


# Concurrent Probabilistic Control Co-Design and Layout Optimization of Wave Energy Converter Farms using Surrogate Modeling

DETC2023-116896

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# Introduction

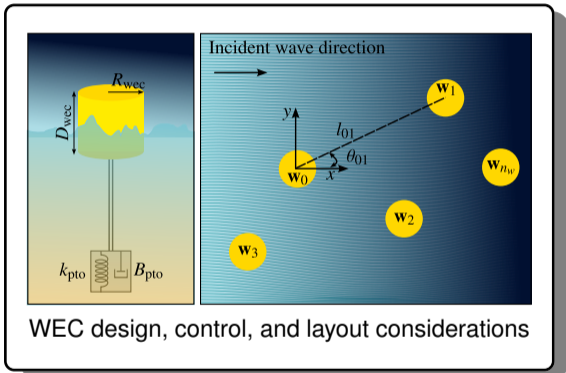
## → Introduction

- **Wave energy** is a promising source of renewable energy due to its temporal and spatial availability, low variability, and high predictability<sup>1</sup>
- Its technology readiness level (TRL), however, is low compared to wind and solar<sup>2</sup>
- Thus, more research and investment is required to improve the techno-economic performance of wave energy converters (WECs)
- **Sizing** (i.e. plant) and **power take-off** (PTO) (i.e. control) have been investigated in the literature through optimization methods<sup>3</sup>
- WECs must be deployed in a farm to reduce installation, maintenance, & operation costs<sup>4</sup>
- The presence of multiple WECs in close proximity results in a **hydrodynamic interaction effect** that can be constructive or destructive
- To ensure constructive effect (maximized power generation), methods from **layout optimization** have been used<sup>5</sup>

<sup>1</sup> Ning and Ding 2022   <sup>2</sup> Straub 2015   <sup>3</sup> McCabe, Murphy, and Haji 2022; Neshat, Sergiienko, et al. 2020; Herber and Allison 2013   <sup>4</sup> Abdulkadir and Abdelkhalik 2023   <sup>5</sup> Abdulkadir and Abdelkhalik 2023; Neshat, Mirjalili, et al. 2022; Mercadé Ruiz et al. 2017

## → Motivation

- In this research, we leverage a **system-level framework** that considers **control co-design** (i.e. plant and control), and **layout** concurrently in an **optimization** problem
- This approach has the potential to improve WEC farm performance since it accounts for the **coupling** between these domains<sup>1</sup>



<sup>1</sup> Ringwood, Zhan, and Faedo 2023

## → Challenges

- One challenge is the **high computational burden** for the accurate estimation of hydrodynamic coefficients ( which entails the calculation of the excitation force, added mass and damping coefficient matrices)
- We address this challenge by constructing **data-driven surrogate models** using **artificial neural networks** (ANNs), and hierarchical interaction decomposition using **many-body expansion** (MBE)<sup>1</sup> principles

<sup>1</sup> Zhang, Taflanidis, and Scruggs 2020

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# Methods

## → Dynamics and Control of WECs

- Using linear potential flow theory, and considering regular waves with radial frequency  $\omega$  and unit amplitude, the equation of motion for  $n_{\text{wec}}$  buoys is described as

$$-\omega^2 \mathbf{M} \hat{\boldsymbol{\xi}}(\omega) = \hat{\mathbf{F}}_{\text{FK}}(\omega) + \hat{\mathbf{F}}_{\text{s}}(\omega) + \hat{\mathbf{F}}_{\text{r}}(\omega) + \hat{\mathbf{F}}_{\text{hs}}(\omega) + \hat{\mathbf{F}}_{\text{pto}}(\omega)$$

- Excitation force:

$$\hat{\mathbf{F}}_{\text{e}}(\omega) = \hat{\mathbf{F}}_{\text{FK}}(\omega) + \hat{\mathbf{F}}_{\text{s}}(\omega)$$

- Radiation force:

$$\hat{\mathbf{F}}_{\text{r}}(\omega) = -i\omega \mathbf{B}(\omega) \hat{\boldsymbol{\xi}}(\omega) + \omega^2 \mathbf{A}(\omega) \hat{\boldsymbol{\xi}}(\omega)$$

where  $\mathbf{A}$  is added mass and  $\mathbf{B}$  is damping coefficient (obtained from NemoH)

