Kinematic Design of Optimally Fault Tolerant Robots for Different Joint Failure Probabilities

Biyun Xie and Anthony A. Maciejewski, *Fellow, IEEE*

***Abstract*—A measure of fault tolerance for different joint failure probabilities is defined based on the properties of the singular values of the Jacobian after failures. Using this measure, methods to design optimally fault tolerant robots for an arbitrary set of joint failure probabilities and multiple cases of joint failure probabilities are introduced separately. Given an arbitrary set of joint failure probabilities, the optimal null space that optimizes the fault tolerant measure is derived, and the associated isotropic Jacobians are constructed. The kinematic parameters of the optimally fault tolerant robots are then generated from these Jacobians. One special case, i.e., how to construct the optimal Jacobian of spatial 7R robots for both positioning and orienting is further discussed. For multiple cases of joint failure probabilities, the optimal robot is designed through optimizing the sum of the fault tolerant measures for all the possible joint failure probabilities. This technique is illustrated on planar 3R robots, and it is shown that there exists a family of optimal robots.**

***Index Terms*—Redundant robots, kinematics, fault tolerant robots.**

I. Introduction

In RECENT years, robots have become increasingly common for a wide range of applications, and the reliability of robots operating in structured and benign environments is quite high. However for dangerous tasks in remote or hazardous environments, where routine maintenance can not be performed, one must plan for the probability of failures. Such applications include space exploration [1], nuclear waste remediation [2], and disaster rescue [3]. One may also want to employ fault tolerant robots in applications requiring high degrees of safety, such as robotic surgery [4], rehabilitation [5], and human robot interaction [6]. In all cases, one needs to employ some type of fault tolerant control scheme, e.g., adaptive [7], Markovian-based [8], or active [9], [10] fault tolerant control, and possibly compute the available workspace after a failure [11].

In addition to improving the reliability of the components of the robot, one can also employ kinematically redundant robots to take advantage of their extra degrees of freedom (DOF) to tolerate different types of joint failures. It has been previously shown that an improperly designed kinematically redundant robot can actually be fault intolerant [12], so that there has been significant effort devoted to identifying fault-tolerant kinematic designs. Most of this work has used a locked joint failure mode, either because the failure exhibits this property or fail-safe brakes are employed. Researchers have explored the number of DOFs that are necessary and sufficient to guarantee fault tolerance, along with how these joints should be distributed [13]. Other work has assumed a certain amount of redundancy, frequently a single additional DOF, and then developed an optimal kinematic design [14], [15]. In [14], researches applied a genetic algorithm to optimize the joint types and link lengths based on the fault tolerant workspace reachability index. In [15], researches incorporate fault tolerance and reliability into the design of robot manipulators using fault tree analysis. Other studies have shown that there exist entire classes of designs with desired optimal fault tolerance properties [16]–[21]. In [16] researchers constructed optimally fault tolerant Jacobians for equal joint failure probabilities. Based on these Jacobians, families of optimally fault tolerant planar 3R robots [17], spatial positioning 4R robots [18] and spatial positioning and orienting 7R robots [19] were designed. The structural characteristics and global pre- and post-failure dexterity performance of the 7! optimally fault tolerant robots generated in [19] were further analyzed in [20]. In [21], a class of orthogonal Gough-Stewart platforms was developed that provide optimal fault tolerant manipulability under a single failure.

All of the above studies [16]–[19] on fault tolerant design only consider the case where all joints are equally likely to fail when designing the optimally fault tolerant robots. While this is typically how the robot components are designed, individual variations within the components or environmental factors will change the failure probabilities over the robot's lifetime. Identical components may also have unequal failure probabilities due to the different loads that they are exposed to, which is typically the case for modular robot designs [22]. Thus, when designing an optimally fault tolerant robot, this factor needs to be considered. This is the focus of this work. Specifically, the main contributions of this letter are as follows: (1) determine the equations for the null space and the canonical form of a Jacobian that optimizes the minimum singular value after a failure for arbitrary joint failure probabilities (2) design classes of robots that are optimal in terms of the resulting minimum singular value after a failure for an arbitrary set of joint failure probabilities, e.g., planar 3R robots, spatial positioning 4R robots and spatial positioning and orienting 7R robots; (3) develop a method to

Manuscript received September 10, 2017; accepted December 31, 2017. Date of publication January 12, 2018; date of current version January 25, 2018. This letter was recommended for publication by Associate Editor A. Dietrich and Editor P. Rocco upon evaluation of the reviewers' comments. This work was supported in part by the National Science Foundation under Contract IIS-0812437. *(Corresponding author: Biyun Xie.)*

The author are with the Department of Electrical and Computer Engineering, Colorado State University, Fort Collins, CO 80523 USA (e-mail: biyunxie@rams.colostate.edu; aam@colostate.edu).

Digital Object Identifier 10.1109/LRA.2018.2792691

design optimally fault tolerant robots for cases where the joint failure probabilities change, and illustrate this method for planar 3R robots. This is in contrast to the work in [12] where unequal joint failure probabilities were first proposed, however, only for the reduced manipulability measure of fault tolerance and not for the minimum singular value. In addition, the kinematic design problem was not explored in [12].

The rest of this letter is organized as follows. In the next section a measure of fault tolerance for different joint failure probabilities is defined. Designing optimally fault tolerant robots for an arbitrary set of joint failure probabilities is presented in Section III and for multiple cases of joint failure probabilities in Section IV. Finally, the conclusions of this work are presented in Section V.

