Lagrange's Equations Procedure

by Dr. Dave

1. Find a set of generalized coordinates:

$$q_i \qquad (i = 1 \text{ to } n)$$

2. Determine constraints (if any) on the generalized coordinates:

$$\phi_j(q_i, \dot{q}, t) = 0 \qquad (j = 1 \text{ to } m)$$

To determine if a coordinate is constrained or not, fix all other coordinates and ask if the coordinate in questions can vary freely or not.

3. Eliminate generalize coordinates by solving holonomic constraints if possible. First try to integrate any constraints expressed in nonholonomic form. For all holonomic constraints that cannot be solved (to eliminate coordinates), convert them to nonholonomic form:

$$\frac{d}{dt}\phi_j(holo) \Rightarrow \phi_j(nonholo)$$

4. Express nonholonomic constraints in standard form:

$$\sum_{i=1}^{n} a_{ji}\dot{q}_i + a_{jt} = 0$$

Identify all coefficients (a_{11}, a_{12}, ..., a_{mn}) in all constraint equations.

5. Find the system kinetic energy (T) and potential energy (V).

6. Determine the generalized nonconservative force (Q_i') for each generalized coordinate:

$$Q_i' = \sum_{nc\ forces} F_{nc}\frac{\partial x}{\partial q_i} + \sum_{nc\ torques} T_{nc}\frac{\partial \theta}{\partial q_i}$$

7. Set up the Lagrange Equation for each generalized coordinate:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = \sum_{j=1}^{m} \lambda_j a_{ji} + Q_i' \qquad (i = 1 \text{ to } n)$$

You now have m constraint equations and n Lagrange Equations that define the equations of motion of the system. The n+m unknowns are q_i and λ_j.

8. If there are constraint equations, and constraint forces are required, calculate them with:

$$C_i = \sum_{j=1}^{m} \lambda_j a_{ji}$$

where C_i is the component of the resultant constraint force (torque) acting in the q_i direction (about the q_i axis).