10.5 PRESS FITS

In a **press fit** the pressure p_f is caused by the interference between the shaft and the hub. This pressure increases the radius of the hole and decreases the radius of the shaft. Section 10.2 described shaft and hub dimensions in terms of tolerance, which results in specific fits. The present section focuses on the stress and strain found in press fits and uses material developed in Sec. 10.3.2 for thick-walled cylinders.

Figure 10.9 shows a side view of interference in a press fit. There is a radial displacement of the hub δ_{rh} and a radial displacement of the shaft δ_{rs}. Figure 10.10 shows the front view of an interference fit. In Fig. 10.10(a) the cylinders are assembled with an interference fit; in Fig. 10.10(b) the hub and shaft are disassembled and the dimensions of each are clearly shown. Figure 10.10(b) also shows the interference pressure being internal for the hub and external for the shaft. The shaft is shown as hollow in order to present the most general case.

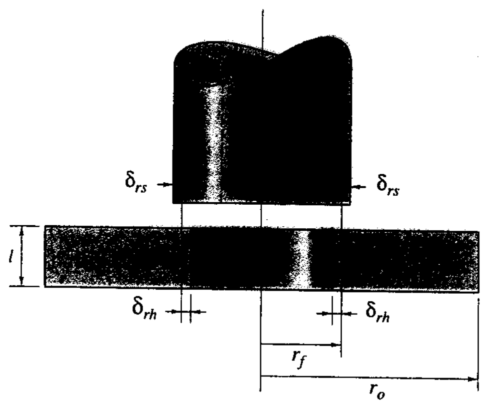

Figure 10.9 Side view showing interference in press fit of hollow shaft to hub.

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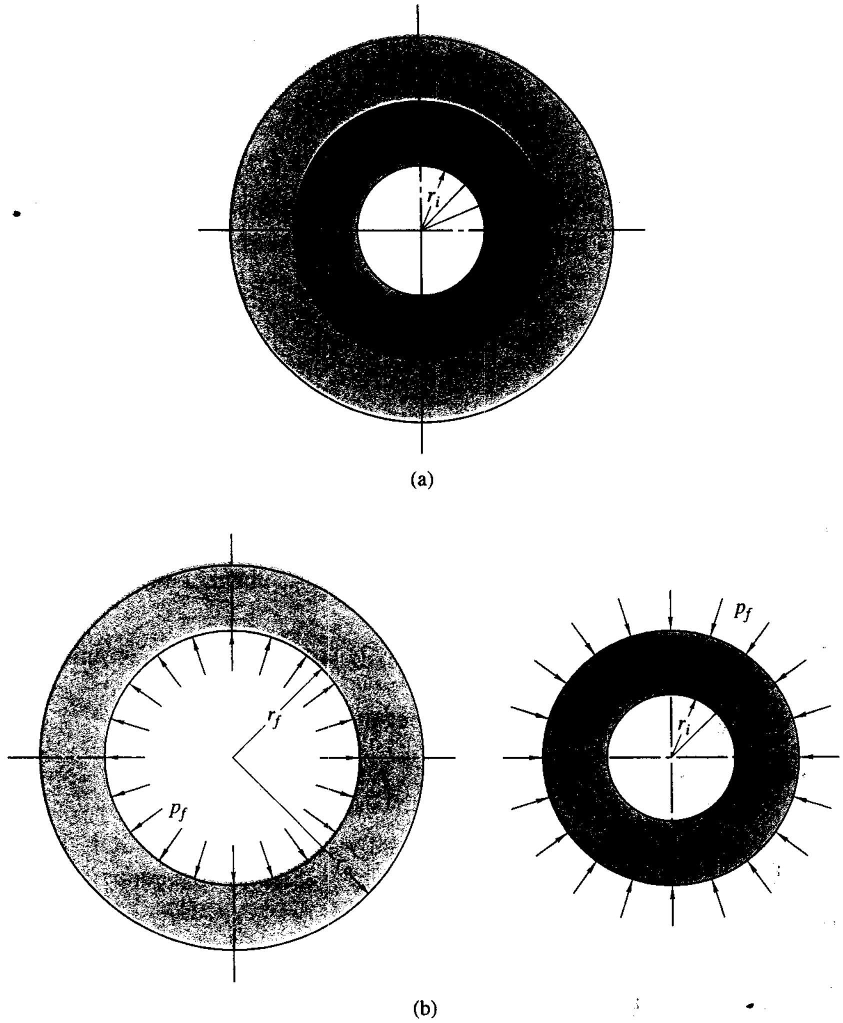

Figure 10.10 Front view showing (a) cylinder assembled with an interference fit and (b) hub and hollow shaft disassembled (also showing interference pressure).