II. A Definition of Fault Tolerance for Different Joint Failure Probabilities

The local kinematic dexterity of a robot is frequently described by the properties of the Jacobian matrix $\mathbf{J}$, which linearly relates joint velocities to end-effector velocities. The Jacobian $\mathbf{J}$ can be written as a collection of columns

$$\mathbf{J}_{m\times n} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_n \end{bmatrix} \tag{1}$$

where m is the dimension of the workspace, n is the number of DOFs of the robot, and $\mathbf{j}_i$ is the contribution of joint i to the end-effector velocity. For redundant robots, the DOFs of the robot n is larger than the workspace dimension m, so that the extra DOFs can be utilized to realize fault tolerance.

If an arbitrary single joint fails and is locked, this joint is no longer able to contribute any motion of the end-effector. Therefore, the reduced Jacobian after joint f fails can be easily obtained by removing the fth column from the original Jacobian, i.e.,

$${}^f\mathbf{J}_{m\times(n-1)} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_{f-1} & \mathbf{j}_{f+1} & \cdots & \mathbf{j}_n \end{bmatrix}. \tag{2}$$

The local kinematic dexterity of the reduced robot can be quantified by combinations of the singular values of the associated reduced Jacobian, such as their ratio (condition number) or product (manipulability). In this work, the minimal singular value of the reduced Jacobian, denoted as ${}^f\sigma_m$, is used to define the dexterity of the reduced robot after joint f is locked. This is because the minimum singular value tends to denominate the behavior of both the condition number and the manipulability, but also because the minimum singular value is a measure of proximity to a singularity, i.e., a measure of worst-case dexterity over all end-effector motion directions [16]. A larger value of ${}^f\sigma_m$ means that the reduced robot maintains a higher motion ability in the worst-case direction, so that a robot in this configuration is more fault tolerant to the failure of joint f. When ${}^f\sigma_m$ equals 0, the reduced robot is in a singular configuration, and this robot in this configuration is fault intolerant to the failure of joint f.

The definition of fault tolerance in this work is based on all the possible ${}^f\sigma_m$ considering all the possible joint failures. A measure of fault tolerance is defined as

$$\mathcal{F} = w_1\,{}^1\sigma_m + w_2\,{}^2\sigma_m + \cdots + w_n\,{}^n\sigma_m \tag{3}$$

where the w_i's are the weighting coefficients, satisfying $\sum_{i=1}^{n} w_i = 1$. In order to weight each reduced Jacobian's minimum singular value with its probability of occurring, the weights are selected as

$$w_i = \frac{p_i}{p_1 + p_2 + \cdots + p_n} \tag{4}$$

where p_i is the failure probability of joint i. When all the joints are equally likely to fail, all the weighting coefficients are equal, which implies that all the ${}^f\sigma_m$ are equally important. At the other extreme, i.e., when one of the joints is much more likely to fail than the others, the weighting coefficient of this joint approaches 1, and the weighting coefficients of the other joints approach 0 so that the optimization will focus on this joint's ${}^f\sigma_m$. Based on this definition of fault tolerance, the kinematic design of optimally fault tolerant robots will be studied in the following sections.

III. Designing For an Arbitrary Set Of Joint Failure Probabilities

A. Method

We assume that robot manufacturers have historical data on the failure probabilities of the various physical components of joints used in the design of their robots. If such data is not available, then one can employ the reliability analysis described in [23] using the component reliability data available in [24]. Therefore, these probabilities are known during the kinematic design of robots where the fault tolerance measure defined in (3) is maximized. This insures that the robot achieves optimally fault tolerant performance after a locked joint failure. In addition, it is desirable to have optimal dexterity before a failure, so that the robot is required to have an isotropic pre-failure Jacobian, i.e., the singular values are all equal. Therefore, the problem in this section can be stated as finding a Jacobian that satisfies

$$\begin{aligned} \max \quad & \mathcal{F} = w_1\,{}^1\sigma_m + w_2\,{}^2\sigma_m + \cdots + w_n\,{}^n\sigma_m \\ \text{s.t.} \quad & \sigma_1 = \sigma_2 = \cdots = \sigma_m = \sigma \end{aligned} \tag{5}$$

where σ is the singular value before a failure. The variables in this optimization are the elements of the Jacobian. Because a Jacobian is a function of a robot's kinematic parameters, one can determine the optimal kinematic design from the Jacobian that is the solution to (5).

Once an optimally fault tolerant Jacobian is identified for a given set of joint failure probabilities, the method developed in [17] can be applied to generate robot kinematic parameters from this Jacobian, and a family of optimally fault tolerant robots can be obtained. In this section, the optimal null space that maximizes the fault tolerance measure in (3) is identified first. Then, the isotropic Jacobian with this optimal null space is constructed.

1) Finding the Optimally Fault Tolerant null Vector: For manipulators with one degree of redundancy, the absolute value of the ith component of its Jacobian's null vector, denoted n_i, is equal to the absolute value of the determinant of the Jacobian after joint i fails, which is the product of the singular values after joint i fails, i.e.,

$$|n_i| = |\det({}^i\mathbf{J})| = {}^i\sigma_1\, {}^i\sigma_2\, \cdots\, {}^i\sigma_m. \quad (6)$$

The singular values before and after a failure satisfy the following inequality [25]

$$\sigma_1 \geqslant {}^i\sigma_1 \geqslant \sigma_2 \geqslant {}^i\sigma_2 \geqslant \cdots \geqslant \sigma_m \geqslant {}^i\sigma_m. \quad (7)$$

Note that the directions of the singular vectors associated with σ_i and ${}^f\sigma_i$ might be quite different. Therefore, although ${}^f\sigma_i > 0$, the robot might still lose the ability to move along the direction of the singular vector associated with σ_i after joint f fails.

