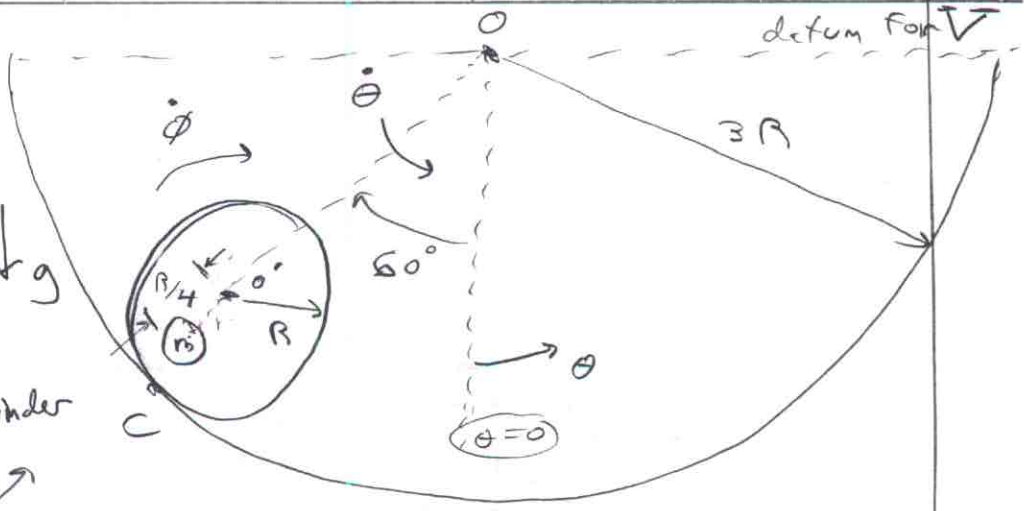
3-24)

massless disk w/ imbedded particle released from rest as shown in cylinder

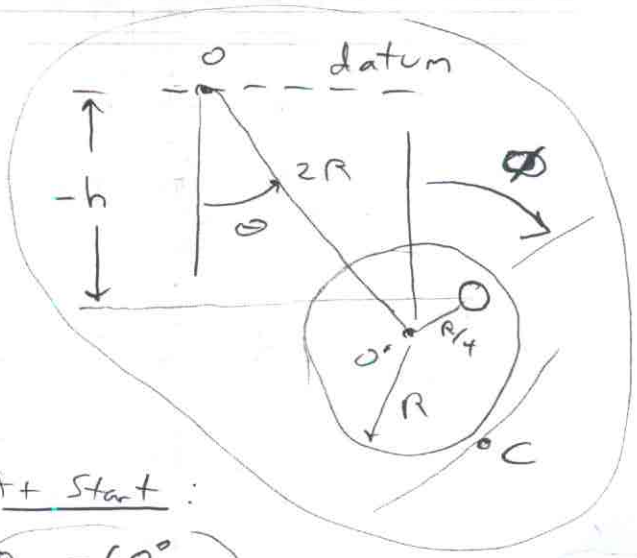


FIND: a) max. speed of mass
b) reaction force @ bottom

NOTE: ENERGY (T+V) CONSERVED SINCE NO NON CONSERVATIVE FORCES

GEOMETRY

GENERAL POSITION:



FROM KINEMATICS ROLLING CONSTRAINT:

$V_{O'/C} = V_{O'/O}$

$R\dot{\phi} = 2R\dot{\theta}$

$\Rightarrow \dot{\phi} = 2\dot{\theta}$

At Start:

$\theta_0 = -60°$

$\phi_0 = -120°$

IC's

$\int dt \Rightarrow \phi - \phi_0 = 2(\theta - \theta_0)$

$\phi = 2\theta$

So @ bottom, $\theta = 0°$, $\phi = 0°$

a.) max speed of mass

ENERGY CONSERVED ⇒ $T + V = \text{const.}$

(CAN FIND KINEMATICS OF ANY STATE FROM IC's)

$$T = \frac{1}{2} m v^2$$

$$V = mgh = mg\left(-2R C_\theta + \frac{R}{4} C_\phi\right)$$

$$\phi = 2\theta \Rightarrow V(\theta) = mgR\left(-2C_\theta + \frac{1}{4} C_{2\theta}\right)$$

max v occur when T_{max} & V_{min} (v_{max}) since $(T+V = \text{const})$

METHOD:

TO FIND V_{min}

$$\frac{dV}{d\theta} = 0, \quad \theta \rightarrow$$

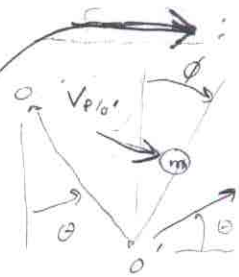
ENERGY CONS:

$$V(y_0) = T + V(\theta), \quad v_{max} = \sqrt{\frac{2}{m} T}$$

1st: ASIDE

"another trick for your bag"

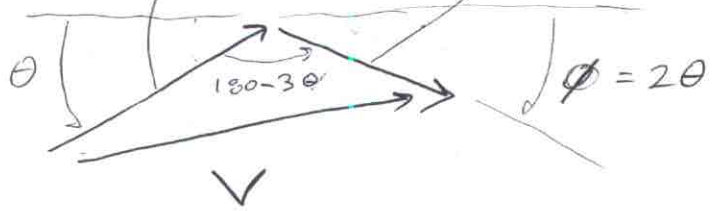
: quick method to find v^2



$$\vec{V} = \vec{V}_{O'/O} + \vec{V}_{P/O'}$$

$\vec{R}$ $\quad$ $(\vec{\rho})_{rel}$

$$V_{O'/O} = 2R\dot{\theta} \qquad V_{P/O'} = \frac{R}{4}\dot{\phi} = \frac{R}{2}\dot{\theta}$$



LAW OF COSINES

$$V^2 = V_{O'/O}^2 + V_{P/O'}^2 = (2R\dot{\theta})^2 + \left(\frac{R}{2}\dot{\theta}\right)^2 - (2R\dot{\theta})\left(\frac{R}{2}\dot{\theta}\right)\cos(180-3\theta)$$

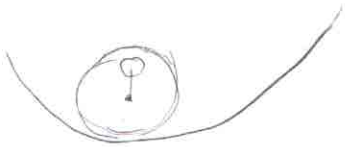
$$V(\theta) = mgR\left(-2\cos\theta + \tfrac{1}{4}\cos 2\theta\right)$$

$$\frac{dV}{d\theta} = 0 \Rightarrow 2\sin\theta - \tfrac{1}{2}\sin 2\theta = 0$$

$$\Rightarrow \theta = 0$$

$\Rightarrow$ min V @ bottom

(THIS ISN'T OBVIOUS)



see graph next page

CONS. OF ENERGY

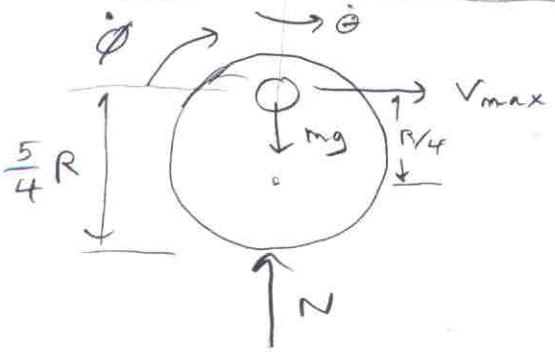
INITIAL

$$V(-60°) = T_{max}(0) + V_{min}(0)$$

$$-\frac{9}{8}mgR = \frac{1}{2}mv_{max}^2 - \frac{7}{4}mgR$$

$$\Rightarrow \boxed{v_{max} = \frac{1}{2}\sqrt{5gR}}$$

b.) Reaction Force @ Bottom



$$\dot{\phi} = \frac{v_{max}}{\frac{5}{4}R} = 2\sqrt{\frac{g}{5R}}$$

$$\dot{\theta} = \tfrac{1}{2}\dot{\phi} = \sqrt{\frac{g}{5R}}$$

$$\vec{a}_P = \underbrace{\vec{a}_{O'/O}}_{\vec{R}} + \vec{a}_{P'/O'} \quad \left(\ddot{\vec{\rho}}\right)_{rel}$$

(non rotating frame)

$$\Sigma F_z = ma_z$$

$$N - mg = m\left(2R\dot{\theta}^2 - \frac{R}{4}\dot{\phi}^2\right)$$

$$\boxed{N = \frac{6}{5}mg}$$

MATH CAD

$$\theta := -\frac{\pi}{3}, -\frac{\pi}{3} + \frac{\pi}{300} .. \frac{\pi}{3}$$

$$h(\theta) := -2 \cdot \cos(\theta) + \frac{1}{4} \cdot \cos(2 \cdot \theta)$$