DYNAMIC CHARACTERISTICS OF MEASUREMENT SYSTEMS (OR ANY)

CAN BE DESCRIBED BY A **DE:**

$$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = \text{func. of } X_{in}$$

"ORDER" OF ODE

CONSTANT COEFFICIENTS (SYSTEM PARAMETERS: R, C, L, b, k, m)

0-ORDER SYSTEM

$$X_{out} = K X_{in}$$

gain or sensitivity

ex: SEE F4.11

1st-ORDER SYSTEM

$$\tau \frac{dX_{out}}{dt} + X_{out} = K X_{in}$$

time constant

1

STANDARD FORM

STEP INPUT: $X_{in}(t) = A_{in}$

$$X_{out}(t) = K A_{in}(1 - e^{-t/\tau})$$

ex: SEE Q4.19 RC CIRCUIT + F4.12

2nd-ORDER SYSTEM

$$m \frac{d^2 X_{out}}{dt^2} + b \frac{dX_{out}}{dt} + k X_{out} = F_{ext}(t)$$

or

$$\frac{d^2 X_{out}}{dt^2} + 2\zeta\omega_n \frac{dX_{out}}{dt} + \omega_n^2 X_{out} = \frac{\omega_n^2}{k} F_{ext}(t)$$

ex: see F4.15

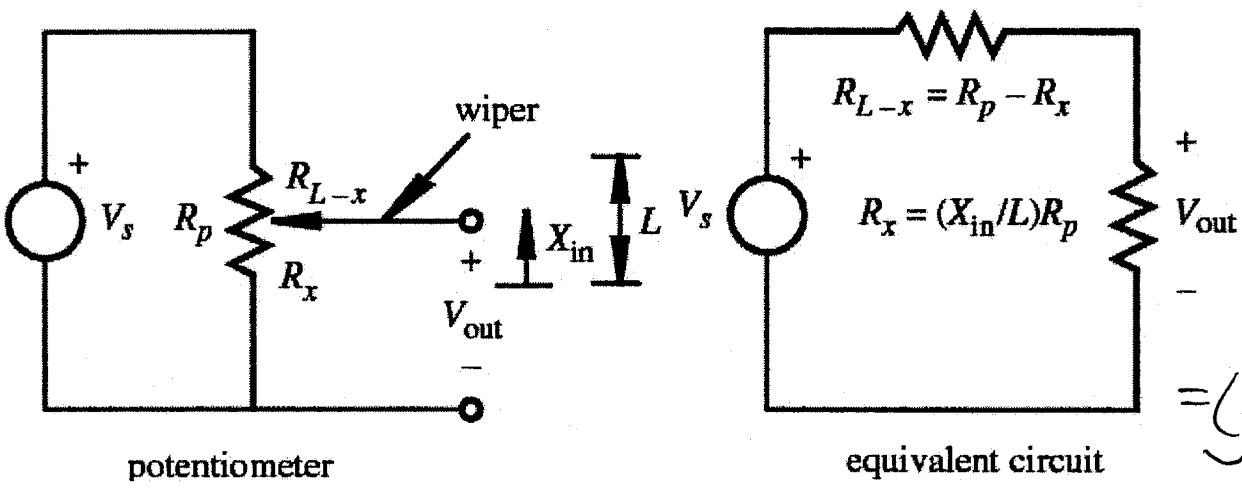
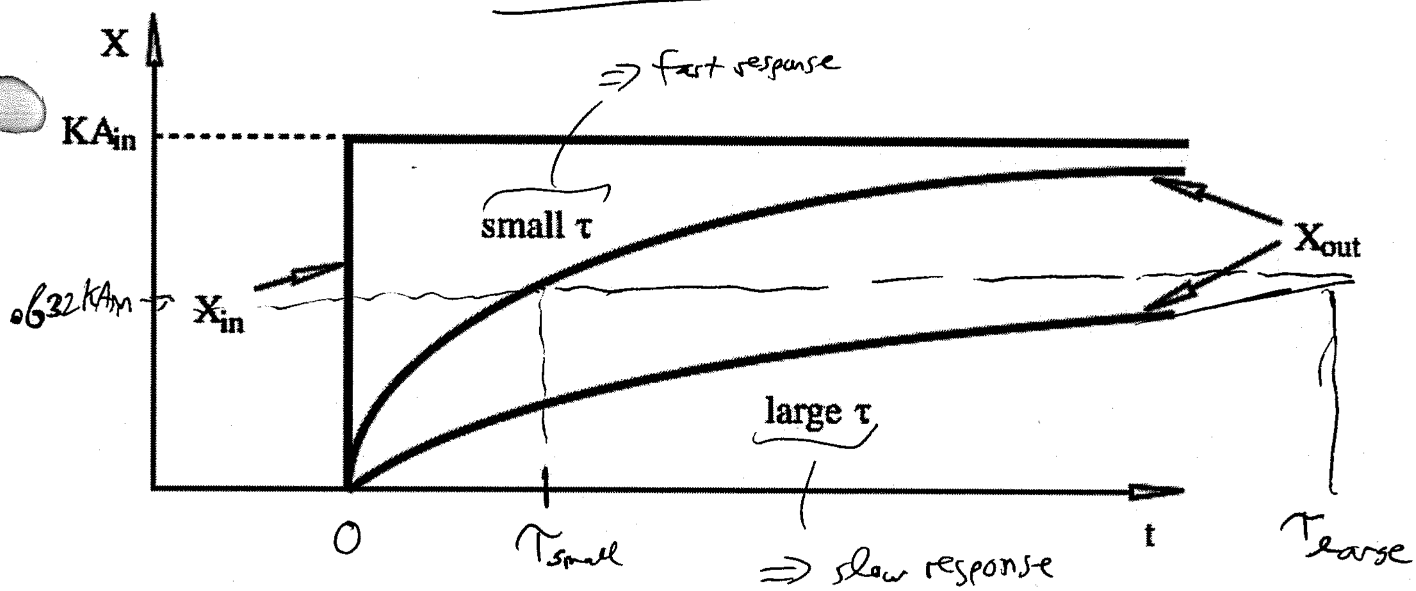