10.5.1 Hub

By making use of Eq. (10.15) the hub displacement is

$$\delta_{rh} = \frac{r_f}{E_h}(\sigma_\theta - \nu_h \sigma_r) \tag{10.43}$$

where

E_h = modulus of elasticity of hub material, Pa
ν_h = Poisson's ratio of hub material

For internally pressurized, thick-walled cylinders, from Eqs. (10.23) and (10.24) the radial and circumferential stresses for the hub while letting $p_i = p_f$, $r = r_f$, and $r_i = r_f$, are

$$\sigma_r = \frac{p_f r_f^2 \left(1 - \frac{r_o^2}{r_f^2}\right)}{r_o^2 - r_f^2} = -p_f \tag{10.44}$$

$$\sigma_\theta = \frac{p_f r_f^2 \left(1 + \frac{r_o^2}{r_f^2}\right)}{r_o^2 - r_f^2} = \frac{p_f \left(r_o^2 + r_f^2\right)}{r_o^2 - r_f^2} \tag{10.45}$$

Substituting Eqs. (10.44) and (10.45) into Eq. (10.43) gives

$$\delta_{rh} = \frac{r_f p_f}{E_h}\left(\frac{r_o^2 + r_f^2}{r_o^2 - r_f^2} + \nu_h\right) \tag{10.46}$$

The positive sign of δ_{rh} indicates that the radial displacement of the hub is outward.

10.5.2 Shaft

By making use of Eq. (10.15) the displacement of the shaft is

$$\delta_{rs} = \frac{r_f}{E_s}(\sigma_\theta - \nu_s \sigma_r) \tag{10.47}$$

where

E_s = modulus of elasticity of shaft material, Pa
ν_s = Poisson's ratio of shaft material

The circumferential and radial stresses for externally pressurized, thick-walled cylinders can be obtained from Eqs. (10.29) and (10.30) while letting $p_o = p_f$, $r_o = r_f$, and $r = r_f$ to give

$$\sigma_r = \frac{p_f r_f^2}{r_f^2 - r_i^2}\left(\frac{r_i^2}{r_f^2} - 1\right) = -p_f \tag{10.48}$$

$$\sigma_\theta = -\frac{p_f r_f^2}{r_f^2 - r_i^2}\left(\frac{r_i^2}{r_f^2} + 1\right) = -\frac{p_f\left(r_f^2 + r_i^2\right)}{r_f^2 - r_i^2} \tag{10.49}$$

Substituting Eqs. (10.48) and (10.49) into Eq. (10.47) gives

$$\delta_{rs} = -\frac{r_f p_f}{E_s}\left(\frac{r_f^2 + r_i^2}{r_f^2 - r_i^2} - \nu_s\right) \tag{10.50}$$

Because the first term in parentheses in Eq. (10.50) is greater than 1 and the Poisson's ratio of any material is less than or equal to 0.5, δ_{rs} is negative. Therefore, shaft displacement is directed inward toward the center of the shaft.