For isotropic Jacobians, the singular values before a failure are all equal, i.e., $\sigma_i = \sigma\ \forall\, i$, so from (7) it is easy to see that

$${}^i\sigma_1 =^i \sigma_2 = \cdots =^i \sigma_{m-1} = \sigma. \quad (8)$$

Substituting (8) into (6), the null space can be obtained as follows

$$|n_i| = |\det({}^i\mathbf{J})| = {}^i\sigma_m \sigma^{m-1}. \quad (9)$$

The normalized version of the above equation is given by

$$|\hat{n}_i| = \frac{{}^i\sigma_m \sigma^{m-1}}{\sqrt{\sum_{i=1}^{n}[\det({}^i\mathbf{J})]^2}}. \quad (10)$$

By applying the Binet-Cauchy Theorem for an isotropic J

$$\det(\mathbf{J}\mathbf{J}^{\mathrm{T}}) = \sum_{i=1}^{n}[\det({}^i\mathbf{J})]^2 = \sigma^{2m}. \quad (11)$$

Equation (10) becomes

$$|\hat{n}_i| = \frac{{}^i\sigma_m}{\sigma} \quad (12)$$

which can be interpreted as the post-failure dexterity divided by the pre-failure dexterity. We can now use this relationship along with the optimization defined by (5) to determine the form of an optimal null vector.

Maximizing the objective function defined in (5) is equivalent to

$$\begin{aligned} \max \quad & w_1|\hat{n}_1| + w_2|\hat{n}_2| + \cdots + w_n|\hat{n}_n| \\ \text{s.t.} \quad & |\hat{n}_1|^2 + |\hat{n}_2|^2 + \cdots + |\hat{n}_n|^2 = 1. \end{aligned} \quad (13)$$

This constrained optimization problem can be solved by the method of Lagrange multipliers. The Lagrange function is defined as

$$\begin{aligned} L(|\hat{n}|, \lambda) &= w_1|\hat{n}_1| + w_2|\hat{n}_2| + \cdots + w_n|\hat{n}_n| \\ &\quad - \lambda(|\hat{n}_1|^2 + |\hat{n}_2|^2 + \cdots + |\hat{n}_n|^2 - 1). \end{aligned} \quad (14)$$

Solving the set of equations resulting from taking partial derivatives yields

$$|\hat{n}_i| = \frac{w_i}{\sqrt{w_1^2 + w_2^2 + \cdots + w_n^2}}. \quad (15)$$

Therefore, when the elements of a Jacobian's null vector are given by (15) the objective function reaches its maximum value and ${}^i\sigma_m = \sigma w_i / \sqrt{w_1^2 + w_2^2 + \cdots + w_n^2}$.

There are two additional items that should be pointed out about the above derivation. First, if the constraint of isotropy in (5) is replaced by a bound on σ_1, i.e., a simple norm constraint on the Jacobian, the optimal solution will still be the same. This is true because to maximize the objective function, each ${}^i\sigma_m\ \forall\, i$ should be as large as possible. However, the singular values before and after a failure satisfy the inequality in (7), which shows that ${}^i\sigma_m$ is bounded by σ_m. Therefore, the minimum singular value before a failure should also be as large as possible. The value of σ_m reaches its maximum when $\sigma_1 = \sigma_2 = \cdots = \sigma_m = \sigma$, i.e., the Jacobian is isotropic, which is the same as the constraint in (5). Second, because of isotropy, maximizing the fault tolerance measure defined in (3) also maximizes the reduced manipulability [12]. This is true because the determinant in (9) is equal to the reduced manipulability. Furthermore, the ratio of the reduced manipulabilities is equal to the ratio of the weighting coefficients.

2) Constructing Isotropic Jacobians According to the Optimally Fault Tolerant null Vector: This subsection will discuss how to construct isotropic Jacobians whose associated null vectors satisfy (15). An isotropic Jacobian can be decomposed into the following two matrices,

$$\mathbf{J} = \mathbf{D}\mathbf{U}, \quad (16)$$

where $\mathbf{D}$ is an $m \times n$ diagonal matrix whose diagonal elements are σ, and $\mathbf{U}$ is an $n \times n$ orthogonal matrix, whose column can be written as

$$\mathbf{u}_i = \begin{bmatrix} \frac{1}{\sigma}\mathbf{j}_i \\ \hat{n}_i \end{bmatrix}. \quad (17)$$

The first m rows of $\mathbf{U}$ are equal to the Jacobian divided by σ, which without loss of generality can be considered to be equal to 1, and the last row is equal to the transpose of the null vector. For optimally fault tolerant Jacobians, the elements of the null vector are given by (15), where for the following derivation we assume each element to be non-negative.

We now construct an orthogonal $\mathbf{U}$ that satisfies these constraints as follows: (1) The weighting coefficients w_i are sorted in an ascending order, and the columns of the Jacobian are permuted accordingly. (2) All the entries in the first column of the permuted Jacobian are set to 0, except for the first entry that is computed using the constraint that $\|\mathbf{u}_1\| = 1$. (3) All the entries in the second column of the permuted Jacobian are set to 0, except the first and second entry, where the first entry is calculated using constraint that $\mathbf{u}_1 \cdot \mathbf{u}_2 = 0$, and the second entry is calculated using the constraint that $\|\mathbf{u}_2\| = 1$. (4) The remaining columns are obtained in a similar way, using the properties of an orthogonal matrix, i.e., that $\|\mathbf{u}_i\| = 1$ and $\mathbf{u}_i \cdot \mathbf{u}_j = 0\ \forall i, j$. The element in the pth row and qth column of the permuted Jacobian matrix $\mathbf{J}$ is now in the form

$$\begin{aligned} j_{pq} &= 0 && \text{where } p > q \\ j_{pq} &= -\sqrt{\sum\nolimits_{i=p+1}^{n} w_i^2 \Big/ \sum\nolimits_{i=p}^{n} w_i^2} && \text{where } p = q \\ j_{pq} &= \frac{w_p w_q}{\sqrt{\sum_{i=p}^{n} w_i^2 \sum_{i=p+1}^{n} w_i^2}} && \text{where } p < q \end{aligned} \quad (18)$$