Figure 4.11 (p. 132)

$$V_{out} = \frac{R_x}{\underbrace{R_x + R_{L-x}}_{R_p}} V = \underbrace{\left(\frac{V_S}{L}\right)}_{K} X_{in}$$

F4.12



$$-t/\tau \,]\, @\, t=\tau$$

$$X_{out}(\tau) = KA_{in}(1-e^{-1}) = .632\,(KA_{in})$$

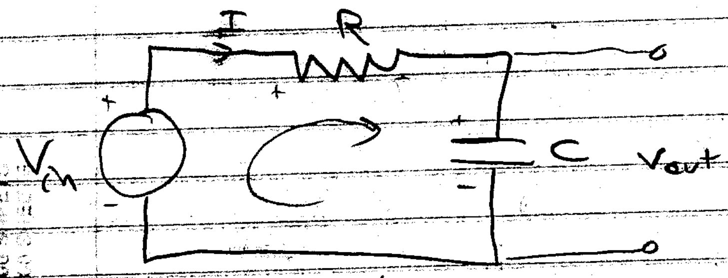
TIME FOR OUTPUT TO REACH 63.2% OF STEP INPUT STEADY STATE VALUE

$$(X_{out}(\infty) = KA_{in}(1-e^{-\infty}) = KA_{in})$$

SKIP

Q 4.19 RC CIRCUIT TRANSIENT RESPONSE

(NOT "STEADY STATE")



FIND: $V_{out}(t)$

KVL:

$IR + \frac{1}{C} q = V_{in}$

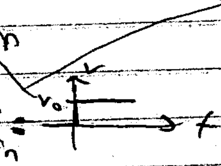
$R \frac{dq}{dt} + \left(\frac{1}{C}\right) q = V_{in}$

$(RC)\, \dot{q} + q = C V_{in}$ — 1st ORDER SYS STANDARD FORM

x_{in}, x_{out}

$\Rightarrow \tau = RC$ (K)

Step: measurement of how long circuit takes to charge the capacitor

Step response: V_{in} = 

IC: $q_c(0) = 0$ (no charge on capacitor)

Eqn 4.44 $\Rightarrow$

$q(t) = CV_o(1 - e^{-t/\tau})$

$V_{out}(t) = \frac{1}{C} q(t) = V_o(1 - e^{-t/\tau})$

SEE RC CIRCUIT CHARGING VIDEO

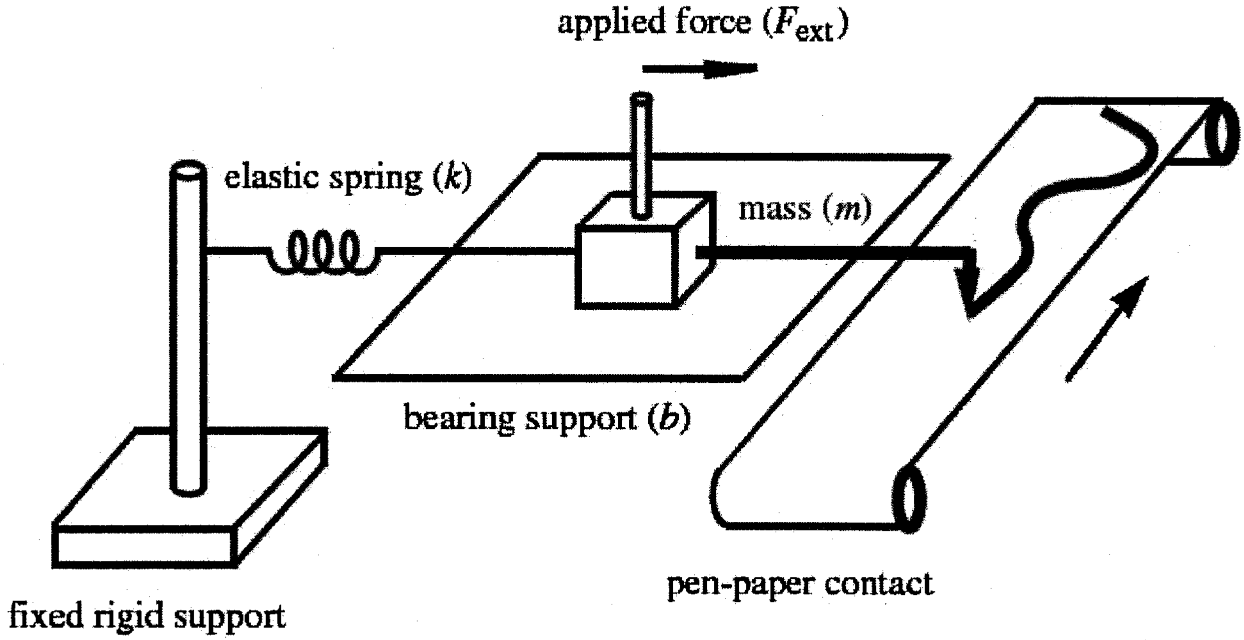
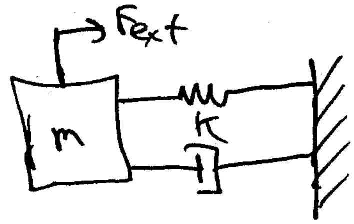


Figure 4.15 (p. 138)

