Referring to Eq. (2–138), we find that the Coriolis acceleration is

$$2\boldsymbol{\omega} \times (\dot{\boldsymbol{\rho}})_r = \frac{2r\Omega^2}{\sin\beta}(-\cos\phi\,\mathbf{i} + \sin\phi\sin\beta\,\mathbf{j}) \tag{2–145}$$

Adding terms from Eqs. (2–141)–(2–145), the acceleration of $P$ is found to be

$$\mathbf{a} = \frac{r\Omega^2}{\sin^2\beta}[-(1+\cos\phi)\sin\beta\cos^2\beta\,\mathbf{i} - \sin\phi\cos^2\beta\,\mathbf{j} - \cos\phi\cos\beta\,\mathbf{k}] \tag{2–146}$$

The preceding examples have illustrated the application of Eq. (2–106) which is the general vector equation for the acceleration of a point in terms of its motion relative to a moving coordinate system. Although this equation is valid for an arbitrary motion of the moving coordinate system, this system should be chosen such that the calculations are made as simple as possible. An unfortunate choice at this point can result in a large increase in the required effort. Roughly speaking, the motion of $P$ relative to the moving system should be of about the same complexity as the absolute motion of $O'$, provided that the angular velocity $\boldsymbol{\omega}$ is constant or varies in a simple fashion. Also, the choice of unit vectors in expressing the result should be made for convenience. In general, they should form an orthogonal set.

## REFERENCES


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2–2 Malvern, L. E., *Engineering Mechanics*, Vol. 2, *Dynamics*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1976.

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## PROBLEMS

**2–1.** A vertical wheel of radius $r$ rolls without slipping along a straight horizontal line. If its angular velocity is given by $\omega = \alpha t$, where $\alpha$ is a constant, solve for the

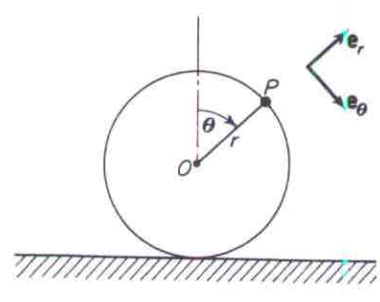


**Figure P2-1**

acceleration of a point $P$ on its rim, assuming that $P$ is at the highest point of its path at $t = 0$. Express the result in terms of the unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$.

**2–2.** Solve for the hodograph of the two-dimensional harmonic motion given by Eqs. (2–64) and (2–69). Sketch the result in the $\dot{x}\dot{y}$ plane.

**2–3.** A particle moves upward along a fixed right-handed helix having a vertical axis, a radius $R$, and an angle of 30° between the tangent vector to the path and the horizontal. Find the acceleration of the particle at an instant when the speed $\dot{s} = v_0$ and $\ddot{s} = a_0$. Use the unit vectors $\mathbf{e}_t$, $\mathbf{e}_n$, and $\mathbf{e}_b$ to express the result.

**2–4.** The lower end of a rigid bar of length $l$ is moved to the right at a constant speed $v_0$ along a horizontal floor. It slides on the corner of a step of height $l/2$. Assuming planar motion with $30° < \theta < 90°$, find: (a) $\dot{\theta}(\theta)$; (b) $\ddot{\theta}(\theta)$; (c) $\mathbf{v}_C(\theta)$ where $C$ is at the center of the bar.

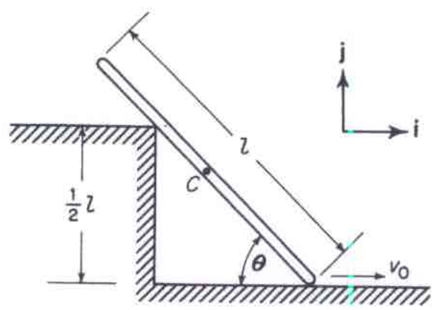


**Figure P2-4**

**2–5.** An airplane flies over a city at 45° north latitude with a relative velocity $v_0$ towards the northwest. Assuming that it flies in a great circle route of radius $R$ relative to the earth which is rotating at $\omega_e$ rad/sec, solve for the airplane's acceleration relative to a nonrotating frame translating with the earth's center. Express the result in terms of the spherical unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$.