- Linear PTO force:

$$\hat{\mathbf{F}}_{\text{pto}}(\omega) = -i\omega \mathbf{B}_{\text{pto}} \hat{\boldsymbol{\xi}}(\omega) - \mathbf{K}_{\text{pto}} \hat{\boldsymbol{\xi}}(\omega)$$

- $\hat{\mathbf{F}}_{\text{e}}$ ,  $\mathbf{A}$ , and  $\mathbf{B}$  are dependent on **plant** and **layout**

## → Dynamics and Control of WECs (continued)

- Time-averaged absorbed mechanical power for a sea state with significant wave height of  $H_s$  and peak period of  $T_p$

$$\mathbf{p}_m(H_s, T_p, \omega) = \frac{1}{2} \omega^2 \hat{\boldsymbol{\xi}}^T \mathbf{B}_{\text{pto}} \hat{\boldsymbol{\xi}}$$

- The mechanical power matrix is estimated by integrating the product of the wave spectrum with the time-averaged power over all frequencies<sup>1</sup>

$$\mathbf{p}_i(H_s, T_p, y) = \sum_{k=0}^{n_w} 2 \Delta \omega_k S_{JS}(H_s, T_p, \omega_k) \mathbf{p}_m(H_s, T_p, \omega_k)$$

<sup>1</sup> Neshat, Mirjalili, et al. 2022; Borgarino, Babarit, and Ferrant 2012



## → Dynamics and Control of WECs (continued)

- Considering the number of years in the study  $n_y$ , and the associated probability matrices, the average power is calculated as

$$p_a = \eta_{\text{pcc}} \eta_{\text{oa}} \eta_t \sum_{y=1}^{n_y} \mathbf{p}_i(H_s, T_p, y) \mathbf{p}_r(H_s, T_p, y)$$

where  $\eta_{\text{pcc}}$  is power conversion chain efficiency,  $\eta_{\text{oa}}$  is operational availability, and  $\eta_t$  is transmission efficiency

- The objective function can then be formulated as the average power per unit volume of the device:

$$p_v = \frac{p_a}{\pi R_{\text{wec}}^2 D_{\text{wec}}}$$

where  $R_{\text{wec}}$  and  $D_{\text{wec}}$  are the radius and draft of the heaving cylinder WEC device, respectively.

## → Array Considerations

- A total of  $n_{\text{wec}}$  WEC devices, fully characterized by the 2-by- $n_{\text{wec}}$  dimensional layout matrix

$$\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_{n_{\text{wec}}}]$$

- Each element of  $\mathbf{w}$  is a vector, composed of  $\mathbf{w}_p = [x_p, y_p]^T$
- The relative distance and angle between  $p$ th and  $q$ th bodies is characterized as  $l_{pq}$  and  $\theta_{pq}$ , respectively

## → Surrogate Models for Hydrodynamic Interactions

- The goal is to efficiently estimate the **hydrodynamic interaction effect**
- Direct surrogate modeling of these coefficients is computationally prohibitive
- **Many-body expansion** (MBE) principles, along with **artificial neural networks** (ANN) can be used to ease the computational cost
- In MBE, the total interaction effect among  $n_{\text{wec}}$  bodies is estimated as the summation of effects corresponding to a finite number of clusters<sup>1</sup>
- MBE systematically captures the effects of a single-, two-, three-, and  $m$ -body clusters<sup>2</sup>
- We use MBE up to second order, i.e. accounting for single- and two-body clusters

<sup>1</sup> E. Suarez, Diaz, and D. Suarez 2009    <sup>2</sup> E. Suarez, Diaz, and D. Suarez 2009

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# Surrogate Modeling

## → Data Processing

- Extreme and unreasonable design combinations are avoided by only considering cases where the radius and draft ratio are within an acceptable range
- A safety distance, proportional to the radius of the WEC, is necessary for the reliable maintenance of WEC devices. This safety distance is also considered when generating data for the training of ANNs.
- All inputs and outputs (shown below) are appropriately normalized.

$$\tilde{\mathbf{F}}_e = \hat{\mathbf{F}}_e / (\rho g \pi R_{\text{wec}}^2 D_{\text{wec}})$$

