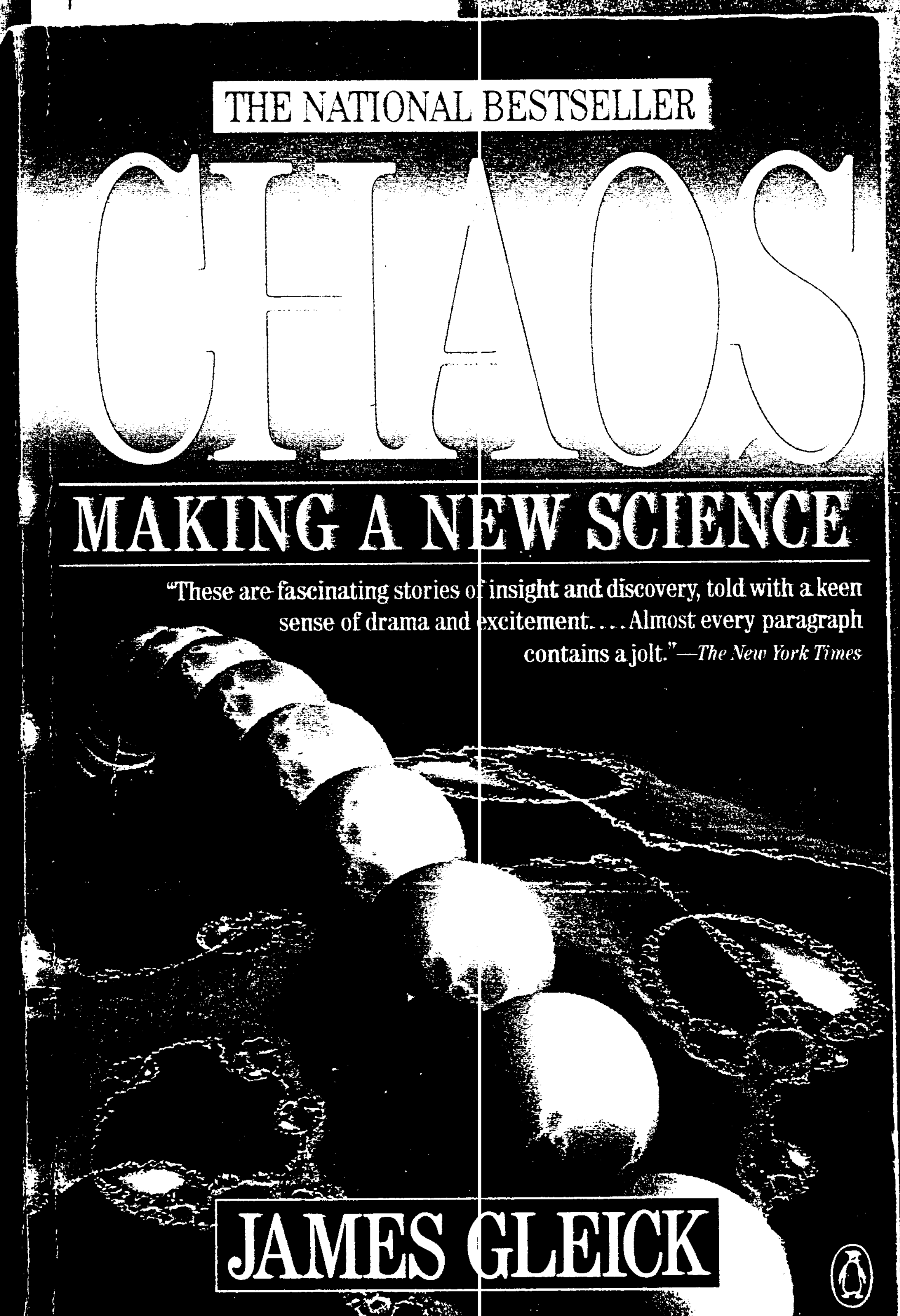

THE NATIONAL BESTSELLER
CHAOS
MAKING A NEW SCIENCE
"These are fascinating stories of insight and discovery, told with a keen sense of drama and excitement. . . . Almost every paragraph contains a jolt."—The New York Times
JAMES GLEICK

CHAOS NOTES

from

CHAOS
by
JAMES GLEICK

1984, Los Alamos Labs
Center for Nonlinear Studies established
"Feigenbaum was working on a problem that was deep: chaos"
"Where chaos begins, classical science stops"

"The new science has spawned its own language, an elegant shop talk of fractals and bifurcations, intermittencies and periodicities, folded-towel diffeomorphisms and smooth noodle maps."