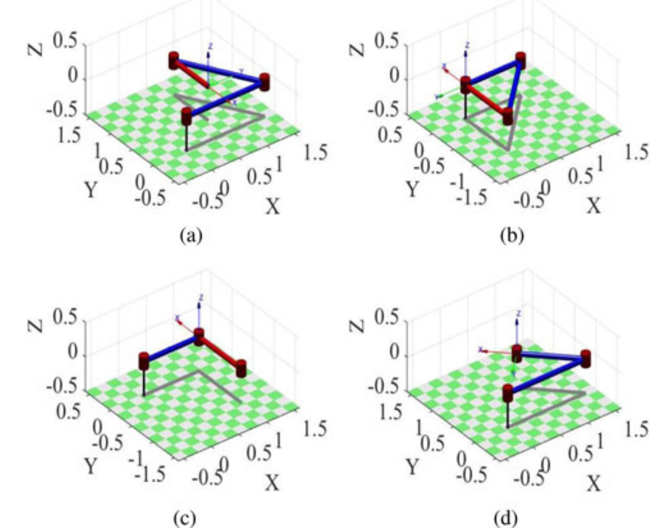

Fig. 1. The optimally fault tolerant planar 3R robots are shown for different joint failure probabilities. The optimal robot when all the joints are equally likely to fail is shown in (a). The optimal robots when each joint is failing are shown in (b)–(d) for joints 1 to 3, respectively. Note that for the robots in (b)–(d), the end-effector is located at the joint axis that is likely to fail. A robot designer can use these extremal cases to determine the desired link lengths based on the relative probability of failure for the different joints.

Finally, the columns of the Jacobian are permuted back so that the null vector of this isotropic Jacobian is

$$\hat{\mathbf{n}} = \begin{bmatrix} \frac{w_1}{\sqrt{\sum_{i=1}^{n} w_i^2}} & \frac{w_2}{\sqrt{\sum_{i=1}^{n} w_i^2}} & \cdots & \frac{w_n}{\sqrt{\sum_{i=1}^{n} w_i^2}} \end{bmatrix}^{\mathrm{T}}. \quad (19)$$

After the optimally fault tolerant Jacobians are constructed, the method developed in [17] can be applied to generate robot kinematic parameters from these Jacobians. It is important to note that the optimally fault tolerant Jacobian is locally optimal and results in a robot design that is optimal at the design configuration. However, robots that have such configurations are more likely to have better performance throughout the workspace. After an optimally fault tolerant robot design is obtained, its workspace can be further evaluated using the method proposed in [19].

B. Results for Positioning Robots

1) Planar 3R Robots: We consider four illustrative examples for this simple case, i.e., equal probability of joint failure and an impending failure in each joint. For the equal probability of failure case, the weighting coefficient is $\mathbf{w} = [1/3 \quad 1/3 \quad 1/3]$. The optimal robot is shown in Fig. 1(a), where the link length vector $\mathbf{l}$ is given by $\mathbf{l} = [\sqrt{2} \quad \sqrt{2} \quad \sqrt{2/3}]$. This is the same as the result obtained in [16]. Now, consider the other extreme cases, i.e., one of the joints is known to have an imminent failure. The weighting coefficients for this case are $w_f = 1$ for the fth joint that is failing and $w_i = 0$ for the other two joints. The optimal robots for these cases are shown in Fig. 1(b)–(d), for $f = 1$ to 3 respectively, with the corresponding link lengths given by: ${}^1\mathbf{l} = [1 \quad \sqrt{2} \quad 1]$, ${}^2\mathbf{l} = [1 \quad 1 \quad 1]$ and ${}^3\mathbf{l} = [\sqrt{2} \quad 1 \quad 0]$ where we have appended the superscript of the failure joint to the link length vector $\mathbf{l}$. Note that when joint i's probability of failure is one, the end-effector of the robot must be located on this joint axis so that the contribution of joint i to the end-effector movement is 0 and its failure does not affect the end-effector motion.

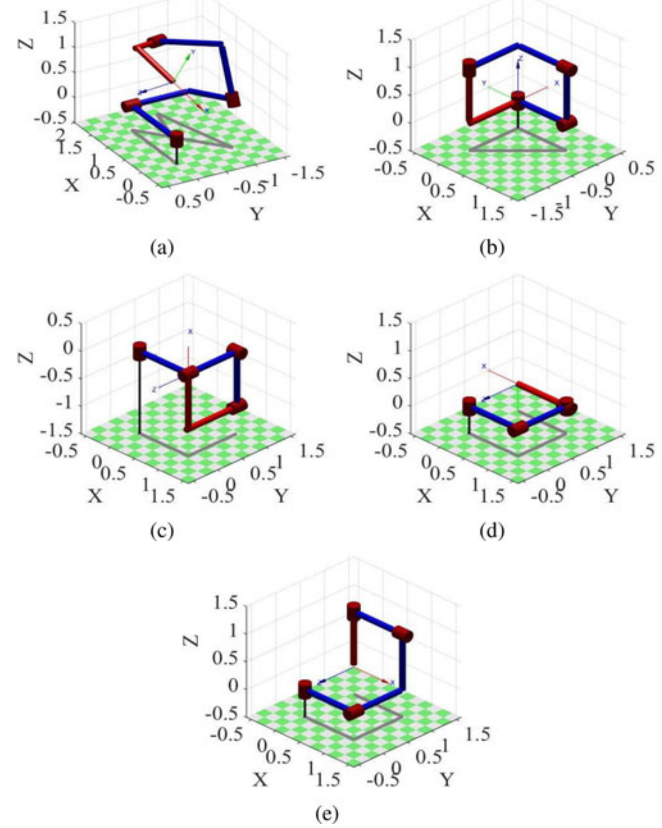

Fig. 2. The optimally fault tolerant spatial 4R robots are shown for different joint failure probabilities. The optimal robot when all the joints are equally likely to fail is shown in (a). The optimal robots when joints 1 to 4 are failing are shown in (b)–(e), respectively. Note that in (b)–(e), the end-effector is located on the joint axis that is failing. A robot designer can use these extremal cases to determine the desired link lengths based on the relative probability of failure for the different joints.