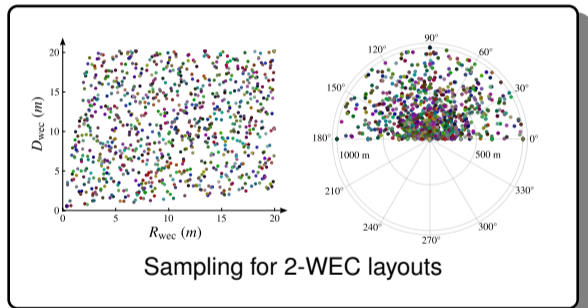
$$\tilde{\mathbf{A}} = \mathbf{A} / (\rho \pi R_{\text{wec}}^2 D_{\text{wec}})$$

$$\tilde{\mathbf{B}} = \mathbf{B} / (\omega \rho \pi R_{\text{wec}}^2 D_{\text{wec}})$$

- To reduce the range of QoI, we transformed each solution set to the range of  $[-1, 1]$  through a linear transformation.
- However, this requires additional ANNs need to be developed in order to estimate the range and offset of these linear transformations.

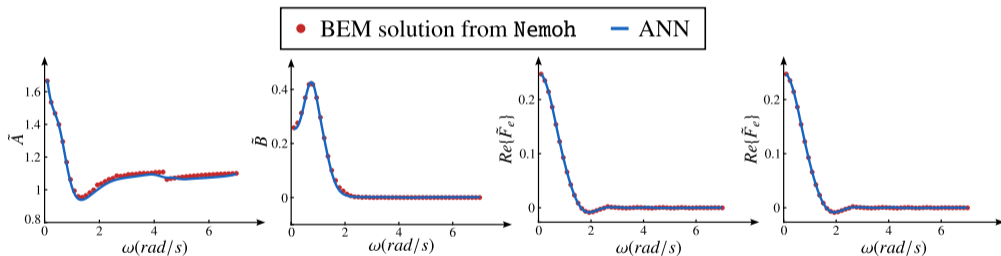
## → Developing Surrogate Models

- We used Latin hypercube sampling
- The first-order term in MBE only needs QoI for a single WEC at the origin, but with a sufficient number of samples for different WEC radius and draft
- The second-order term in MBE needs QoI calculated for two WECs with various ( $R_{wec}$ ), draft ( $D_{wec}$ ), relative distance ( $l_{pq}$ ), and relative angle ( $\theta_{pq}$ )



## → First-Order Surrogate Models

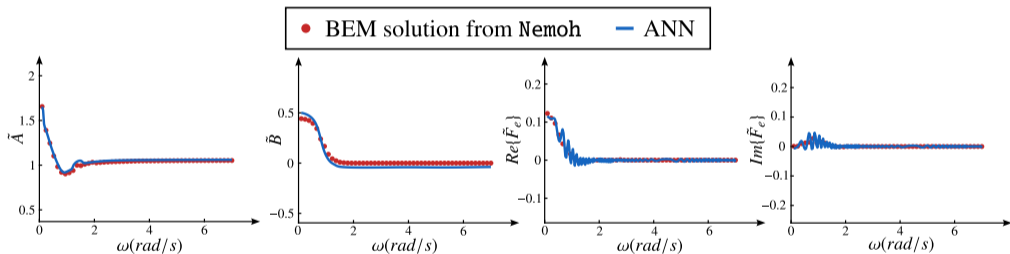
- The input to the first-order surrogate model is  $\tilde{\mathbf{v}}_1 = [\tilde{R}_{\text{wec}}, \tilde{D}_{\text{wec}}, \tilde{\omega}]^T$
- The output is  $\tilde{\mathbf{y}}_1 = [\tilde{a}, \tilde{b}, \text{Re}\{\tilde{f}_e\}, \text{Im}\{\tilde{f}_e\}]^T$



1-WEC surrogate models with  $R_{\text{wec}} = 8 \text{ m}$  and  $D_{\text{wec}} = 4 \text{ m}$

## → Second-Order Surrogate Models

- The input to the second-order surrogate model is  $\tilde{\mathbf{v}}_2 = [\tilde{R}_{\text{wec}}, \tilde{D}_{\text{wec}}, \tilde{l}_{pq}, \tilde{\theta}_{pq}, \tilde{\omega}]^T$
- The output is  $\tilde{\mathbf{y}}_2 = [\tilde{a}_{11}, \tilde{a}_{12}, \tilde{b}_{11}, \tilde{b}_{12}, \text{Re}\{\tilde{f}_{e_{11}}\}, \text{Im}\{\tilde{f}_{e_{11}}\}]^T$



(a) Added mass      (b) Damping coefficient      (c) Excitation force (r)      (d) Excitation force (i)

2-WEC surrogate models  $R_{\text{wec}} = 8$  m,  $D_{\text{wec}} = 4$  m,  $l_{pq} = 200$  m, and  $\theta_{pq} = 0.078$