**2–6.** A satellite moves in a circular polar orbit of radius $r$ with a constant speed $v_0$. Assume that the orbital plane of the satellite is fixed in space while the earth of radius $R$ rotates at $\omega_e$ rad/sec. As the satellite moves toward the equator, it passes directly over a radar station at 30° north latitude. This station measures the satellite's relative velocity and acceleration. Find this relative velocity and acceleration, expressing the results in terms of $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$, where $\mathbf{e}_\theta$ points due south and $\mathbf{e}_\phi$ due east.

ans

**2–7.** A water particle $P$ moves outward along the impeller vane of a centrifugal pump with a constant tangential velocity of 20 m/sec relative to the impeller, which is rotating at a uniform rate of 1200 rpm in the direction shown. Find the acceleration of the particle at the point where it leaves the impeller. Use the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ to express the result.

**2–8.** A cyclist rides around a circular track ($R = 30$ m) such that the point of contact of the wheel on the track moves at a constant speed of 10 m/sec. The bicycle is banked at 15° inward from the vertical. Find the acceleration of a tack in the tire (0.4 m radius) as it passes through the highest point of its path. Use cylindrical unit vectors in expressing the answer.

ans

**2–9.** An airplane flies with a constant speed $v$ in a level turn to the left at a constant radius $R$. The propeller is of radius $r$ and rotates about its axis in a clockwise sense

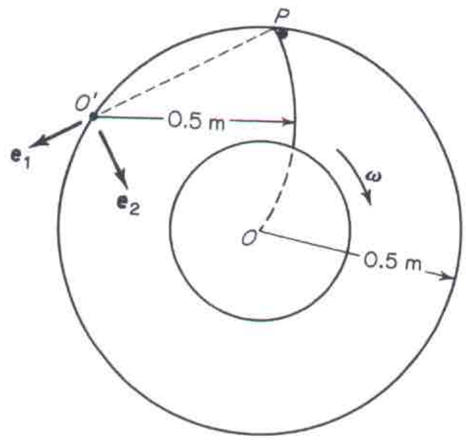


Figure P2-7

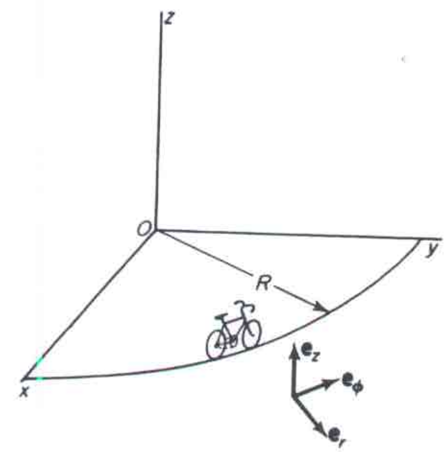


Figure P2-8

(as viewed from the rear) with a constant angular velocity $\Omega$. Find the total acceleration of a point $P$ at the tip of the propeller, assuming that its axis is always aligned with the flight path. Use cylindrical unit vectors and assume that the velocity of $P$ relative to the airplane is vertically upward at $t = 0$.

**2–10.** A particle moves in a uniform gravitational field with a constant downward acceleration $g$. At a certain time, the particle velocity $\mathbf{v}$ is at an angle $\theta$ above the horizontal. At this time, find: (a) the normal acceleration $a_n$; (b) the tangential acceleration $a_t$; (c) the radius of curvature $\rho$; (d) $\dot{\theta}$; (e) $\dot{\rho}$; and (f) $\ddot{\theta}$; all in terms of $v$, $g$, and $\theta$.

**2–11.** Two disks, each of radius $r$, roll without slipping on each other and on the floor. Assuming that the angular rates $\Omega$ and $\dot{\theta} = \omega$ are each constant, find: (a) $\dot{\phi}$ as a function of $\Omega$ and $\omega$; (b) the acceleration of point $P$.

ans

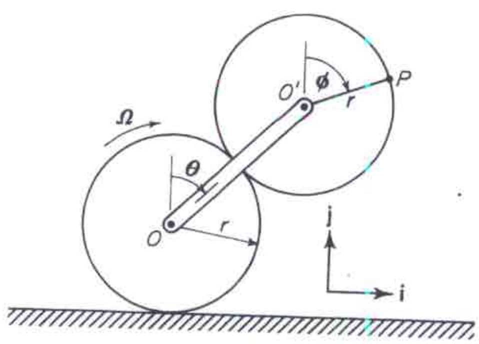


Figure P2-11

**2–12.** A circular disk of radius $r_2$ rolls in its plane on the inside of a fixed circular cylinder of radius $r_1$. Find the acceleration of a point $P$ on the wheel at a distance $b$ from its hub $O'$. Assume that $\dot{\phi}$ is not constant, where the angle $\phi$ is measured between $O'P$ and the line of centers $O'O$.