$\equiv$



JUST SAY MOST SENSORS (& ANY MECH. SYS) W/ MOVING PARTS HAVE THIS SAME MODEL

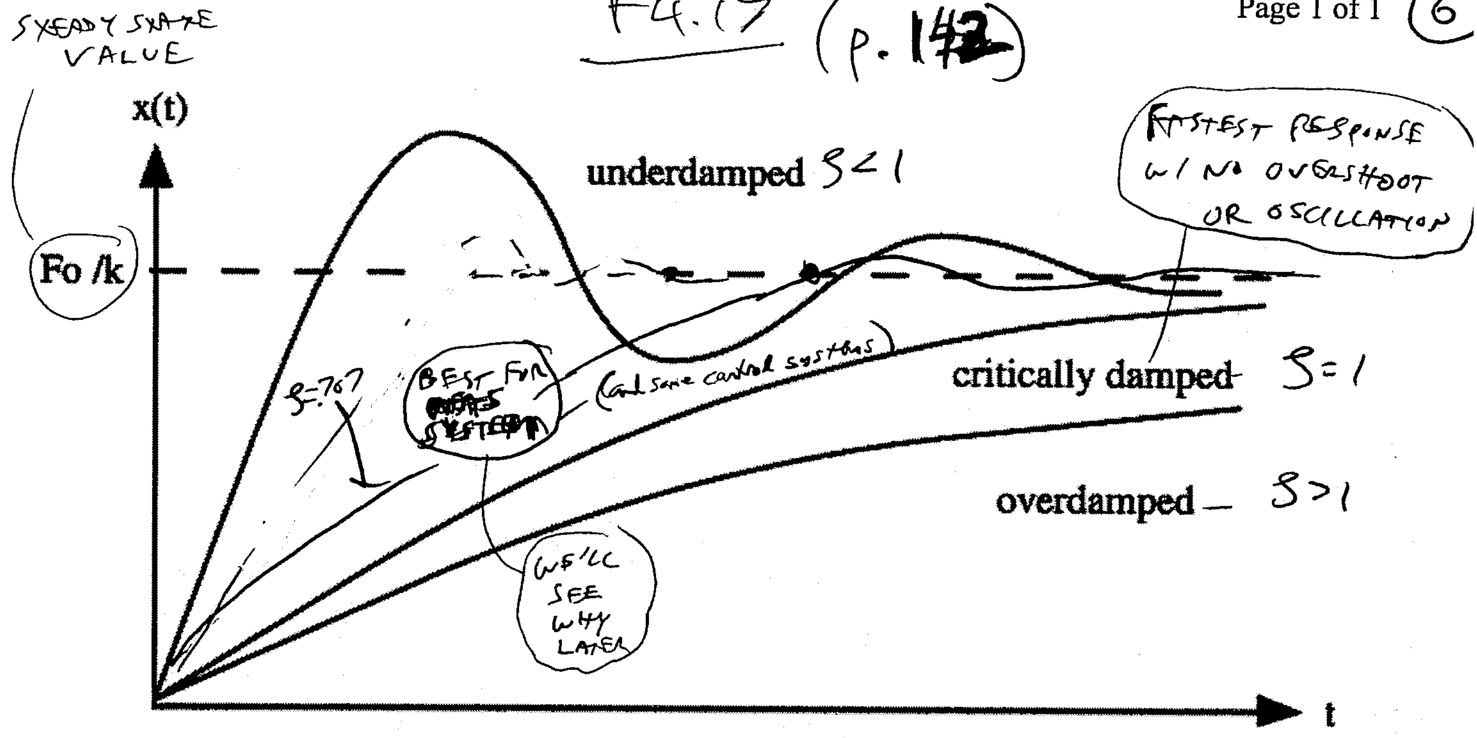
NAT. FREQ: $\omega_n = \sqrt{\frac{k}{m}} = 2\pi \underbrace{f_n}_{\text{cycles/sec}}$