## → MBE using Surrogate Models

- Defining the 1WEC surrogate models as  $[f_1^a, f_1^b, f_1^{fr}, f_1^{fim}]^T$  and the 2-WEC surrogate model functions as  $[f_2^{a11}, f_2^{a12}, f_2^{b11}, f_2^{b12}, f_2^{fr}, f_2^{fim}]^T$ , the interaction effect is estimated as:

$$\Delta \tilde{a}_{11} = \tilde{a}_{11} - \tilde{a} = f_2^{a11}(\tilde{\mathbf{v}}_2) - f_1^a(\tilde{\mathbf{v}}_1)$$

$$\Delta \tilde{a}_{12} = f_2^{a12}(\tilde{\mathbf{v}}_2)$$

$$\Delta \tilde{b}_{11} = \tilde{b}_{11} - \tilde{b} = f_2^{b11}(\tilde{\mathbf{v}}_2) - f_1^b(\tilde{\mathbf{v}}_1)$$

$$\Delta \tilde{b}_{12} = f_2^{b12}(\tilde{\mathbf{v}}_2)$$

- For excitation force, the additive effect is captured as:

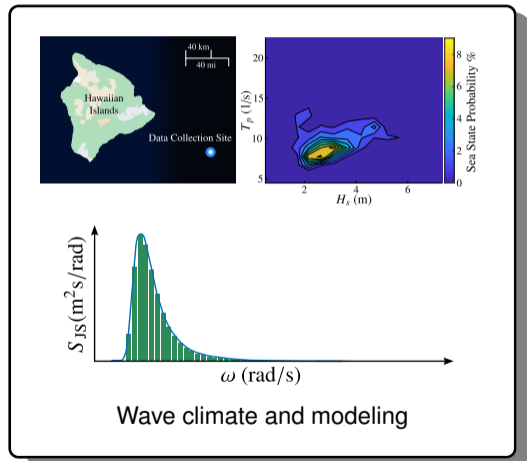
$$\begin{aligned} \Delta \tilde{f}_{e11} &= (\tilde{f}_e - \tilde{f}_{e11}) \exp(ikL) \\ &= ([f_{fr}^1(\tilde{\mathbf{v}}_1) + if_{fim}^1(\tilde{\mathbf{v}}_1)] - [f_{fr}^2(\tilde{\mathbf{v}}_2) + if_{fim}^2(\tilde{\mathbf{v}}_2)]) \exp(ikL) \end{aligned}$$

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# Formulation & Results

## → Wave Climate and Modeling

- Data collected off the coast of the Hawaiian Islands for 30 years (1976-2005) was used in this study
- Gaussian quadrature with  $n_{pq}$  points in each dimension was used to approximate the probability for various significant wave heights  $H_s$  and wave periods  $T_p$
- The Gauss quadrature nodes/weights were used in MATLAB's *ksdensity* (kernel distribution characterized by a smoothing function and a bandwidth value) to represent the (non-parametric) joint probability distribution function
- JONSWAP spectrum was used with the superposition of  $n_r$  regular waves



## → Problem Formulation

- $R_{wec}$  and  $D_{wec}$  are optimized for the farm
- $\mathbf{w}$ , along with  $[\mathbf{K}_{pto}, \mathbf{B}_{pto}]$  are optimized for each individual device
- $s_d = (R_{wec}/5) \times 50$  m is safe distance (to allow maintenance ships to pass)<sup>1</sup>
- The farm area is restricted to a box with dimensions of  $\pm 0.5 \times \sqrt{20000n_{wec}}$  m in  $x$  and  $y$  axes<sup>2</sup>

$$\text{minimize: } \quad -p_v(\mathbf{p}, \mathbf{u}, \mathbf{w})$$

$$\text{subject to: } \quad 2R_{wec} + s_d - L_{pq} \leq 0$$

$$\quad \quad \quad \forall p, q = 1, 2, \dots, n_{wec} \quad p \neq q$$

$$\underline{\mathbf{p}} \leq \mathbf{p} \leq \bar{\mathbf{p}}$$

$$\underline{\mathbf{u}} \leq \mathbf{u} \leq \bar{\mathbf{u}}$$

$$\underline{\mathbf{w}} \leq \mathbf{w} \leq \bar{\mathbf{w}}$$

$$\text{where: } \quad \mathbf{p} = [R_{wec}, D_{wec}]^T \in \mathbb{R}^2$$

$$\mathbf{u} = [\mathbf{K}_{pto}, \mathbf{B}_{pto}]^T \in \mathbb{R}^{2n_{wec}}$$

$$\mathbf{w} = [\mathbf{x}, \mathbf{y}] \in \mathbb{R}^{2(n_{wec}-1)}$$

<sup>1</sup> Neshat, Mirjalili, et al. 2022    <sup>2</sup> Neshat, Mirjalili, et al. 2022