"Believers in chaos...feel that they are turning back a trend in science toward reductionism, the analysis of systems in terms of their constituent parts...they are looking for the whole."

"Relativity eliminated the Newtonian illusion of absolute space and time; quantum theory eliminated the Newtonian dream of a controllable measurement process; and chaos eliminates the Laplacian fantasy of deterministic predictability."

"in a universe ruled by entropy, drawing inexorably toward greater and greater disorder, how does order arise? At the same time, objects of everyday experience like fluids and mechanical systems...seem so basic and ordinary that physicists had a natural tendency to assume they were well understood. It was not so."

"The modern study of chaos began with the creeping realization in the 1960s that quite simple mathematical equations could model systems every bit as violent as a waterfall. Tiny differences in input could quickly become overwhelming differences in output – a phenomenon given the name "sensitive dependence of initial conditions. In weather [this is known as]...the Butterfly Effect – the notion that a butterfly stirring the air today in Peking can transform storm systems next month in New York."

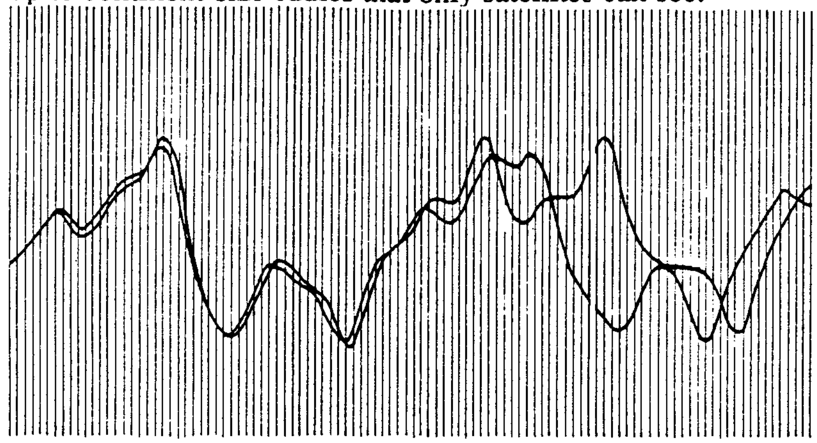
quote from Richard P. Feynman: "Physicists like to think that all you have to do is say, these are the conditions, now what happens next?"

"Laplace...caught the Newtonian fever like no one else: ...'Such an intelligence would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.' In these days of Einstein's relativity and Heisenberg's uncertainty, Laplace seems almost buffoon-like in his optimism, but much of modern science has pursued his dream."

typical Laplace statement conjured by Alciatore: "Give me the present speed and velocity of every particle in the universe and I will predict the future"

"Given an approximate knowledge of a system's initial conditions and an understanding of natural law, one can calculate the approximate behavior of the system...'The basic idea of Western science is that you don't have to take into account the falling of a leaf on some planet in another galaxy when you're trying to account for the motion of a billiard ball on a pool table on earth. Very small influences can be neglected."

Edward Lorenz (MIT, winter, 1961) "decided that long-range weather forecasting must be doomed....'any physical system that behaved nonperiodically would be unpredictable.'...The Butterfly Effect was the reason. For small pieces of weather – and to a global forecaster, small can mean thunderstorms and blizzards – any prediction deteriorates rapidly. Errors and uncertainties multiply, cascading upward through a chain of turbulent features, from dust devils and squalls up to continent-size eddies that only satellites can see."



Edward N. Lorenz / Adolph E. Brotman

How TWO WEATHER PATTERNS DIVERGE. From nearly the same starting point, Edward Lorenz saw his computer weather produce patterns that grew farther and farther apart until all resemblance disappeared. (From Lorenz's 1961 printouts.)

