PROBLEMS

B.1. Using the defining integral of the Laplace transform, (B-1), derive the Laplace tansform of
(a) $f(t) = u(t - 25)$
(b) $f(t) = e^{-4t}$
(c) $f(t) = t$
(d) Verify the results in (b) and (c) with MATLAB.

B.2. Use the Laplace transform tables to find the transforms of each function.
(a) $f(t) = -3te^{-t}$
(b) $f(t) = -5 \cos t$
(c) $f(t) = t \sin 3t$
(d) $f(t) = 7e^{-0.5t} \cos 3t$
(e) $f(t) = 5 \cos (4t + 30°)$
(f) $f(t) = 6e^{-2t} \sin (t - 45°)$
(g) Verify all the results with MATLAB.

B.3. Find the inverse Laplace transform of each.
(a) $F(s) = \dfrac{5}{s(s + 1)(s + 2)}$
(b) $F(s) = \dfrac{1}{s^2(s + 1)}$
(c) $F(s) = \dfrac{2s + 1}{s^2 + 2s + 10}$
(d) $F(s) = \dfrac{s - 30}{s(s^2 + 4s + 29)}$
(e) Verify all the results with MATLAB.

B.4. Given the Laplace transform

$$F(s) = \frac{s + 5}{s^2 + 4s + 13}$$

(a) Express the inverse transform as the sum of two complex exponentials.
(b) Using Euler's identity, manipulate the result of (a) into the form $f(t) = Be^{-at} \sin(bt + \Phi)$.
(c) Use the procedure of Section B.1 to express the inverse transform as $f(t) = Ae^{-at} \cos (bt + \Phi)$.
(d) Verify all the results with MATLAB.

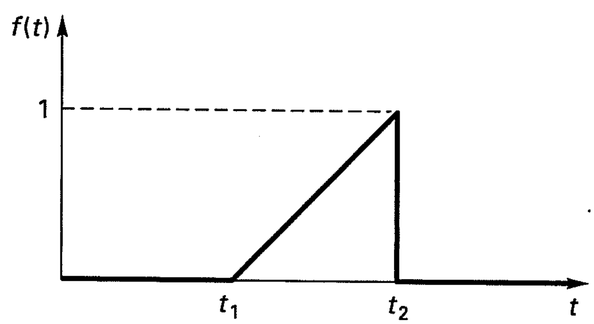

FIGURE PB.5

B.5. **(a)** Find and plot $f(t)$ if its Laplace transform is given by

$$F(s) = \frac{e^{-t_1 s} - e^{-t_2 s}}{s} \qquad t_2 > t_1$$

(b) The time function in part (a) is a rectangular pulse. Find the Laplace transform of the triangular pulse shown in Figure PB.5.

B.6. Given that $f(t) = 4e^{-2(t-3)}$

(a) Find $\mathcal{L}[df(t)/dt]$ by differentiating $f(t)$ and then using the Laplace transform tables.

(b) Find $\mathcal{L}[df(t)/dt]$ using the theorem for differentiation.

(c) Repeat (a) and (b), for the case that $f(t) = 4e^{-2(t-3)}u(t-3)$.

B.7. Find the Laplace transforms of

(a) $f_1(t) = 5e^{-2(t-1)}$

(b) $f_2(t) = 5e^{-2(t-1)}u(t-1)$

(c) Sketch the two time functions.

(d) Why are the two transforms different?

B.8. For the functions of Problem B.3,

(a) Which inverse Laplace transforms do not have final values; that is, for which of the inverse transforms do the $\lim_{t\to\infty} f(t)$ exist?

(b) Find the final values for those functions that have final values.

B.9. Given the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 10u(t)$$

(a) Find $x(t)$ for the case that the initial conditions are zero.

(b) Find $x(t)$ for the case that $x(0) = 1$ and $\dot{x}(0) = 1$. Show that your solution yields the correct initial conditions, that is, solve for $x(0)$ and $\dot{x}(0)$ using your solution.

(c) Verify all the results with MATLAB.

B.10. Given the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 5 \qquad t \geq 0$$

(a) Find $x(t)$ for the case that the initial conditions are zero.

(b) Find $x(t)$ for the case that $x(0) = 0$ and $\dot{x}(0) = 2$. Show that your solution yields the correct initial conditions; that is, solve for $x(0)$ and $\dot{x}(0)$ using your solution.

(c) Verify all the results with MATLAB.

B.11. For each of the systems, find the system differential equation if $G(s) = C(s)/R(s)$ is given by

(a) $G(s) = \dfrac{60}{s^2 + 10s + 60}$

(b) $G(s) = \dfrac{3s + 20}{s^2 + 4s^2 + 8s + 20}$

(c) $G(s) = \dfrac{s + 1}{s^2}$

(d) $G(s) = \dfrac{7e^{-0.2s}}{s^2 + 5s + 32}$

B.12. Give the characteristic equations for the systems of Problem B.11.