CRITICAL DAMPING: $b_c = 2 m \omega_n$

DAMPING RATIO: $\zeta = \frac{b}{b_c}$: $\begin{cases} >1 & \text{"OVERDAMPED"} \\ =1 & \text{"CRITICALLY DAMPED"} \\ <1 & \text{"UNDERDAMPED"} \end{cases}$

"ZETA"

F4.17 (p. 142)



F_0 APPLIED @ $t=0$ w/ $\underbrace{x(0)=\dot{x}(0)=0}_{\text{from rest}}$

(p. 143)

2nd ORDER SYSTEM

2nd order system

FOR CIRCUITS AC CIRCUIT ANALYSIS

4.10.2 Frequency Response of a System

[redacted] (4.72)

[redacted] (4.73)

[redacted]

1. *Find the Laplace transform of the system differential equation assuming initial conditions are zero: $x(0) = \mathrm{d}x/\mathrm{d}t(0) = 0$. The Laplace transform converts the differential equation into an algebraic equation that is related to the frequency response of the system.*

$$\frac{1}{\omega_n^2}\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} + \frac{2\zeta}{\omega_n}\frac{\mathrm{d}x(t)}{\mathrm{d}t} + x(t) = \frac{1}{k}F_{\mathrm{ext}}(t) \qquad \text{time domain} \tag{4.74}$$

$\frac{d^i x}{dt^i} \to s^i X(s)$

more in EE 4417

$$\left(\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1\right)X(s) = \frac{1}{k}F_{\mathrm{ext}}(s) \qquad \text{s-domain (freq. domain)} \tag{4.75}$$

2. *Find the* **transfer function** *of the system, which is the ratio of the output and input Laplace transforms.*

$G(s) = \frac{X_{out}(s)}{X_{in}(s)}$

$$G(s) = \frac{X(s)}{F_{\mathrm{ext}}(s)} = \frac{1/k}{\left(\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1\right)} \tag{4.76}$$

out / in

3. *To simulate a harmonic input, replace s with jω in the transfer function. This yields the frequency response behavior of the system.*

$s \to j\omega$

$$G(\mathrm{j}\omega) = \frac{1/k}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + \mathrm{j}\left(2\zeta\frac{\omega}{\omega_n}\right)} = \left(\frac{1}{k}\right)\frac{1}{a+jb} \tag{4.77}$$

JUST SAY: "Euler magic" because $e^{j\omega t} = \cos\omega t + j\sin\omega t$

4. *Find the desired amplitude ratio between the output and input by determining the magnitude of the complex transfer function:*

$$\mathrm{mag}[G(\mathrm{j}\omega)] = |G(\mathrm{j}\omega)| \tag{4.78}$$

$$\frac{A_{out}}{A_{in}} = \frac{\boxed{X_o}}{\boxed{F_o}/k} = \frac{1}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}^{1/2}} = \frac{1}{\sqrt{a^2+b^2}} \tag{4.79}$$

