and velocities at the damper location.

\(\dot{z}_0\). The road data has an IRI of 7.37, corresponding to a maintained unpaved road [38, p. 170]. The weights on road handling and passenger comfort objectives are \(r_1\) and \(r_2\).

Problem 1 (P1) is solved first to obtain the actuator force trajectory that minimizes \(\Phi_\alpha(\cdot)\) (a Stage (1) study). No assumptions are made yet on actuator structure or force bounds (unrestricted OLC). The P1 solution serves as a benchmark for maximum system performance (see its minimal objective value \(\Phi_\alpha(\cdot)\) in Table I).

The open-loop optimal actuator force trajectory obtained in P1 may be realized using electric, pneumatic, or hydraulic actuators [29], which make it an actively controlled suspension. These actuators typically have prohibitively high power requirements that hinder their widespread use in practice. An alternative is a semi-active suspension using magneto-rheological (MR) damping [39]. MR dampers can achieve comparable performance (to active suspensions) with near-zero power consumption, and are inherently BIBO stable [40]. To make an informed decision on the active/semi-active actuator selection, P2 was solved assuming an ideal semi-active actuator. This was accomplished by constraining the control force such that the energy is always dissipated in the system, and assuming that we can achieve any damper coefficient at a given velocity \(\dot{z}_s - \dot{z}_{us}\). This structures the OLC and force as \(F(u) = -u(t) \cdot (\dot{z}_s - \dot{z}_{us})\), \(u(t) \geq 0\).

The \(\Phi_\alpha(\cdot)\) value for P2 is only \(1.80 \times 10^3\) worse than for P1. Since an ideal semi-active component has similar performance to the active system and previously mentioned advantages, this actuation strategy was chosen. Selection and sizing of the specific semi-active actuator can also be guided by the P2 solution. Dampers are typically characterized by the performance in the velocity vs. force space (which is shown in Figs. 3g–3i). Using the maximum force from P2 (about 900 N) and regions of attained velocities and forces (see Fig. 3h), we can quantify force demands required of the damper to produce optimal semi-active system performance. Additionally, the sprung mass displacement and road disturbance vs. time plots for all the problem are shown in Figs. 3a–3c and sprung mass acceleration vs. time plots are shown in Figs. 3d–3f.

At this step, we select an MR damper sized using the P2 solution (specifically, a Lord 8041-1). The continuous current operation range for this damper is 0A–1A, and it has a maximum stroke of 74 mm. The damper behavior was then
characterized in the laboratory to obtain the data needed to construct a smooth surrogate model that estimates damper force as a function of damper velocity and input current: \( F(I, z_s - \dot{z}_{us}) \) (see Fig. 3i). MR damper use is challenging due to its inherently hysteretic and nonlinear dynamics. P3 seeks the optimal OLC damper input current trajectory that is within saturation bounds. The \( \Phi_s(\cdot) \) value for P3 is about 2.57× worse than P2, which can be expected since we are moving from an ideal damper to a highly structured MR damper model (constraining available force for given velocities). This concludes Stage 1 studies, yielding a specific damper architecture and knowledge of maximum system performance.

Finally, in P4, we move toward a more realizable CLC based on a full-state feedback controller with optimized gains, \( K \) (now a Stage 2 study). We assume that all the states are measurable. Specifically, mass accelerations \((\ddot{z}_s, \dot{z}_{us})\) are measured using accelerometers, displacements \((z_{us} - z_0, \dddot{z}_s - \dddot{z}_{us})\) are obtained using linear encoders, and velocities \((\dot{z}_s, \dot{z}_{us})\) are estimated by integrating acceleration trajectories. A feedback control law is determined for damper input current: \( I(t) \), and a path constraint was added to enforce damper saturation limits: \( 0 \leq K\dot{z}(t) \leq 1 \).

At this final step, the \( \Phi_s(\cdot) \) value for P4 is 2.08× worse than for P3. There may exist a better performing implementable control law that could be extracted from the P4 solution using the techniques described in Sec. III, which is left as a future task. We would then expect the \( \Phi_s(\cdot) \) value for P4 to be closer to the optimal objective for P3, but never better. A new architecture design (or different damper) would need to be selected to surpass this performance limit and attempt to arrive at the ideal performance points defined by \( \Phi_s(\cdot) \) for P1 and for P2. Lastly, for comparison, we also provide a solution to P0 in which we solve a dynamic system design optimization problem with an idealized passive linear damping coefficient as a design variable. The effective passive damping in the system however is nonlinear due to geometric nonlinearities of the system. The idealized damper (P0) exhibits slightly better performance than P3 and P4 as expected. It should be noted however that such an idealized damper can not be realized in practice.

The optimal objective function value \( \Phi_s(\cdot) \) degrades as we move from P1 to P4 (Table I). This is congruent with the intuition that as we gradually add detail and refine the constraints and structure of the problem, we increase the realism at the cost of performance degradation. However, the end result of this process is an implementable control law (with corresponding system-optimal physical design) that we know is only 9.70× worse than the maximum performance predicted by the unstructured problem P1.

V. Conclusions

In traditional co-design there exists a gap between OLC trajectories and implementable CLC. Here we proposed a framework to bridge this gap. In this framework we laid out a step-by-step approach to the design of controlled dynamic systems, wherein well-informed design and controller architecture decisions are made throughout the process, culminating in an implementable control law and optimal plant design. Proceeding with this framework necessitates novel formulation considerations to properly address the challenges associated with dynamic system design. Four linked optimization problems were solved sequentially in designing a trailing-arm type quarter car suspension, where solutions from previous problems inform the subsequent one. The results show the potential of this framework in helping systematic selection of optimal plant design variables, controller architecture, and implementable control laws.

Further work is required to bring more rigor and automation to this design process, making possible the use of co-design in design practice. In the co-design paradigm, the choice of design variables \( x_p \) directly modifies the system matrices \( A, B_1 \) and \( B_2 \), hence a deeper analysis of structural properties of dynamic systems such as stability, controllability and robustness under uncertainties is necessary. Moreover a more exhaustive review of available techniques for moving from OLC solutions to CLC in Sec. III is still required and new approaches suitable for this task need to be investigated. In addition, formalizing the trade-offs between the system performance and implementability of each of the design studies is desired. Specifically the implementability considerations of co-design solutions for nonlinear systems (for example using standard feedback linearization) need to be investigated. Lastly, the validation of our control laws will be performed using a physical reconfigurable trailing-arm suspension testbed (Fig. 4).

ACKNOWLEDGMENTS

We would like to thank the following University of Illinois students for their contributions toward the construction of the semi-active suspension testbed: Adam Cornell, Johnny Ho, Dhruv Kanwal, Danny Lohan, Kevin Lohan, Jason McDonald, and Insuck Suh.