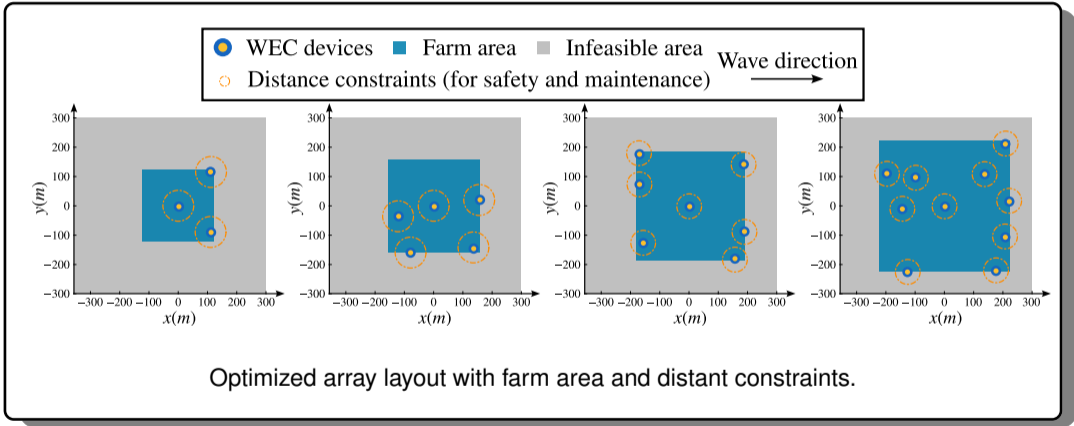
## → Results

Using MATLAB's *surrogateopt* with 300 function evaluations:

- + Practical framework for system-level WEC farm investigations
- + Computational tractability
- + Unique Control design
- Accuracy is affected by
  - Order of MBE
  - Approximation error from surrogate models
  - Limit on function evaluations in global optimization

Case Study	$R_{wec}$ [m]	$D_{wec}$ [m]	$B_{pto}$ [Ns/m]	$K_{pto}$ [N/m]	$w$ [m]	$p_v$ [MW/m <sup>3</sup> ]	Time [s]
3-WEC	8.83	0.54	$1.7 \times 10^8$	$-2.97 \times 10^8$	[0, 0]	175.58	97
			$1.65 \times 10^8$	$2.53 \times 10^8$	[109, -89.52]		
			$1.1 \times 10^8$	$-4.21 \times 10^6$	[114.07, 115.89]		
5-WEC	7.12	0.5	$2.64 \times 10^8$	$1.44 \times 10^8$	[0, 0]	566.02	143
			$1.67 \times 10^8$	$5.26 \times 10^6$	[-138.76, -119.09]		
			$1.67 \times 10^8$	$2.56 \times 10^8$	[-24.07, -154.43]		
			$2.94 \times 10^8$	$-2.95 \times 10^8$	[-41.54, 129.69]		
			$2.72 \times 10^8$	$2.73 \times 10^8$	[-49.97, -70.61]		
7-WEC	6.71	0.62	$8.86 \times 10^7$	$-2.95 \times 10^8$	[0, 0]	$2.21 \times 10^3$	219
			$1.94 \times 10^8$	$-3.24 \times 10^7$	[154, -181.38]		
			$1.16 \times 10^8$	$1.32 \times 10^8$	[-169.51, 73.9]		
			$6.66 \times 10^7$	$-8.66 \times 10^6$	[-171.96, 178.79]		
			$2.84 \times 10^8$	$2.32 \times 10^8$	[185.47, 142.23]		
			$2.67 \times 10^8$	$1.69 \times 10^8$	[187.08, -86.12]		
			$2.67 \times 10^8$	$4.4 \times 10^7$	[-156.86, -124.98]		
10-WEC	7.32	0.5	$2.24 \times 10^8$	$-1.73 \times 10^8$	[0, 0]	$4.12 \times 10^3$	379
			$2.99 \times 10^8$	$-2.56 \times 10^7$	[-100.66, 98.45]		
			$1.29 \times 10^8$	$-2.83 \times 10^8$	[-127.97, -223.61]		
			$6.9 \times 10^7$	$1.82 \times 10^8$	[-145.75, 8.36]		
			$2.41 \times 10^8$	$-2.08 \times 10^7$	[219.94, 18.84]		
			$2.23 \times 10^8$	$-6.74 \times 10^7$	[-198.02, 112.55]		
			$1.89 \times 10^8$	$2.14 \times 10^7$	[206.39, 214.89]		
			$2.68 \times 10^7$	$1.45 \times 10^8$	[204.82, -102.91]		
			$1.81 \times 10^8$	$-2.27 \times 10^8$	[174.45, -218.4]		
$1.66 \times 10^8$	$1.49 \times 10^7$	[135.33, 109.09]					