2) Spatial 4R Robots: Similar to the planar 3R robots, we consider five illustrative examples for this case. Note that as described in [18], there exists a family of optimal robots that can realize a given Jacobian, because the linear velocity represented by a column of the Jacobian can be realized by a one-dimensional set of joint axes oriented circularly around the end effector. In [18], the entire family of optimal robot designs were parameterized, however, here we simply select one example design where all the twist angles are selected to be $\pm 90^\circ$.

The optimal robots are shown in Fig. 2, where Fig. 2(a) is the case when the joints are equally likely to fail. The three singular values after joint i fails are ${}^i\sigma_1 = {}^i\sigma_2 = 1.00$, ${}^i\sigma_m = 0.50 \; \forall \, i$, which are also equal to the length of the semi axes of the manipulability ellipsoid after a failure. Fig. 2(b)–(e) are the cases when joint one to four is failing, respectively. Similarly to the 3R case, the end-effector is located on the joint axis that is failing in Fig. 2(b)–(e) so that there is no linear motion associated with the failing joint. Therefore, the manipulability ellipsoid after a failure is still a unit sphere, i.e., isotropic, the same as before a failure.

C. Results for Spatial 7DOF Robots for Positioning and Orienting

The optimally fault tolerant Jacobians of spatial 7DOF robots described by (18) can not be realized by rotational joints. This is true because for a rotational joint i, the associated column of a Jacobian is given by

$$\mathbf{j}_i = \begin{bmatrix} \mathbf{v}_i \\ \boldsymbol{\omega}_i \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{a}}_i \times \mathbf{p}_i \\ \hat{\mathbf{a}}_i \end{bmatrix} \tag{20}$$

where $\mathbf{v}_i$ and $\boldsymbol{\omega}_i$ are the linear velocity and angular velocity, respectively, $\hat{\mathbf{a}}_i$ is the unit vector of the rotational joint axis, and $\mathbf{p}_i$ is a position vector from the joint axis to the end-effector. Therefore, $\|\boldsymbol{\omega}_i\| = 1$ and the linear velocity must be orthogonal to the angular velocity, i.e., $\mathbf{v}_i \perp \boldsymbol{\omega}_i$. These constraints are not true for the Jacobian described by (18). In this section, we discuss modifications to the above approach so that spatial 7R robots can be designed.

For isotropic spatial 7R Jacobians (17) becomes

$$\mathbf{u}_i = \begin{bmatrix} \sqrt{\frac{3}{7}}\mathbf{v}_i \\ \sqrt{\frac{3}{7}}\boldsymbol{\omega}_i \\ \hat{n}_i \end{bmatrix} \tag{21}$$

where the factor of $\sqrt{\frac{3}{7}}$ is due to the constraint that $\mathbf{U}$ is orthogonal. To be optimally fault tolerant, the elements of the null vector must satisfy (15). In order to enforce the constraints that $\|\boldsymbol{\omega}_i\| = 1$ and $\mathbf{v}_i \perp \boldsymbol{\omega}_i$, one can represent the ith column of the Jacobian as

$$\mathbf{j}_i = \begin{bmatrix} \mathbf{v}_i \\ \hdashline \boldsymbol{\omega}_i \end{bmatrix} = \begin{bmatrix} \|\mathbf{v}_i\| \begin{bmatrix} \cos\alpha_i \sin\beta_i \cos\gamma_i + \sin\alpha_i \sin\gamma_i \\ \sin\alpha_i \sin\beta_i \cos\gamma_i - \cos\alpha_i \sin\gamma_i \\ -\cos\beta_i \cos\gamma_i \end{bmatrix} \\ \hdashline \begin{bmatrix} \cos\alpha_i \cos\beta_i \\ \sin\alpha_i \cos\beta_i \\ \sin\beta_i \end{bmatrix} \end{bmatrix}. \tag{22}$$

where α_i, β_i and γ_i are the parameters defining joint i. Using (15) and the fact that $\mathbf{u}_i$ is a unit vector (because $\mathbf{U}$ is orthogonal) one can show that the norm of the linear velocity is given by

$$\|\mathbf{v}_i\| = \sqrt{\frac{4}{3} - \frac{7w_i^2}{3\sum_{i=1}^{7} w_i^2}}. \tag{23}$$

By imposing the constraints that the rows of $\mathbf{U}$ are unit norm and orthogonal one can determine the values of the α_i's, β_i's and γ_i's to identify the optimally fault tolerant 7R manipulator Jacobian.

Unfortunately, it is not possible to satisfy all these constraints for an arbitrary set of joint failure probabilities. In particular, the optimal 7R manipulator Jacobian for equal joint failure probabilities has not been identified [16]. However, it is possible to solve the set of nonlinear constraint equations using the Levenberg-Marquardt algorithm to find one set of weighting coefficients, i.e., failure probabilities, that is close to equally likely, i.e., $\mathbf{w} = \begin{bmatrix} 0.15 & 0.11 & 0.11 & 0.12 & 0.19 & 0.19 & 0.12 \end{bmatrix}$,[1] where the associated 7R manipulator Jacobian is

$$\mathbf{J} = \begin{bmatrix} -0.38 & -1.02 & 0.52 & -0.39 & -0.10 & 0.48 & 0.69 \\ 0.04 & -0.30 & 0.39 & 0.79 & -0.70 & 0.59 & -0.79 \\ -0.91 & -0.16 & -0.84 & 0.57 & 0.52 & 0.41 & 0.01 \\ 0.68 & -0.28 & -0.79 & 0.44 & -0.65 & -0.03 & 0.74 \\ -0.66 & 0.51 & 0.56 & 0.66 & -0.39 & -0.55 & 0.65 \\ -0.31 & 0.81 & -0.23 & -0.61 & -0.66 & 0.83 & 0.17 \end{bmatrix} \tag{24}$$

