Example 6.6

The Routh–Hurwitz criterion will be utilized to perform a simple design for the control system shown in Figure 6.2, in which a proportional compensator is employed. The system is type 0 (see Section 5.5), and the steady-state error for a constant input of unity is, from (5-35),

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K}$$

where the position error constant K_p is, for this example,

$$K_p = \lim_{s\to 0} G_c(s)G_p(s) = \lim_{s\to 0} \frac{2K}{s^3+4s^2+5s+2} = K$$

Suppose that a design specification is that e_{ss} must be less than 2 percent of a constant input. Thus

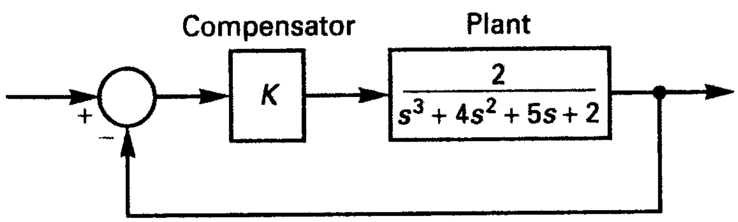

Figure 6.2 System for Example 6.6.

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} < \frac{1}{50}$$

or, K must be greater than 49. The calculation of steady-state errors is based on the assumption of stability; thus we must ensure that the system is stable for the range of K required. The system characteristic equation is given by

$$1 + G_c(s)G_p(s) = 1 + \frac{2K}{s^3+4s^2+5s+2} = 0$$

or

$$Q(s) = s^3 + 4s^2 + 5s + 2 + 2K = 0$$

The Routh array for this polynomial is then

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 4 & 2+2K \\ s^1 & \frac{18-2K}{4} & \\ s^0 & 2+2K & \end{array} \qquad \begin{array}{l} b_1 = -\frac{1}{4}(2+2K-20) \\ \\ \Rightarrow \quad K < 9 \\ \Rightarrow \quad K > -1 \end{array}$$

Thus the system is stable only for the compensator gain K greater than -1 but less than 9. The steady-state error criterion cannot be met with the proportional compensator; it will be necessary to use a dynamic compensation such that $G_c(s)$ is a function of s and not simply a pure gain. Of course, a PI compensator with

$$G_c(s) = K_P + \frac{K_I}{s}$$

will make the system type 1, since $G_c(s)G_p(s)$ has a pole at the origin. Hence the steady-state error for a constant input is then zero, *provided* that the compensated system is stable.