In summary, the six rules for sketching the root locus of a closed-loop system with the characteristic equation

$$1 + KG(s)H(s) = 0$$

are given in Table 7.2. Recall that the root locus is a plot of the roots of the *system characteristic equation* (poles of the *closed-loop system*) as the parameter K is varied. Some examples will be given now to illustrate the use of these rules.

TABLE 7.2 RULES FOR ROOT LOCUS DEVELOPMENT

1. The root locus is symmetrical with respect to the real axis.
2. The root locus originates on the poles of $G(s)H(s)$ (for $K = 0$) and terminates on the zeros of $G(s)H(s)$ (as $K \to \infty$), including those at infinity.
3. If the open-loop function has α zeros at infinity $\alpha \geq 1$, the root locus will approach α asymptotes as K approaches infinity. The asymptotes are located at the angles

   $$\theta = \frac{r180°}{\alpha} \qquad r = \pm 1, \pm 3, \ldots$$

   and those asymptotes intersect the real axis at the point

   $$\sigma_a = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

   where the symbol # denotes number.
4. The root locus includes all points on the real axis to the left of an odd number of real critical frequencies (poles and zeros).
5. The breakaway points on a root locus will appear among the roots of the polynomial obtained from either

   $$\frac{d[G(s)H(s)]}{ds} = 0$$

   or, equivalently,

   $$N(s)D'(s) - N'(s)D(s) = 0$$

   where $N(s)$ and $D(s)$ are the numerator and denominator polynomials, respectively, of $G(s)H(s)$.
6. Loci will depart from a pole p_j (arrive at a zero z_j) of $G(s)H(s)$ at the angle $\theta_d(\theta_a)$, where

   $$\theta_d = \sum_i \theta_{zi} - \sum_{\substack{i \\ i \neq j}} \theta_{pi} + r(180°)$$

   $$\theta_a = \sum_i \theta_{pi} - \sum_{\substack{i \\ i \neq j}} \theta_{zi} + r(180°)$$

   and where $r = \pm 1, \pm 3, \ldots$ and θ_{pi} (θ_{zi}) represent the angles from pole p_i (zero z_i), respectively, to $p_j(z_j)$.