D. Extension to Multiple Degrees of Redundancy and Joint Failures

1) Multiple Degrees of Redundancy: The above approach can be extended to the case of robots with multiple degrees of redundancy in a manner analogous to that in [12]. For the Jacobian of a robot with multiple degrees of redundancy, the matrix $\mathbf{U}$ in (16) can be decomposed into the following two matrices,

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} \tag{25}$$

where $\mathbf{U}_1$ is equal to the Jacobian divided by σ, and $\mathbf{U}_2$ is a matrix of $n - m$ orthonormal n-dimensional vectors in the null space of $\mathbf{J}$. Let $\hat{\mathbf{n}}_i$ be the ith column of $\mathbf{U}_2$, and its norm can be calculate as

$$\|\hat{\mathbf{n}}_i\| = \frac{\det({}^i\mathbf{J})}{\det(\mathbf{J})} = \frac{{}^i\sigma_1 \, {}^i\sigma_2 \, \cdots \, {}^i\sigma_m}{\sigma_1 \, \sigma_2 \, \cdots \, \sigma_m}. \tag{26}$$

Substituting (8) into (26), the relationship between the null space and the minimum singular value after a failure is given by

$$\|\hat{\mathbf{n}}_i\| = \frac{{}^i\sigma_m}{\sigma}. \tag{27}$$

Equation (12) can be considered as a special case of this equation, in which each column has only one element. Likewise, the generalization of the constraint in (13) becomes

$$\|\hat{\mathbf{n}}_1\|^2 + \|\hat{\mathbf{n}}_2\|^2 + \cdots + \|\hat{\mathbf{n}}_n\|^2 = n - m. \tag{28}$$

Similarly, applying the method of Lagrange multipliers, the null space of the optimally fault tolerant Jacobian for the robots with multiple degrees of redundancy needs to satisfy

$$\|\hat{\mathbf{n}}_i\| = w_i \sqrt{\frac{n - m}{w_1^2 + w_2^2 + \cdots + w_n^2}} \leqslant 1. \tag{29}$$

[1]The elements of w do not add up to 1 due to rounding.

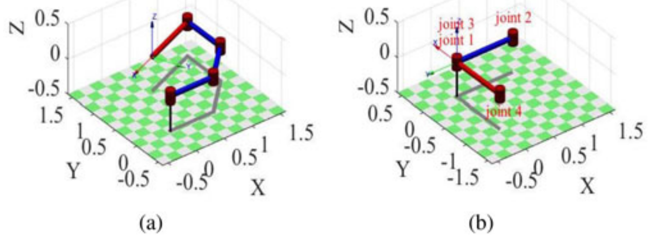

Fig. 3. The optimally fault tolerant planar 4R robot designs for two simultaneous joint failures are shown. In (a) all joints are equally likely to fail. In (b) joint 1 and joint 3 are guaranteed to fail, so that their axes are coincident with the end effector.

For the cases where $\|\hat{\mathbf{n}}_i\| > 1$, the value of $\|\hat{\mathbf{n}}_i\|$ is set to be 1, and the norms of the other columns of the null space can be identified according to the weighting coefficients.

2) Multiple Joint Failures: For the case of multiple joint failures, the minimum singular value after these failures is not only affected by the norm of the null space matrix columns, but also by the angles between them. This is best illustrated through a simple example. Consider the case of a planar 4R manipulator that experiences two simultaneous joint failures. If all joints are equally likely to fail then all possible pairs of joint failures are also equally likely to fail. This means that the optimal design is one where the remaining two columns of the Jacobian are as orthogonal as possible after the locking of any two arbitrary joints. This results in a design like the one shown in Fig. 3(a), which is the same as the optimal planar 4R robot obtained in [12]. The situation of unequal joint failure probabilities, which is not discussed in [12], requires the consideration of all six possible combinations of pairs of joint failures. Consider the extremal case where it is known which pair of joints will fail, e.g., joints one and three. Similar to the one joint failure case, the end-effector must be located on the joint axes of the two joints that are going to fail, as shown in Fig. 3(b).

IV. Designing for Multiple Cases of Joint Failure Probabilities

A. Problem Formulation

In the previous section, a method for designing an optimally fault tolerant robot for any set of joint failure probabilities was presented. However, joint failure probabilities are usually changing while performing a task. In particular, one common case is that a robot is designed so that all the joints are equally likely to fail when manufactured, however, during use one of the joints becomes more likely to fail either due to original component variation or the characteristics of the tasks. Unfortunately, one can not guarantee at design time which joint is the one that is more likely to fail, so that all cases should be considered. This scenario is discussed in this section.

