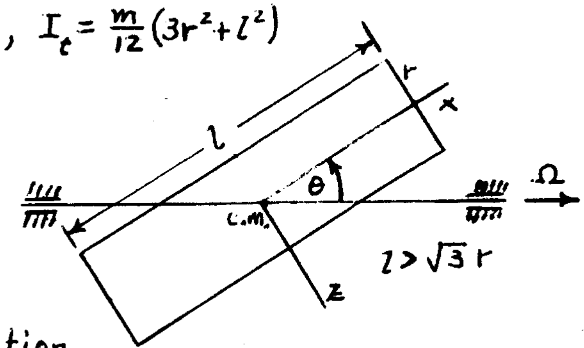
8-5. $I_a = \frac{1}{2}mr^2$, $I_t = \frac{m}{12}(3r^2+l^2)$

$\omega_x = \Omega \cos\theta$

$\omega_y = \dot\theta$

$\omega_z = \Omega \sin\theta$



Use Euler's equation.

$$I_{yy}\dot\omega_y + (I_{xx} - I_{zz})\omega_x\omega_z = M_y = 0$$

This leads to

$$\frac{m}{12}(3r^2+l^2)\ddot\theta + \frac{m}{12}(3r^2-l^2)\Omega^2 \sin\theta\cos\theta = 0$$

The perturbation equation about $\theta = \frac{\pi}{2}$ is

$$\frac{m}{12}(3r^2+l^2)\delta\ddot\theta - \frac{m}{12}(3r^2-l^2)\Omega^2\delta\theta = 0$$

which results in the circular frequency $\omega = \sqrt{\frac{l^2-3r^2}{l^2+3r^2}}\,\Omega$

(b) Let $\ddot\theta = \dot\theta \frac{d\dot\theta}{d\theta}$ and integrate the differential equation.

$$\int_0^{\dot\theta} \dot\theta\, d\dot\theta = \frac{l^2-3r^2}{l^2+3r^2}\int_0^{\theta} \Omega^2 \sin\theta\cos\theta\, d\theta$$

or

$$\frac{1}{2}\dot\theta^2 = \frac{1}{2}\left(\frac{l^2-3r^2}{l^2+3r^2}\right)\Omega^2 \sin^2\theta$$

At $\theta = \frac{\pi}{2}$, $\quad \dot\theta^2 = \left(\frac{l^2-3r^2}{l^2+3r^2}\right)\Omega^2$

or

$$\ddot\theta = \sqrt{\frac{l^2-3r^2}{l^2+3r^2}}\,\Omega$$

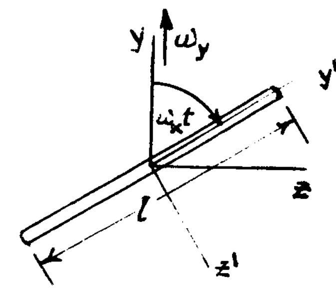
8-7. $x'y'z'$ is a principal axis system.

Use the Euler equations

$$I_{x'x'}\dot\omega_{x'} + (I_{z'z'} - I_{y'y'})\omega_{y'}\omega_{z'} = M_{x'}$$

where $\omega_{x'} = \omega_x = \text{const}$ so $\dot\omega_x = 0$

$\omega_{y'} = \omega_y \cos\omega_x t$, $\quad \omega_{z'} = -\omega_y \sin\omega_x t$



8-7. (cont'd.) $\quad I_{x'x'} = I_{z'z'} = \frac{ml^2}{12}$, $\quad I_{y'y'} = 0$

Hence we obtain $\quad M_x = M_{x'} = -\frac{ml^2}{12}\omega_y^2 \sin\omega_x t\cos\omega_x t$

$$= -\frac{ml^2}{24}\omega_y^2 \sin 2\omega_x t$$

Also,

$$M_{y'} = I_{y'y'}\dot\omega_{y'} + (I_{x'x'} - I_{z'z'})\omega_{z'}\omega_{x'} = 0$$

