Euler Angle Angular Velocity Derivation

$$R_x(\alpha) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad R_y(\alpha) := \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \quad R_z(\alpha) := \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Euler Angle Rotations: $R_z(\psi),\ R_y(\theta),\ R_x(\phi)$

Note: $R^{-1} = R^{\mathrm{T}}$

$$\omega_\phi(\phi_{dot}) := \phi_{dot} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \phi_{dot} \\ 0 \\ 0 \end{bmatrix}$$

$$\omega_\theta(\theta_{dot}, \phi) := R_x(\phi)^{\mathrm{T}} \cdot R_y(\theta)^{\mathrm{T}} \cdot \begin{bmatrix} 0 \\ \theta_{dot} \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ \theta_{dot} \cdot \cos(\phi) \\ -(\theta_{dot} \cdot \sin(\phi)) \end{bmatrix}$$

$$\omega_\psi(\psi_{dot}, \theta, \phi, \psi) := R_x(\phi)^{\mathrm{T}} \cdot R_y(\theta)^{\mathrm{T}} \cdot R_z(\psi)^{\mathrm{T}} \cdot \begin{bmatrix} 0 \\ 0 \\ \psi_{dot} \end{bmatrix} \rightarrow \begin{bmatrix} -(\psi_{dot} \cdot \sin(\theta)) \\ \psi_{dot} \cdot \cos(\theta) \cdot \sin(\phi) \\ \psi_{dot} \cdot \cos(\phi) \cdot \cos(\theta) \end{bmatrix}$$

total angular velocity

$$\omega_\phi(\phi_{dot}) + \omega_\theta(\theta_{dot}, \phi) + \omega_\psi(\psi_{dot}, \theta, \phi, \psi) \rightarrow \begin{bmatrix} \phi_{dot} - \psi_{dot} \cdot \sin(\theta) \\ \theta_{dot} \cdot \cos(\phi) + \psi_{dot} \cdot \cos(\theta) \cdot \sin(\phi) \\ \psi_{dot} \cdot \cos(\phi) \cdot \cos(\theta) - \theta_{dot} \cdot \sin(\phi) \end{bmatrix}$$