When all the joints are equally likely to fail, one would like to optimize $\frac{1}{n}({}^1\sigma_m + {}^2\sigma_m + \cdots + {}^n\sigma_m)$. When joint i is failing, one would like to optimize ${}^i\sigma_m$. To consider all of these cases simultaneously, the objective function is the sum of the equally likely case along with all of the ${}^i\sigma_m$'s. The design problem can be stated as finding the optimal robot kinematic parameters and the $n+1$ different optimal configurations for each component of the objective function that will maximize the overall objective function. It can be mathematically expressed as

$$\begin{aligned} \max\ \mathcal{F} = &\frac{1}{n}({}^1\sigma_{m,c_0} + {}^2\sigma_{m,c_0} + \cdots + {}^n\sigma_{m,c_0}) \\ &+ {}^1\sigma_{m,c_1} + {}^2\sigma_{m,c_2} + \cdots + {}^n\sigma_{m,c_n} \\ \text{s.t.}\ \ & l_1 + l_2 + \cdots + l_n = l \end{aligned} \tag{30}$$

where the notation ${}^f\sigma_{m,c_i}$, indicates the minimum singular value of the Jacobian at the configuration c_i for a failure in joint f. Here, the $n+1$ different configurations may be at different locations, so the robot would need to adjust the relative position between its base and the task once a joint failure is imminent. In cases where one does not want to change the location of the task, one can impose the additional constraint that all the $f(c_i)$'s are equal where $f()$ denotes the forward kinematics function of the robot.

There are many different ways of solving the above constrained optimization problem. The approach used here is to first discretize the space of kinematic parameters to identify candidate robot designs. For each candidate design we discretize its workspace and find the configuration that optimizes each component of the objective function for every workspace location. This approach makes it easy to determine the optimal configurations with or without the constraint that the $f(c_i)$'s are equal.

B. Illustrative Example for Planar 3R Robots

The above optimization problem is solved for a simple 3R robot to illustrate the properties of the various robot designs. As described above, the kinematic parameters, i.e, the three link lengths are discretized in the interval [0 1] using an increment of 0.02 under the constraint that the overall length is equal to one. There are no joint limits imposed, so that the resulting circular workspaces is then uniformly sampled using 5000 points. At each workspace location, all the self-motion manifolds are calculated, and each component of the objective function $\mathcal{F}$ in (30), i.e., $\frac{1}{3}({}^1\sigma_{m,c_0} + {}^2\sigma_{m,c_0} + {}^3\sigma_{m,c_0})$, ${}^1\sigma_{m,c_1}$, ${}^2\sigma_{m,c_2}$ and ${}^3\sigma_{m,c_3}$ are evaluated along all the self-motion manifolds. The maximum values of each component of the objective function are saved, and the sum of them is the optimal objective function value at each workspace location. Finally, the maximal objective function value over all the sampled points in the workspace is saved as the optimal objective function value for each set of link lengths.

Fig. 4 shows the value of the objective function $\mathcal{F}$ in (30) for each set of link lengths. These results are shown for the version of the optimization that includes the constraint that all the $f(c_i)$'s are equal. However, it is interesting to point out that if one solves the unconstrained problem, these same configurations will be part of a larger set of equally optimal configurations. Thus one does not lose any optimality by constraining the task to be at the same location before and after a joint failure. From Fig. 4 one can also see that there is a family of optimal robots that have the

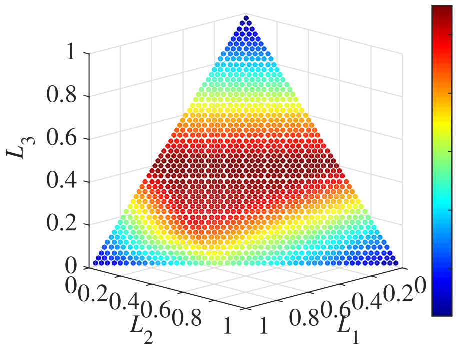

Fig. 4. The objective function values $\mathcal{F}$ for all possible link lengths, which have been normalized to equal one, is shown. Note that there is a family of optimal robots that have the same value of the objective function $\mathcal{F}$ along the line where $l_3 = 0.4$.

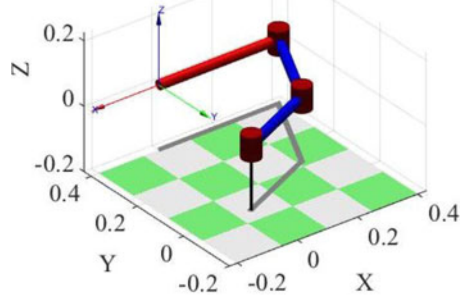

Fig. 6. The optimally fault tolerant planar 3R robot where $\mathbf{l} = [0.30, 0.30, 0.40]$ is shown at the optimal configuration for the equal failure probability case, i.e., $\frac{1}{3}({}^1\sigma_m + {}^2\sigma_m + {}^3\sigma_m)$.

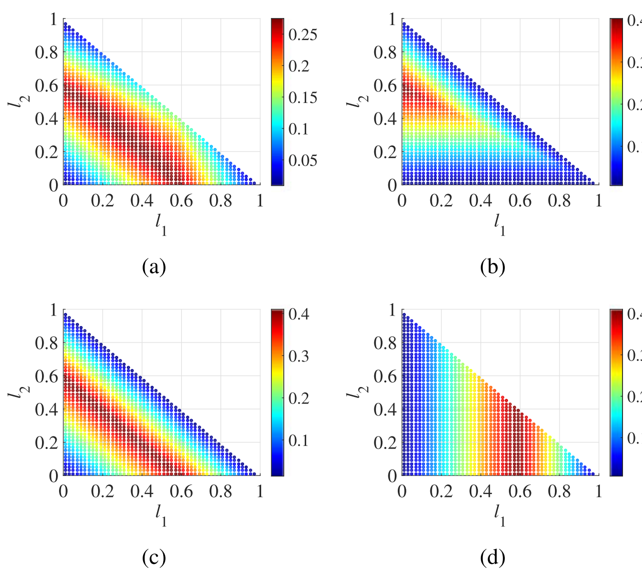

Fig. 5. The value of each component of the objective function $\mathcal{F}$ is plotted for all the sets of link lengths, with the first component, i.e., $\frac{1}{3}({}^1\sigma_m + {}^2\sigma_m + {}^3\sigma_m)$, shown in (a) and the components for the individual joint failures, i.e., ${}^1\sigma_m$, ${}^2\sigma_m$ and ${}^3\sigma_m$, shown in (b)–(d), respectively. Note that in (a), for the robots with link lengths $l_1 + l_2 < 0.6$, the optimal value of $\frac{1}{3}({}^1\sigma_m + {}^2\sigma_m + {}^3\sigma_m)$ is a function of the sum $l_1 + l_2$. In (b), the functional dependence of the optimal value of ${}^1\sigma_m$ depends on whether the link lengths are above or below the line defined by $l_1 = 1 - (\sqrt{2}/2 + 1)l_2$. If below, then ${}^1\sigma_m$ is only a function of l_2 and if above it is a function of the sum $l_1 + l_2$. In (c), for all the robots, the optimal value of ${}^2\sigma_m$ is only a function of the sum $l_1 + l_2$. In (d), for all the robots, the optimal value of ${}^3\sigma_m$ is only a function of l_1.