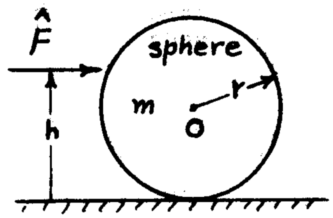
$$M_{z'} = I_{z'z'}\dot\omega_{z'} + (I_{y'y'} - I_{x'x'})\omega_{x'}\omega_{y'}$$

$$= -\frac{ml^2}{12}\omega_x\omega_y\cos\omega_x t - \frac{ml^2}{12}\omega_x\omega_y\cos\omega_x t = -\frac{ml^2}{6}\omega_x\omega_y\cos\omega_x t$$

Now $\quad M_y = M_{y'}\cos\omega_x t - M_{z'}\sin\omega_x t = \frac{ml^2}{6}\omega_x\omega_y \sin\omega_x t\cos\omega_x t$

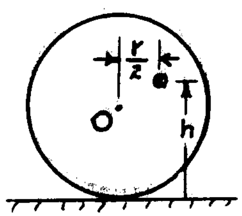
or $\quad M_y = \frac{ml^2}{12}\omega_x\omega_y \sin 2\omega_x t$

8-9. For no slipping, $\hat{F}$ must pass through the center of percussion relative to the contact point.



$$(h-r)r = \frac{2}{5}r^2, \qquad \underline{h = \frac{7}{5}r}$$

(b) Use angular impulse and momentum about the c.m. The horizontal component of $\bar{\omega}$ is



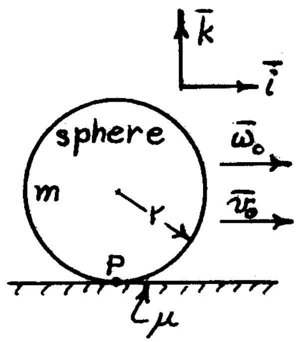
$$\frac{2}{5}r\hat{F} = \frac{2}{5}mr^2\omega_h \quad \text{or} \quad \omega_h = \frac{\hat{F}}{mr}$$

The vertical component of $\bar{\omega}$ is

$$\frac{r}{2}\hat{F} = \frac{2}{5}mr^2\omega_v \quad \text{or} \quad \omega_v = \frac{5\hat{F}}{4mr}$$

Then $\omega = \sqrt{\omega_h^2 + \omega_v^2} = \sqrt{1+\frac{25}{16}}\,\frac{\hat{F}}{mr} = \underline{1.6008\,\frac{\hat{F}}{mr}}$

8-10. In general, the velocity of the contact point P on the ball is



$$\bar{v}_p = \bar{v} + \bar{\omega}\times(-r\bar{k})$$

At $t=0$, we have

$$\bar{v}_p = v_o\bar{i} + r\omega_o\bar{j}$$

Friction force $\bar{F}_f = -\mu mg\,\frac{\bar{v}_p}{v_p}$

Acceleration $\dot{\bar{v}} = \frac{\bar{F}_f}{m} = \underline{-\mu g\,\frac{v_o\bar{i}+r\omega_o\bar{j}}{\sqrt{v_o^2+r^2\omega_o^2}}}$

Friction moment $\bar{M}_f = -r\bar{k}\times\bar{F}_f = \mu mgr\,\frac{v_o\bar{j}-r\omega_o\bar{i}}{\sqrt{v_o^2+r^2\omega_o^2}}$

Then $\dot{\bar{\omega}} = \frac{\bar{M}_f}{\frac{2}{5}mr^2} = \underline{\frac{5\mu g\,(-r\omega_o\bar{i}+v_o\bar{j})}{2r\sqrt{v_o^2+r^2\omega_o^2}}}$

8-10. (cont'd.) (b) In general, we see that

$$\dot{\bar{v}}_p = \dot{\bar{v}} + \dot{\bar{\omega}}\times(-r\bar{k}) = -\mu g\frac{\bar{v}_p}{v_p} + \frac{(-r\bar{k}\times\bar{F}_f)\times(-r\bar{k})}{I}$$

$$= -\mu g\frac{\bar{v}_p}{v_p} + \frac{5}{2mr^2}(r^2\bar{F}_f) = \frac{-7}{2}\mu g\,\frac{\bar{v}_p}{v_p}$$

Since $\dot{\bar{v}}_p$ is opposite in direction to $\bar{v}_p$, the direction of $\bar{v}_p$ will not change, and therefore the direction of $\bar{F}_f$ will be unchanged as long as sliding continues.