10.5.3 Interference Fit

The total radial displacement is shown in Fig. 10.9. Recall that outward deflection (expanding the inside diameter of the hub) is positive in sign and inward deflection (reducing the outside diameter of the shaft) is negative. Thus, the total radial interference is

$$\delta_r = \delta_{rh} - \delta_{rs} = r_f p_f \left[\frac{r_o^2 + r_f^2}{E_h\left(r_o^2 - r_f^2\right)} + \frac{\nu_h}{E_h} + \frac{r_f^2 + r_i^2}{E_s\left(r_f^2 - r_i^2\right)} - \frac{\nu_s}{E_s} \right] \tag{10.51}$$

If the shaft and the hub are made of the same material, $E = E_s = E_h$ and $\nu = \nu_s = \nu_h$ and Eq. (10.51) reduces to

$$\delta_r = \frac{2 r_f^3 p_f \left(r_o^2 - r_i^2\right)}{E\left(r_o^2 - r_f^2\right)\left(r_f^2 - r_i^2\right)} \tag{10.52}$$

Furthermore, if the shaft is solid rather than hollow, $r_i = 0$ and Eq. (10.52) further reduces to

$$\delta_r = \frac{2 r_f p_f r_o^2}{E\left(r_o^2 - r_f^2\right)} \tag{10.53}$$

In these equations, if displacement is known, the interference pressure may be an unknown and these equations can readily be used.

10.5.4 Force and Torque

The maximum force P_{max} to assemble a press fit varies directly as the thickness of the outer member, the length of the outer member, the difference in diameters of the mating shaft and hub, and the coefficient of friction μ. The maximum stress is

$$\tau_{max} = p_f \mu = \frac{P_{max}}{A} = \frac{P_{max}}{2\pi r_f l} \tag{10.54}$$

The torque is

$$T = P_{max} r_f = 2\pi \mu r_f^2 l p_f \tag{10.55}$$

The axial and circumferential stresses are related to the maximum stress by

$$\tau_a^2 + \tau_c^2 = \tau_{max}^2$$

where

$$\tau_a = \frac{P_a}{2\pi r_f l} = \text{axial stress}$$

$$\tau_c = \frac{P_c}{2\pi r_f l} = \text{circumferential stress}$$

Example 10.7

GIVEN A 6-in.-diameter steel shaft is to have a press fit with a 12-in.-outside-diameter cast iron hub. Both the hub and the shaft are 10 in. long. The maximum circumferential stress is to be 5000 psi. The moduli of elasticity are 30×10^6 psi for steel and 15×10^6 psi for cast iron. The Poisson's ratio for both steel and cast iron is 0.3 and the coefficient of friction for the two materials is 0.12. That is,

$$r_f = 3 \text{ in.} \qquad r_i = 0 \qquad r_o = 6 \text{ in.}$$

$$E_s = 30\times10^6 \text{ psi} \qquad E_h = 15\times10^6 \text{ psi} \qquad \nu_s = \nu_h = 0.3$$

$$\mu = 0.12 \qquad l = 10 \text{ in.} \qquad \sigma_{\theta,\max} = 5000 \text{ psi}$$

FIND Determine

a) The interference

b) The axial force required to press the hub on the shaft

c) What torque this press fit can transmit

Solution

a) From Eq. (10.45) the interference pressure is

$$p_f = \frac{\sigma_{\theta,\max}\left(r_o^2 - r_f^2\right)}{r_o^2 + r_f^2} = \frac{5000\left(6^2 - 3^2\right)}{6^2 + 3^2} = 3000 \text{ psi}$$

From Eq. (10.51) the maximum permissible radial interference is

$$\delta_r = r_f p_f\left[\frac{r_o^2 + r_f^2}{E_h\left(r_o^2 - r_f^2\right)} + \frac{\nu_h}{E_h} + \frac{r_f^2 + r_i^2}{E_s\left(r_f^2 - r_i^2\right)} - \frac{\nu_s}{E_s}\right]$$

$$= \frac{3(3000)}{15(10^6)}\left[\frac{6^2 + 3^2}{6^2 - 3^2} + 0.3 + \frac{1}{2} - \frac{0.3}{2}\right] = 1.390\times10^{-3} \text{ in.}$$

b) From Eq. (10.54) the force required for the press fit is

$$P_{\max} = 2\pi\mu r_f l p_f = (2)(\pi)(0.12)(3)(10)(3000) = 67\ 860 \text{ lbf}$$

c) From Eq. (10.55) the torque is

$$T = P_{\max} r_f = (67\ 860)(3) = 203\ 600 \text{ lbf-in.}$$

Example 10.8

GIVEN A wheel hub is press fit on a 105-mm-diameter solid shaft. The coefficient of friction is 0.11 and the hub and shaft material is AISI 1080 steel. The hub's outer diameter is 160 mm and its width is 120 mm. The radial interference between the shaft and the hub is 65 μm (the shaft diameter is 130 μm larger than the inside diameter of the hub).