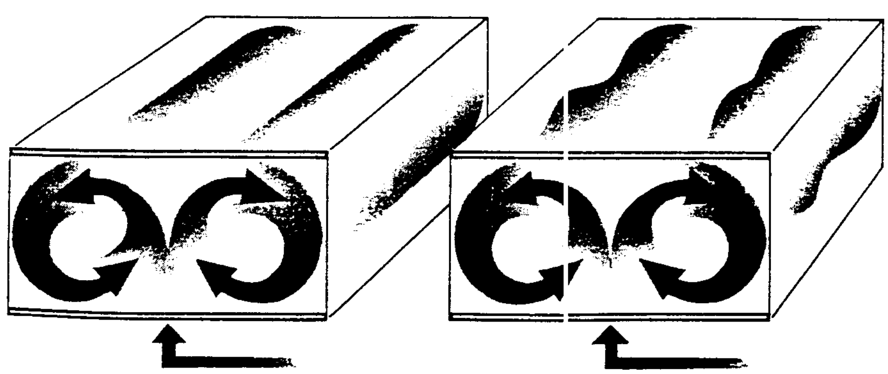
"sensitive dependence on initial conditions[:]...a chain of events can have a point of crisis that could magnify small changes. But chaos meant that such points were everywhere"

example in folklore:

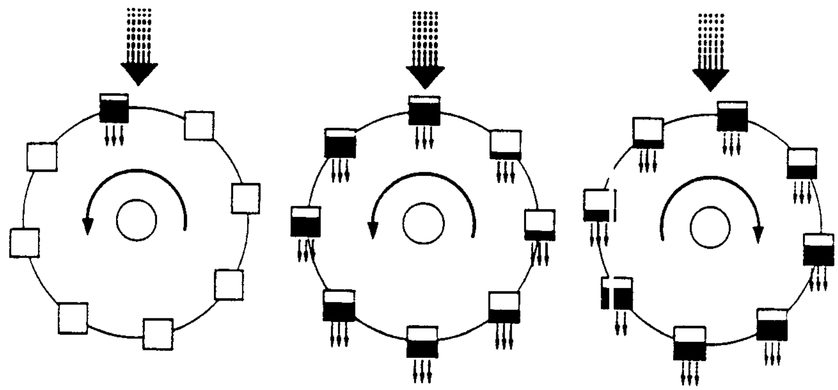
'For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse , the rider was lost;
For want of a rider , the battle was lost;
For want of a battle , the kingdom was lost!'

"The world would be a different place – and science would not need chaos – if only the Navier-Stokes equation did not contain the demon of nonlinearity."

Adolph E. Brotman



A ROLLING FLUID. When a liquid or gas is heated from below, the fluid tends to organize itself into cylindrical rolls (*left*). Hot fluid rises on one side, loses heat, and descends on the other side—the process of convection. When the heat is turned up further (*right*) an instability sets in, and the rolls develop a wobble that moves back and forth along the length of the cylinders. At even higher temperatures, the flow becomes wild and turbulent.



Adolph E. Brotman

THE LORENZIAN WATERWHEEL. The first, famous chaotic system discovered by Edward Lorenz corresponds exactly to a mechanical device: a waterwheel. This simple device proves capable of surprisingly complicated behavior.

The rotation of the waterwheel shares some of the properties of the rotating cylinders of fluid in the process of convection. The waterwheel is like a slice through the cylinder. Both systems are driven steadily—by water or by heat—and both dissipate energy. The fluid loses heat; the buckets lose water. In both systems, the long-term behavior depends on how hard the driving energy is.

Water pours in from the top at a steady rate. If the flow of water in the waterwheel is slow, the top bucket never fills up enough to overcome friction, and the wheel never starts turning. (Similarly, in a fluid, if the heat is too low to overcome viscosity, it will not set the fluid in motion.)

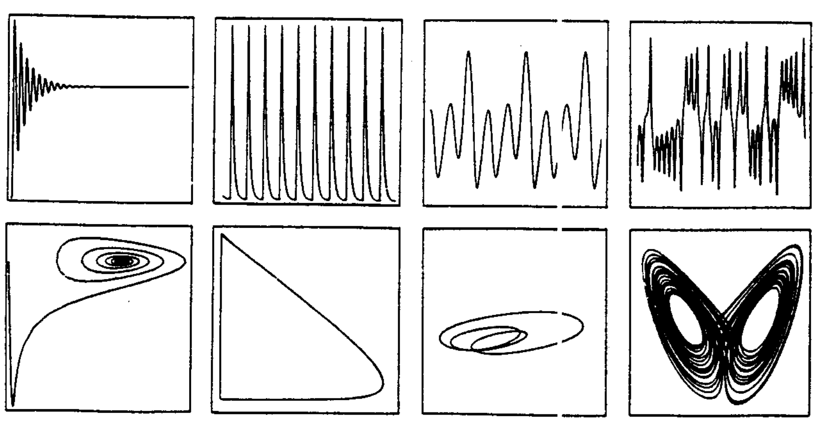
If the flow is faster, the weight of the top bucket sets the wheel in motion (*left*). The waterwheel can settle into a rotation that continues at a steady rate (*center*).