## → Results (continued)



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# Conclusions

## → Conclusions and Future Works

- Due to nonlinear and complex dynamics, the sizing, control, and array layout optimization of wave energy converters (WECs) are **coupled** disciplines and must be approached concurrently from the early stages of the design process.
- Using surrogate modeling and MBE principles (up to second order), the wave energy converter farm problem was solved in a **computationally tractable** manner.
- Optimized solutions points out to the importance of **individual control** of each WEC device.
- Results may improve by using a higher number of terms in MBE, and running the optimization algorithm for a higher number of function evaluation, and improving performance of surrogate models.
- Techniques from machine learning, including selective sampling and active learning can improve the performance of the resulting surrogate models.
- Variations in depth and geographical locations are important to consider in future work.



## → Acknowledgment

This research was supported by NSF under grant number CMMI- 2034040



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# Questions?

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# Appendix

## → Wave-Structure Interactions

- In linear potential flow theory, the fluid velocity potential  $\phi$  is divided into potentials corresponding to incident  $\phi_i$ , scattered  $\phi_s$ , and radiated  $\phi_r$ , such that  $\phi = \phi_i + \phi_s + \phi_r$ <sup>1</sup>
  - **Incident** waves represent the propagation of the wave in the absence of any structure
  - **Scattered** waves appear as a result of the interaction of the waves and motionless structures
  - **Radiated** waves result from the motion of the structure
- The real part of the radiation force is **added mass**, and the imaginary part is **damping coefficient**
- Scattered and radiated waves are very important in WEC farm design because they propagate in all directions (affecting all nearby devices)
- This leads to strong **coupling** between plant, PTO control, and farm layout<sup>2</sup>
- Boundary element method (BEM) software NEMOH is used to generate hydrodynamic coefficients for the data-driven surrogate model development<sup>3</sup>

<sup>1</sup> Ning and Ding 2022   <sup>2</sup> Babarit 2013   <sup>3</sup> Babarit and Delhommeau 2015; Kurnia, Ducrozet, and Gilloteaux 2022

## → Settings

- Array investigations with 3, 5, 7, and 10 WECs are carried out
- MATLAB's *surrogateopt* solver with 300 function evaluations is used for global optimization

Option	Value	Option	Value
$\bar{R}_{wec}$	0.5 m	$\bar{R}_{wec}$	10 m
$\bar{D}_{wec}$	0.5 m	$\bar{D}_{wec}$	10 m
$\bar{\mathbf{k}}_{pto}$	$-3 \times 10^8$ N/m	$\bar{\mathbf{k}}_{pto}$	$3 \times 10^8$ N/m
$\bar{\mathbf{B}}_{pto}$	0 Ns/m	$\bar{\mathbf{B}}_{pto}$	$3 \times 10^8$ Ns/m
$\bar{\mathbf{x}}$	$-0.5\sqrt{2n_{wec} \times 10^4}$ m	$\bar{\mathbf{x}}$	$0.5\sqrt{2n_{wec} \times 10^4}$ m
$\bar{\mathbf{y}}$	$-0.5\sqrt{2n_{wec} \times 10^4}$ m	$\bar{\mathbf{y}}$	$0.5\sqrt{2n_{wec} \times 10^4}$ m
$\rho$	1025 kg/m <sup>3</sup>	$g$	9.81 m/s <sup>2</sup>
$s_d$	$50 \times R_{wec}/5$ m	$n_{wec}$	[3, 5, 7, 10]
$n_{yr}$	30 years	$n_r$	200
$n_{gq}$	20	$\eta_{pcc}$	0.8
$\eta_{oa}$	0.95	$\eta_t$	0.98