Particle Kinematics in Various Standard Coordinate Systems:

Cartesian:

$$\bar{v} = \frac{d\bar{r}}{dt} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

Cylindrical:

$$\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z$$

$$\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta + \ddot{z}\hat{e}_z$$

Tangential/Normal:

$$\bar{v} = \dot{s}\hat{e}_t$$

$$\bar{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

Spherical:

$$\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + r\dot{\phi}\sin\theta\hat{e}_\phi$$

$$\bar{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta\right)\hat{e}_\theta + \left(r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta\right)\hat{e}_\phi$$

Constant Acceleration Relations:

$a = \text{constant}$ $\quad v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\quad v^2 = v_0^2 + 2a(x - x_0)$

Particle Kinematics in General Moving Coordinate System:

$$\bar{v} = \dot{\bar{R}} + \bar{\omega}\times\bar{\rho} + \left(\dot{\bar{\rho}}\right)_{rel}$$

$$\bar{a} = \ddot{\bar{R}} + \bar{\alpha}\times\bar{\rho} + \bar{\omega}\times\bar{\omega}\times\bar{\rho} + 2\bar{\omega}\times\left(\dot{\bar{\rho}}\right)_{rel} + \left(\ddot{\bar{\rho}}\right)_{rel}$$

Particle Dynamics:

$\bar{F} = \dot{\bar{p}} = m\bar{a}$ $\quad a_x = \frac{dv_x}{dt} = \frac{dx}{dt}\frac{dv_x}{dx} = v_x\frac{dv_x}{dx}$

Particle Linear Impulse and Momentum:

$\bar{p} = m\bar{v}$ $\quad \hat{F} = \int_0^t \bar{F}dt = p_t - p_0$

Particle Angular Momentum:

$\bar{H}_o = \bar{r}\times\bar{p} = \bar{r}\times m\bar{v}$ $\quad \bar{M}_o = \bar{r}\times\bar{F} = \dot{\bar{H}}_o$ $\quad \bar{H}_p = \bar{\rho}\times m\bar{v}$ $\quad \bar{M}_p = \dot{\bar{H}}_p + \bar{\rho}\times m\bar{a}_p$

o: fixed (inertial) point $\quad$ p: moving (accelerating) point

Particle Work and Energy:

$W_{AB} = \int_A^B \bar{F}\cdot d\bar{r} = \int_A^B F_s ds = \int_A^B F_s(t)v_s(t)dt$ $\quad P = \frac{dW}{dt} = \bar{F}\cdot\bar{v} = F_s v_s$

$T = \frac{1}{2}mv^2$ $\quad V_{gravity} = mgh$ $\quad V_{spring} = \frac{1}{2}kx^2$ $\quad W_{friction} = -\int_A^B F_\mu ds = -\int_A^B \mu N ds$

$$V_A + T_A = V_B + T_B - W_{AB_{nc}}$$

System of Particles:

$\bar{F}_{ext} = Ma_{cm}$ $\quad M = \sum_{i=1}^n m_i$ $\quad \bar{p} = \sum_{i=1}^n \bar{p}_i = \sum_{i=1}^n m_i\bar{v}_i = M\bar{v}_{cm}$ $\quad T = \sum_{i=1}^n T_i = \sum_{i=1}^n \frac{1}{2}m_i v_i^2 = \frac{1}{2}Mv_{cm}^2 + \sum_{i=1}^n \frac{1}{2}m_i\dot{\rho}_i^2$

$\bar{H}_o = \sum_{i=1}^n \bar{H}_{o_i} = \sum_{i=1}^n \bar{r}_i\times m_i\bar{v}_i$ $\quad \bar{M}_o = \sum_{i=1}^n \bar{r}_i\times\bar{F}_i = \dot{\bar{H}}_o$ $\quad \bar{H}_c = \sum_{i=1}^n \bar{\rho}_i\times m_i\dot{\bar{\rho}}_i$ $\quad \bar{M}_c = \sum_{i=1}^n \bar{\rho}_i\times\bar{F}_i = \dot{\bar{H}}_c$