$= |G(j\omega)|$

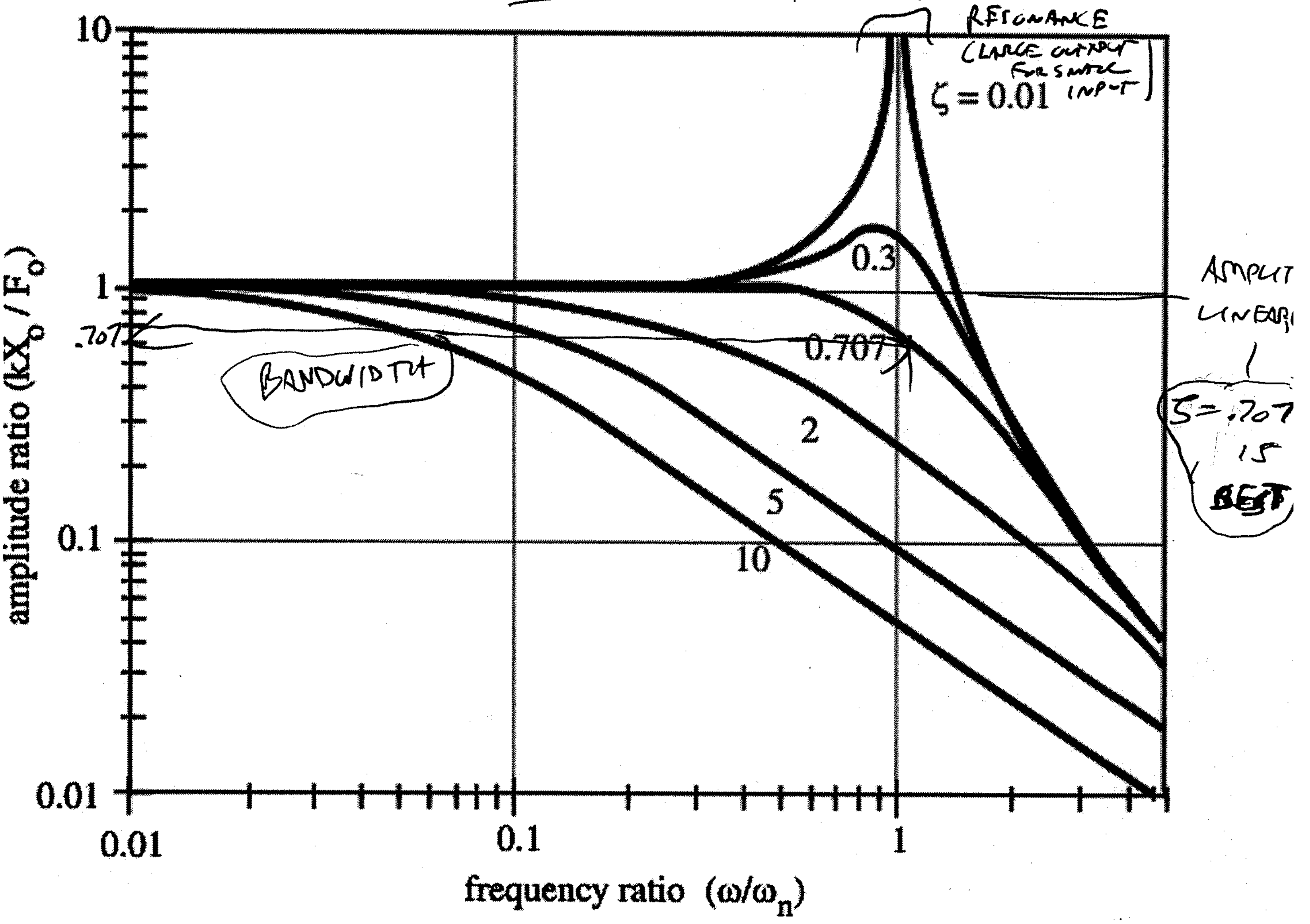
5. *Find the phase angle φ between the output and input by determining the argument of the complex transfer function:*

$$\phi = \arg[G(\mathrm{j}\omega)] = \angle G(\mathrm{j}\omega) = \angle 1/k - \angle(a+jb) = 0 - \tan^{-1}\left(\frac{b}{a}\right) \tag{4.80}$$

$\phi = \angle G(j\omega)$

$$\phi = 0 - \tan^{-1}\left\{\frac{2\zeta\frac{\omega}{\omega_n}}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]}\right\} = -\tan^{-1}\left\{\frac{2\zeta}{\frac{\omega_n}{\omega} - \frac{\omega}{\omega_n}}\right\} \tag{4.81}$$

F4.19 (p.148)