FIND The axial force necessary to dismount the hub.

Solution

Equation (10.53) gives the relationship between radial displacement and pressure.

$$\delta_r = \frac{2r_f p_f r_o^2}{E\left(r_o^2 - r_f^2\right)} = (65)\left(10^{-6}\right) = \frac{(2)(0.0525)p_f(0.080)^2}{(2.07)\left(10^{11}\right)\left[(0.080)^2 - (0.0525)^2\right]}$$

$$\therefore p_f = 72.96 \text{ MPa}$$

The axial force necessary to dismount the hub is

$$P = \mu p_f A = (0.11)(72.96)\left(10^6\right)\pi(0.105)(0.120) = 317\ 700 \text{ N} = 317.7 \text{ kN}$$

10.6 Shrink Fits

In producing a **shrink fit** it is common to heat the outer component (hub) in order to expand it beyond the interference and then slip it over the inner component (shaft). Cooling will then contract the outer component. Temperature change produces a strain, called thermal strain, even in the absence of stress. Although thermal strain is not exactly linear with temperature change, for temperature changes of 100 or 200°F the actual variation can be closely described by a linear approximation. According to this linear relationship the temperature difference through which the outer component (hub) must be heated to obtain the required expansion over the undeformed solid shaft is

$$\Delta t_m = \frac{\delta_r}{\bar{a} r_f} \qquad \textbf{(10.56)}$$

where

$\bar{a}$ = coefficient of linear thermal expansion (see Table 3.5 and Fig. 3.14)

Equation (10.56) can be expressed in terms of radial strain as

$$\varepsilon_r = \frac{\delta_r}{r_f} = \bar{a}\ \Delta t_m \qquad \textbf{(10.57)}$$

The deformation is

$$\delta_r = \varepsilon_r r_f = \bar{a}\ \Delta t_m r_f \qquad \textbf{(10.58)}$$

These equations are valid not only for shrink fits of shaft and hub but for a wide range of thermal problems, as illustrated by the following example.

The strain due to a temperature change may be added algebraically to a local strain by using the *principle of superposition.* The principle states that stresses and strains (at a point on a given plane) due to different loads may be computed separately and added algebraically provided that the sum does not exceed the proportionality limit of the material and that the structure remains stable. The method of superposition for different types of loading was covered in Sec. 5.4. Thus, the normal strain due to normal load and temperature effects is

$$\varepsilon = \varepsilon_\sigma + \varepsilon_{t_m} \tag{10.59}$$

where

ε_σ = strain due to normal stress

ε_{t_m} = strain due to temperature change

Thus, the general (triaxial stress state) stress-strain relationship developed in Appendix B, Eq. (B.44), may be expressed while considering thermal strain as

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \bar{a}\,\Delta t_m$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] + \bar{a}\,\Delta t_m$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \bar{a}\,\Delta t_m \tag{10.60}$$

Example 10.9

GIVEN A 10-in.-long steel tube has a cross-sectional area of 1 in.^{2} that expands by 0.008 in. from a stress-free condition at 80° F when the tube is heated to 480°F.

FIND The load and stress acting on the steel tube.

Solution

From Table 3.5 for steel alloy $\bar{a} = 6.1 \times 10^{-6}$ °F. From Eq. (10.57)

$$\varepsilon = \bar{a}\,\Delta t_m = (6.1)(10^{-6})(400) = 2.44 \times 10^{-3}$$

$$\Delta l = l\varepsilon = (10)(2.44)(10^{-3}) = 0.0244 \text{ in.}$$

Because the measured expansion was only 0.008 in., the constraint due to compressive normal loading must apply a force sufficient to deflect the tube axially by the following amount:

$$\delta = \frac{Pl}{AE} \rightarrow P = \frac{AE\delta}{l}$$

From Table A.1 the modulus of elasticity is 30×10^6 psi. Therefore,

$$P = \frac{(1)(30)(10^6)(0.0164)}{10} = 4.92 \times 10^4 \text{ lbf} = 49\ 200 \text{ lbf}$$

This then is the compressive, normal, axial load being exerted on the steel tube.