But if the flow is faster still (*right*), the spin can become chaotic, because of nonlinear effects built into the system. As buckets pass under the flowing water, how much they fill depends on the speed of spin. If the wheel is spinning rapidly, the buckets have little time to fill up. (Similarly, fluid in a fast-turning convection roll has little time to absorb heat.) Also, if the wheel is spinning rapidly, buckets can start up the other side before they have time to empty. As a result, heavy buckets on the side moving upward can cause the spin to slow down and then reverse.

In fact, Lorenz discovered, over long periods, the spin can reverse itself many times, never settling down to a steady rate and never repeating itself in any predictable pattern.

"Student [learn]...that nonlinear systems were usually unsolvable, which was true, and that they tended to be exceptions – which was not true"

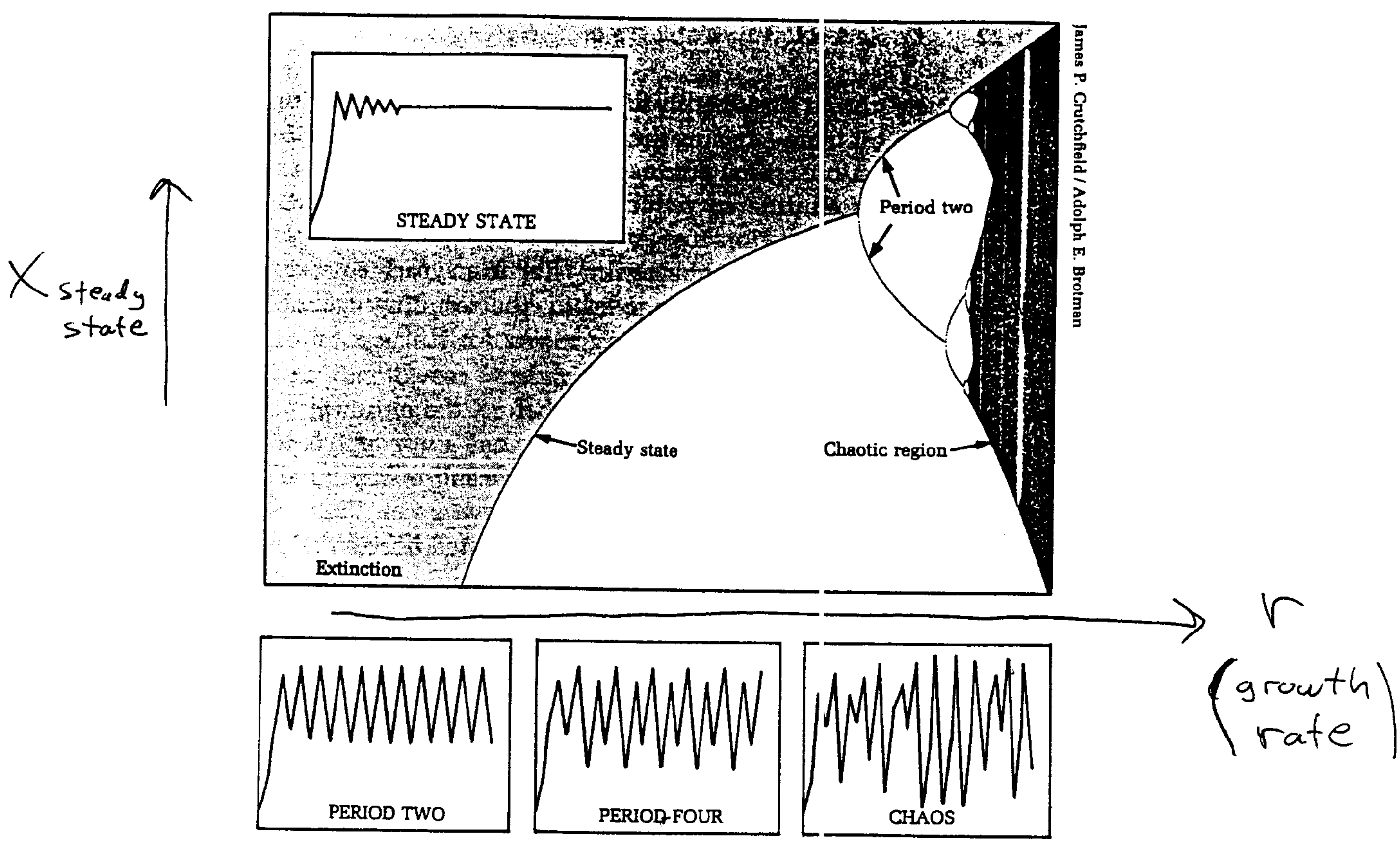
"Traditionally, a dynamicist would believe that to write down a system's equations is to understand the system...But because of the little bits of nonlinearity in these equations, a dynamicist would find himself helpless to answer the easiest practical questions about the future of the system...simulation brings its own problem: the tiny imprecision built into each calculation rapidly takes over, because this is a system with sensitive dependence on initial conditions.