(c) $|\dot{\bar{v}}_p| = \frac{7}{2}\mu g = \text{const}$

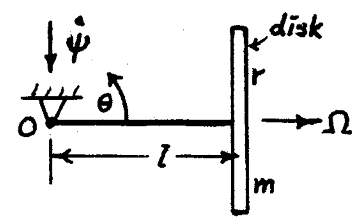
Time to stop sliding is $t_s = \frac{v_p(0)}{|\dot{\bar{v}}_p|} = \frac{\sqrt{v_o^2+r^2\omega_o^2}}{\frac{7}{2}\mu g}$

The velocity when sliding stops is

$$\bar{v}' = v_o\bar{i} + \dot{\bar{v}}\,t_s = v_o\bar{i} - \mu g\frac{(v_o\bar{i}+r\omega_o\bar{j})}{\sqrt{v_o^2+r^2\omega_o^2}}\;\frac{2\sqrt{v_o^2+r^2\omega_o^2}}{7\mu g}$$

$$\bar{v} = \frac{5}{7}v_o\bar{i} - \frac{2}{7}r\omega_o\bar{j}$$

8-16. $\theta(0)=0$, $\dot{\theta}(0)=0$, $\dot{\phi}(0)=\Omega$



Because there is axial symmetry and no axial moment, the total spin Ω is constant.
Conserve the vertical component of $\bar{H}$ about O.

At $\theta=0$, $H_{vert} = \left(\frac{mr^2}{4}+ml^2\right)\dot{\psi}(0)$. At $\theta=\frac{-\pi}{2}$, $H_{vert} = \frac{mr^2}{2}\Omega$

where we take positive downward. Hence we obtain

$$\frac{m}{4}(4l^2+r^2)\dot{\psi}(0) = \frac{mr^2}{2}\Omega \quad \text{or} \quad \underline{\dot{\psi}(0) = \frac{2r^2\Omega}{4l^2+r^2}}$$

From conservation of energy,

$$E' = \frac{m}{2}\left(l^2+\frac{r^2}{4}\right)\dot{\theta}^2 - mgl = \frac{m}{2}\left(l^2+\frac{r^2}{4}\right)\dot{\psi}^2(0) = \frac{mr^4\Omega^2}{2(4l^2+r^2)}$$

Then $\dot{\theta}^2 = \frac{8}{4l^2+r^2}\left[gl + \frac{r^4\Omega^2}{2(4l^2+r^2)}\right]$ or,

at $\theta = \frac{-\pi}{2}$, $\underline{|\dot{\theta}| = \frac{2r^2\Omega}{4l^2+r^2}\sqrt{1+\frac{2gl(4l^2+r^2)}{r^4\Omega^2}}}$

8-28. $I_a = \frac{2}{5}mr^2$, $I_t = \frac{83}{320}mr^2$, $\hat{F} = m\sqrt{gr}$

$\hat{M}_a = \hat{F}r = mr\sqrt{gr}$, $\omega_a = \frac{\hat{M}_a}{I_a} = \frac{5}{2}\sqrt{\frac{g}{r}}$

$\hat{M}_t = \frac{3}{8}\hat{F}r = \frac{3}{8}mr\sqrt{gr}$, $\omega_t = \frac{\hat{M}_t}{I_t} = \frac{120}{83}\sqrt{\frac{g}{r}}$

$v_{cm} = \frac{\hat{F}}{m} = \sqrt{gr}$. Now $v_p = v_{cm} + r\omega_a + \frac{3}{8}r\omega_t$

$$= \left(1 + \frac{5}{2} + \frac{45}{83}\right)\sqrt{gr} = \underline{4.0422\sqrt{gr}}$$

(b) Total energy is conserved after the impulse.

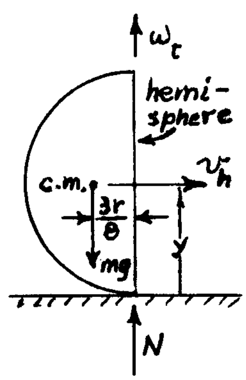
$T = \frac{1}{2}m v_{cm}^2 + \frac{1}{2}I_a\omega_a^2 + \frac{1}{2}I_t\omega_t^2$, $V = mgy$

Immediately after the impulse,

$T(0^+) = \left(\frac{1}{2} + \frac{5}{4} + \frac{45}{166}\right)mgr = 2.0211\ mgr$

which, we note, equals $\frac{1}{2}\hat{F}v_p$.