GIVEN A block of aluminum alloy is placed between two rigid jaws of a clamp, and the jaws are brought up snug. The temperature of the entire assembly is raised 250° C in an oven. The cross-sectional areas are 65 mm^2 for the block and 160 mm^2 for the stainless steel screws.

Example 10.10

FIND How much stress is induced in the screws and the block?

Solution

Figure 10.11 shows the block-and-screw assembly as well as the forces on these components. From force equilibrium

$$P_a = 2P_s \tag{a}$$

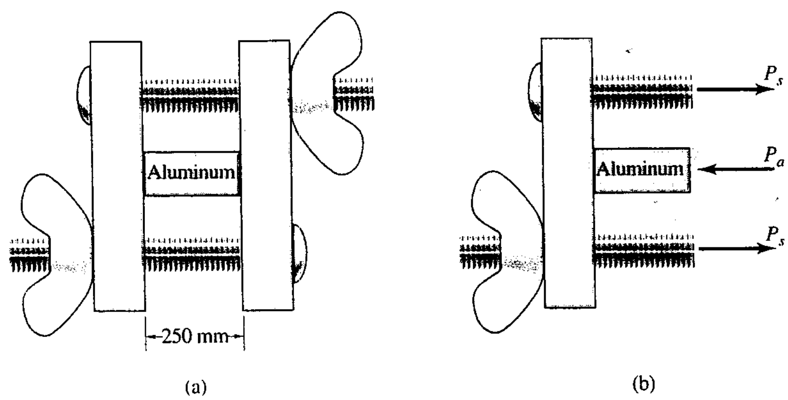

Figure 10.11 Block placed between two rigid jaws of clamp (a) and forces acting (b).

Here subscript a refers to the aluminum block and subscript s refers to the stainless steel screws. Compatibility requires that the length changes of the block and the screws be the same, or

$$\delta_a = \delta_s \tag{b}$$

Thermal expansion will induce an axial force as shown in Fig. 10.11. The displacements of the block and screws are

$$\delta_a = \bar{a}_a l \,\Delta t_m - \frac{P_a l}{E_a A_a} \tag{c}$$

$$\delta_s = \bar{a}_s l \,\Delta t_m + \frac{P_s l}{E_s A_s} \tag{d}$$

Substituting Eqs. (a), (c), and (d) into Eq. (b) gives

$$P_s = \frac{\Delta t_m(\bar{a}_a - \bar{a}_s)}{\dfrac{1}{E_s A_s} + \dfrac{2}{E_a A_a}} \tag{e}$$

From Tables 3.2 and 3.5

$$E_a = 70\ \text{GPa} \qquad E_s = 193\ \text{GPa}$$

$$\bar{a}_a = 24 \times 10^{-6}\ °\text{C} \qquad \bar{a}_s = 17 \times 10^{-6}\ °\text{C}$$

We are given that $A_a = 65\ \text{mm}^2$ and $A_s = 160\ \text{mm}^2$. Substituting these values and the above into Eq. (e) gives the force acting on each screw as

$$P_s = \frac{(250)(24-17)(10^{-6})}{\dfrac{1}{(193)(10^9)(160)(10^{-6})} + \dfrac{2}{(70)(10^9)(65)(10^{-6})}} = 3708\ \text{N}$$

The force acting on the aluminum block is

$$P_a = 2P_s = 7416\ \text{N}$$

The axial stresses of the block and screw are

$$\sigma_a = \frac{P_a}{A_a} = -\frac{7416}{(65)(10^{-6})}\ \text{Pa} = -114.1\ \text{MPa}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{3708}{(160)(10^{-6})}\ \text{Pa} = 23.18\ \text{MPa}$$

The stress acting on the aluminum block is compressive, and that acting on the screws is tensile.