$\bar{H}_p = \sum_{i=1}^n \bar{\rho}_i\times m_i\dot{\bar{\rho}}_i$ $\quad \bar{M}_p = \sum_{i=1}^n \bar{\rho}_i\times\bar{F}_i = \dot{\bar{H}}_p + \bar{r}_{cm}\times M\bar{a}_p$

o: fixed (inertial) point, p: moving (accelerating) point, cm: center of mass

Conservation Laws:
momentum: if $\vec{F}_{ext} = 0$, $\vec{p}$ = constant
angular momentum: if $\vec{M} = 0$, $\vec{H}$ = constant
energy: if no nonconservative forces, $T + V$ = constant

Particle Collisions (elastic: e=1, plastic: e=0):

$$m_1 v_{1_n} + m_2 v_{2_n} = m_1 v'_{1_n} + m_2 v'_{2_n}$$

$$v'_{1_t} = v_{1_t} \qquad v'_{2_t} = v_{2_t}$$

$$v'_{2_n} - v'_{1_n} = e\left(v_{1_n} - v_{2_n}\right)$$

if object 2 is stationary:

$$v'_{1_t} = v_{1_t} \qquad v'_{1_n} = -e v_{1_n}$$

if friction:

$$\hat{F}_n = m_1\left(v_{1_n} - v'_{1_n}\right) \qquad v'_{1_t} = \max\left[\left(v_{1_t} - \frac{\mu \hat{F}_n}{m_1}\right),\ 0\right]$$

System Gaining/Losing Mass:

$$\vec{F} = m\frac{d\vec{v}}{dt} + \frac{dm_{out}}{dt}\vec{v}_{leave} - \frac{dm_{in}}{dt}\vec{v}_{enter} \qquad \vec{v}_{leave} = \vec{v}_{out} - \vec{v} \qquad \vec{v}_{enter} = \vec{v}_{in} - \vec{v}$$

Rigid Body:

$$\vec{H} = [I]\vec{\omega} = (I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z)\hat{i} + (I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z)\hat{j} + (I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z)\hat{k}$$

$$T = \frac{1}{2}mv^2 + \frac{1}{2}I_{xx}\omega_x^2 + \frac{1}{2}I_{yy}\omega_y^2 + \frac{1}{2}I_{zz}\omega_z^2 + I_{xy}\omega_x\omega_y + I_{xz}\omega_x\omega_z + I_{yz}\omega_y\omega_z$$

$$\vec{F} = m\vec{a} = m\left(\dot{\vec{v}}_r + \vec{\omega}\times\vec{v}\right) \qquad \vec{M} = \dot{\vec{H}} = \dot{\vec{H}}_r + \vec{\omega}\times\vec{H}$$

$$F_x = m\left(\dot{v}_x + v_z\omega_y - v_y\omega_z\right) \qquad F_y = m\left(\dot{v}_y + v_x\omega_z - v_z\omega_x\right) \qquad F_z = m\left(\dot{v}_z + v_y\omega_x - v_x\omega_y\right)$$

$$M_x = I_{xx}\dot{\omega}_x + \left(I_{zz} - I_{yy}\right)\omega_y\omega_z \quad M_y = I_{yy}\dot{\omega}_y + \left(I_{xx} - I_{zz}\right)\omega_x\omega_z \quad M_z = I_{zz}\dot{\omega}_z + \left(I_{yy} - I_{xx}\right)\omega_y\omega_x$$

Planar Motion:

$$H = I\omega \qquad M = I\alpha \qquad T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad I_P = I_o + md^2$$

Lagrange's Equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \sum_{j=1}^{m}\lambda_j a_{ji} + Q'_i$$

$$L = T - V \qquad Q'_i = \sum_{n}^{\text{nc forces}}\frac{\partial}{\partial q_i}\left(\vec{F}_n\cdot\vec{r}_n\right) = \sum_{n}^{\text{nc forces}} F_n\frac{\partial x_n}{\partial q_i} + \sum_{n}^{\text{nc torques}} T_n\frac{\partial \theta_n}{\partial q_i} \qquad \sum_{i=1}^{n} a_{ji}\dot{q}_i + a_{jt} = 0$$