$V(0^+) = \frac{5}{8}mgr$. Hence $E(0^+) = 2.6461\ mgr$.



Now suppose that the rim just touches the floor and the flat face is vertical. Then, since $H_{vert} = \text{const}$, the value of ω_t at this time is found from $I_t\omega_t = \hat{F}r = mr\sqrt{gr}$ or $\omega_t = \frac{320}{83}\sqrt{\frac{g}{r}}$.

Noting that the horizontal component of v_{cm} and also the axial angular velocity component ω_a are constant throughout the motion for $t>0$, we obtain a final kinetic energy

$$T_f = \left(\frac{1}{2} + \frac{5}{4} + \frac{160}{83}\right)mgr = 3.6777\ mgr$$

Also $V_f = mgr$ so $E_f = 4.6777\ mgr$.

Since $E_f > E(0^+)$, the required final energy is greater than the available energy $E(0^+)$, and so it is impossible for the face to become vertical.

8-35. Same figure as in 8-34, and also the same rocket and gyro parameters. Rocket $\begin{cases} I_a = 10^3\ kg\cdot m^2 \\ I_t = 10^4\ kg\cdot m^2 \end{cases}$ Gyro $\begin{cases} I_a = 0.1\ kg\cdot m^2 \\ I_t = 0.05\ kg\cdot m^2 \end{cases}$

$\omega_r = 10$ rad/sec and $\omega_g = 10^4$ rad/sec before clamping. $\bar{H}$ is constant in space throughout the motion.

$$\bar{H} = 10^4\,\bar{i} + 10^3\,\bar{k} = (10^3 + 0.05)\omega_x\,\bar{i} + 10^3\,\bar{k} \text{ during clamping}$$

$\omega_x = 9.9995 \cong 10$ rad/sec just after clamping. $\omega_y = 10^4 =$ const since there is no axial moment applied to the gyro. The angular impulse applied to the rocket is the negative of that applied to the gyro. Due to the sudden change of ω_t of the gyro, the rocket angular impulse is

$$\hat{M} = -(0.05)(10)\,\bar{i} = \underline{-0.5000\,\bar{i}\ \text{N-m-sec}}$$

8-35. (cont'd.)(b) Assuming small motion of the symmetry axis (x axis) of the rocket, let us use the complex notation method with $\Omega = \omega_x = 10$ rad/sec. The moment applied to the rocket is $-\dot{\bar{H}}_{gyro} = (0.1)(10^4)(10)\,\bar{j}$ and rotates with the rocket. Hence, in the fixed $y'z'$ complex plane,

$$M_t = 10^4\, e^{i10t}$$

Using (8-207), and recalling that $w = y' + iz'$, we have

$$\ddot{w} - i\frac{I_a\Omega}{I_t}\dot{w} = -i\frac{M_t}{I_t} \quad \text{or} \quad \ddot{w} - i\dot{w} = -i\, e^{i10t}$$

with the initial conditions $w(0) = 0$, $\dot{w}(0) = 0$.

The solution for $\dot{w}$ has the form

$$\dot{w} = A e^{it} + B e^{i10t}$$

where $B = -\frac{1}{9}$ in steady-state solution (particular integral) and $A = -B = \frac{1}{9}$ from $\dot{w}(0) = 0$. Then

$$\dot{w} = \frac{1}{9}\left(e^{it} - e^{i10t}\right)$$

and, upon integration,

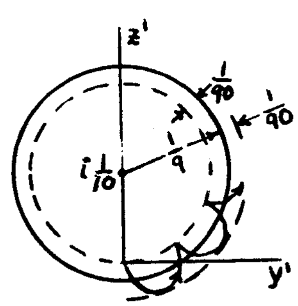
$$w = i\left(\frac{1}{10} - \frac{1}{9}e^{it} + \frac{1}{90}e^{i10t}\right)$$

The maximum deviation after half a precession cycle is

$$|w|_{max} = \frac{1}{10} + \frac{1}{9} + \frac{1}{90} = \frac{2}{9}$$

Maximum angular deviation $= \frac{2}{9}$ rad

$= \underline{12.73°}$



(c) The precession period $\underline{T = 2\pi \text{ sec}}$