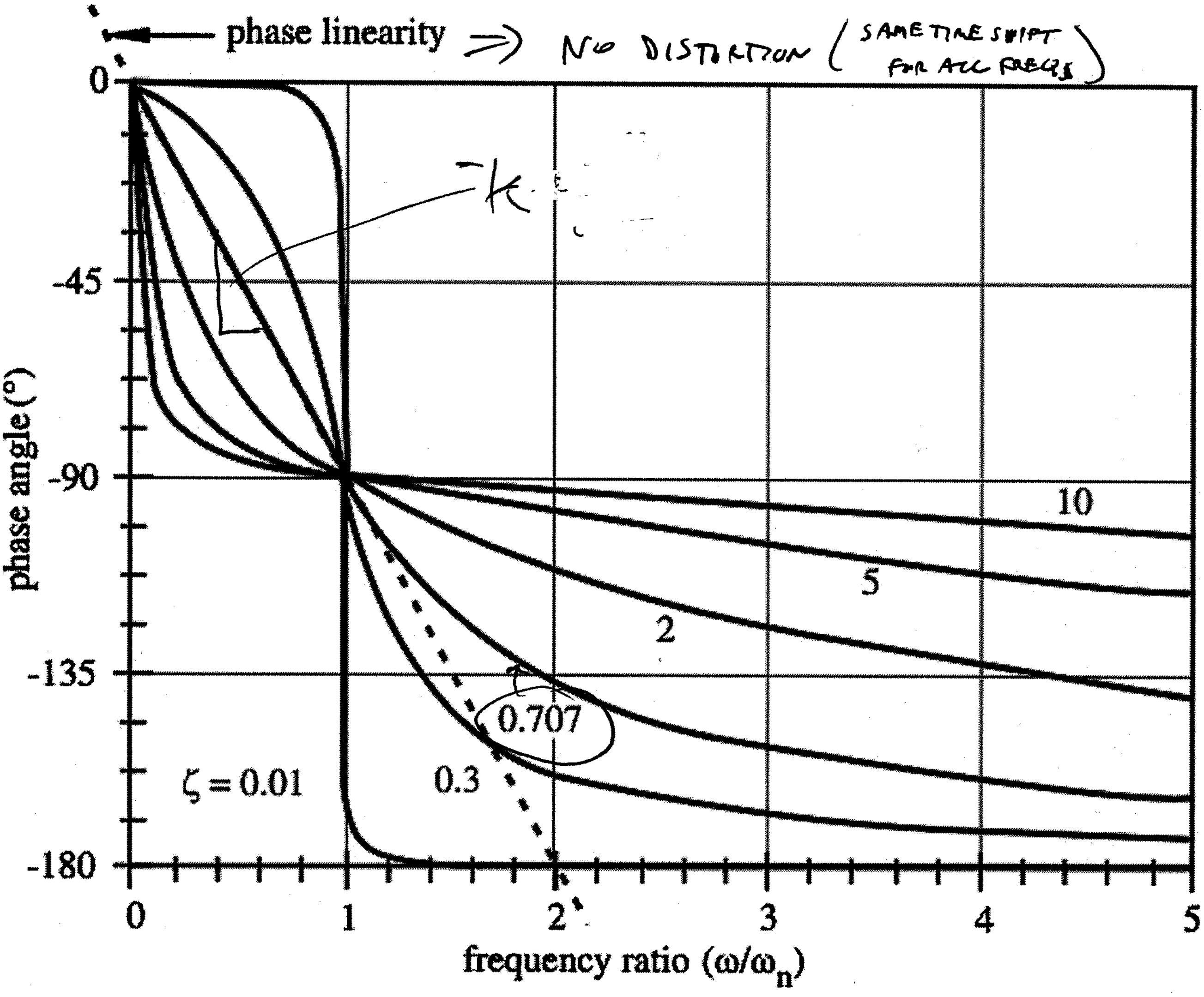
SEE SPRING-MASS

2nd ORDER SYSTEM

VIDEO

(10)

F4.20



$\zeta = .707$ IS GOOD FOR MEAS. SYS SINCE

⇒ AMP. LIN OVER LARGEST BANDWIDTH

+ BEST PHASE LINEARITY