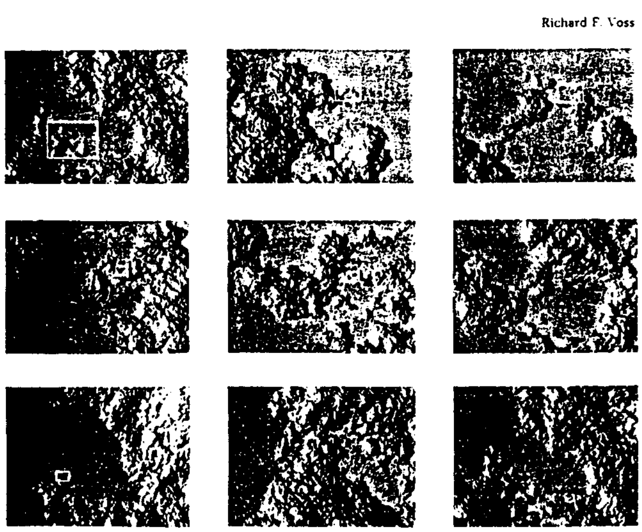
Irving R. Epstein

Making portraits in phase space. Traditional time series (*above*) and trajectories in phase space (*below*) are two ways of displaying the same data and gaining a picture of a system's long-term behavior. The first system (*left*) converges on a steady state—a point in phase space. The second repeats itself periodically, forming a cyclical orbit. The third repeats itself in a more complex waltz rhythm, a cycle with "period three." The fourth is chaotic.

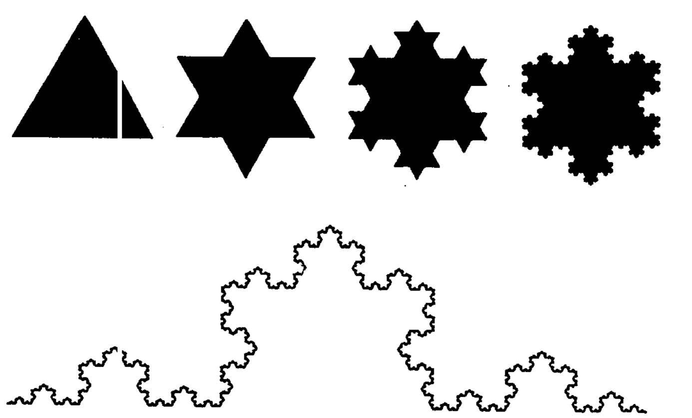
Example of bifurcation instability: $x_{next} = r\,x(1-x)$



PERIOD-DOUBLINGS AND CHAOS. Instead of using individual diagrams to show the behavior of populations with different degrees of fertility, Robert May and other scientists used a "bifurcation diagram" to assemble all the information into a single picture.



A FRACTAL SHORE. A computer-generated coastline: the details are random, but the fractal dimension is constant, so the degree of roughness or irregularity looks the same no matter how much the image is magnified.



THE KOCH SNOWFLAKE. "A rough but vigorous model of a coastline," in Mandelbrot's words. To construct a Koch curve, begin with a triangle with sides of length 1. At the middle of each side, add a new triangle one-third the size; and so on. The length of the boundary is $3 \times 4/3 \times 4/3 \times 4/3 \ldots$ —infinity. Yet the area remains less than the area of a circle drawn around the original triangle. Thus an infinitely long line surrounds a finite area.