Stability Homework

For the fly-ball governor below with fly-ball masses m and collar mass M, massless links, frictionless hinges, and frictionless collar motion, the equation of motion for constant rotational rate ω is:

$$(2m + 4Ms_\theta^2)L^2\ddot{\theta} + 4ML^2 s_\theta c_\theta \dot{\theta}^2 - 2mL^2\omega^2 s_\theta c_\theta + 2(m + M)gLs_\theta = 0$$

where $s_\theta = \sin(\theta)$ and $c_\theta = \cos(\theta)$, and the dynamic equilibrium steady state position is:

$$c_{\theta_e} = \frac{(m + M)g}{mL\omega^2}$$

Use perturbation analysis to determine the range of values for ω which exhibit stable dynamic equilibrium states.