In summary, the six rules for sketching the root locus of a closed-loop system with the characteristic equation

$$1 + KG(s)H(s) = 0$$

are given in Table 7.2. Recall that the root locus is a plot of the roots of the *system characteristic equation* (poles of the *closed-loop system*) as the parameter K is varied. Some examples will be given now to illustrate the use of these rules.

TABLE 7.2 RULES FOR ROOT LOCUS DEVELOPMENT

1. The root locus is symmetrical with respect to the real axis.
2. The root locus originates on the poles of $G(s)H(s)$ (for $K = 0$) and terminates on the zeros of $G(s)H(s)$ (as $K \to \infty$), including those at infinity.
3. If the open-loop function has α zeros at infinity $\alpha \geq 1$, the root locus will approach α asymptotes as K approaches infinity. The asymptotes are located at the angles

   $$\theta = \frac{r180^\circ}{\alpha} \qquad r = \pm 1, \pm 3, \ldots$$

   and those asymptotes intersect the real axis at the point

   $$\sigma_a = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

   where the symbol # denotes number.
4. The root locus includes all points on the real axis to the left of an odd number of real critical frequencies (poles and zeros).
5. The breakaway points on a root locus will appear among the roots of the polynomial obtained from either

   $$\frac{d[G(s)H(s)]}{ds} = 0$$

   or, equivalently,

   $$N(s)D'(s) - N'(s)D(s) = 0$$

   where $N(s)$ and $D(s)$ are the numerator and denominator polynomials, respectively, of $G(s)H(s)$.
6. Loci will depart from a pole p_j (arrive at a zero z_j) of $G(s)H(s)$ at the angle $\theta_d(\theta_a)$, where

   $$\theta_d = \sum_i \theta_{zi} - \sum_{\substack{i \\ i \neq j}} \theta_{pi} + r(180^\circ)$$

   $$\theta_a = \sum_i \theta_{pi} - \sum_{\substack{i \\ i \neq j}} \theta_{zi} + r(180^\circ)$$

   and where $r = \pm 1, \pm 3, \ldots$ and θ_{pi} (θ_{zi}) represent the angles from pole p_i (zero z_i), respectively, to $p_j(z_j)$.

```
>> help margin
 MARGIN  Gain and phase margins and crossover frequencies.

    [Gm,Pm,Wcg,Wcp] = MARGIN(SYS) computes the gain margin Gm, the
    phase margin Pm, and the associated frequencies Wcg and Wcp,
    for the SISO open-loop model SYS (continuous or discrete).
    The gain margin Gm is defined as 1/G where G is the gain at
    the -180 phase crossing.  The phase margin Pm is in degrees.

    The gain margin in dB is derived by
       Gm_dB = 20*log10(Gm)
    The loop gain at Wcg can increase or decrease by this many dBs
    before losing stability, and Gm_dB<0 (Gm<1) means that stability
    is most sensitive to loop gain reduction.  If there are several
    crossover points, MARGIN returns the smallest margins (gain
    margin nearest to 0 dB and phase margin nearest to 0 degrees).

    For a S1-by...-by-Sp array SYS of LTI models, MARGIN returns
    arrays of size [S1 ... Sp] such that
       [Gm(j1,...,jp),Pm(j1,...,jp)] = MARGIN(SYS(:,:,j1,...,jp)) .

    [Gm,Pm,Wcg,Wcp] = MARGIN(MAG,PHASE,W) derives the gain and phase
    margins from the Bode magnitude, phase, and frequency vectors
    MAG, PHASE, and W produced by BODE.  Interpolation is performed
    between the frequency points to estimate the values.

    MARGIN(SYS), by itself, plot the open-loop Bode plot with
    the gain and phase margins marked with a vertical line.

    See also ALLMARGIN, bode, ltiview, ltimodels.

    Overloaded functions or methods (ones with the same name in other directories)
       help lti/margin.m

    Reference page in Help browser
       doc margin

>>
```