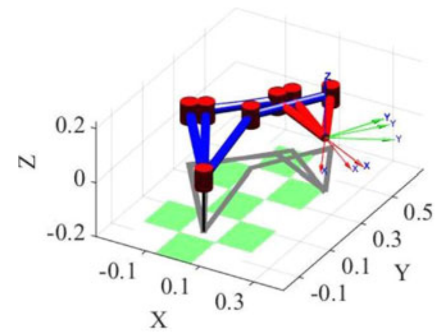

Fig. 7. The optimal configurations for each component of the objective function are shown for the robot with $\mathbf{l} = [0.43, 0.29, 0.28]$. Note that the configuration for optimizing one of the cases is quite different from the others, which was not the case for the robot depicted in Fig. 6.

same value of the objective function. All these optimal robots have the property that the third link length is equal to 0.4, i.e., $l_1 + l_2 = 0.6$.

To distinguish between the robots in this optimal family, the values for each component of the objective function $\mathcal{F}$ for all possible link lengths are shown in Fig. 5. These figures are obtained by the same method as describe above, only replacing the objective function by each component. Because $l_3 = 1 - l_1 - l_2$, the value of each component of the objective function is only plotted as a function of l_1 and l_2. From Fig. 5, one can see that the optimization of $\mathcal{F}$ also results in the optimization of the equal failure probability case, i.e., $\frac{1}{3}({}^1\sigma_m + {}^2\sigma_m + {}^3\sigma_m)$, shown in (a) and the joint two failure case, i.e., ${}^2\sigma_m$, shown in (c) individually. Unfortunately, the joint one and three cases, i.e., ${}^1\sigma_m$ and ${}^3\sigma_m$, shown in (b) and (d), respectively, are directly competing with each other. Obviously, one can select the weighting between these two cases based on failure probabilities, but if failures in joint one and three are equally likely, then the optimal link lengths are when $l_1 = l_2 = 0.3$ (with $l_3 = 0.4$). It turns out that the optimal configurations for each of the four components of $\mathcal{F}$ are very close to each other, i.e., optimization of the equal probability and joint two failure cases result in $\theta = [59^\circ, 39^\circ, 115^\circ]$ (shown in Fig. 6) and for the joint one and three failure cases result in $\theta = [55^\circ, 44^\circ, 111^\circ]$. This is fortunate, because this means that the robot only needs to slightly adjust its configuration when the joint failure probabilities are changing. However, this will not always be true, as shown in Fig. 7, which depicts the optimal configurations for the robot with $\mathbf{l} = [0.43, 0.29, 0.28]$.

V. Conclusion and Future Work

This work presented methods to design optimally fault tolerant robots for different joint failure probabilities. It was shown that for an arbitrary set of joint failure probabilities, the optimal null space satisfies $|\hat{n}_i| = w_i/\sqrt{w_1^2 + w_2^2 + \cdots + w_n^2}$, where the ratio of each element is equal to the ratio of the failure probabilities of the different joints. Based on the optimal null space, isotropic Jacobians for an arbitrary number of joints are constructed, and the kinematic parameters of the robots are generated from these optimal Jacobians. In addition, a method for designing an optimal spatial 7R manipulator Jacobian is derived, and it is shown that such optimal 7R designs do not exist for all possible joint failure probabilities. An example of such a case is that of equal joint failure probabilities. However, an optimal Jacobian for nearly equal joint failure probabilities is given. Furthermore, a method for designing optimally fault tolerant robots for multiple cases of joint failure probabilities is introduced and the design of optimal planar 3R robots is given as an illustrative example. It was shown that there exists a family of optimal robots whose last link length is equal to 40% of the total link length. This family of robots are also shown to be optimal for the case of equally likely joint failures, as well as failures in joint two. However, the optimization for failures in joints one and three are directly competing, so that one must select a particular design based on the failure probabilities of these joints.

There are a wide range of potential issues to be explored as future work. For example, determining for which sets of joint failure probabilities that optimal 7R designs exist is an open problem. In addition, the characterization of the families of optimal robots with respect to practical considerations such as joint limits and self collision, and how they affect the resulting workspace, is an important design factor. Finally, our work can be extended to consider a measure of fault tolerance with respect to a certain set of tasks that one would like to guarantee that the robot can perform after a failure.

References

[1] Z. Mu, L. Han, W. Xu, B. Li, and B. Liang, “Kinematic analysis and fault-tolerant trajectory planning of space manipulator under a single joint failure,” *Robot. Biomimetics*, vol. 3, no. 1, 2016, Art. no. 16.

[2] J. Trevelyan, W. R. Hamel, and S.-C. Kang, “Robotics in hazardous applications,” in *Springer Handbook of Robotics.* Berlin, Germany: Springer-Verlag, 2016, pp. 1521–1548.

[3] R. Phuengsuk and J. Suthakorn, “A study on risk assessment for improving reliability of rescue robots,” in *Proc. 2016 IEEE Int. Conf. Robot. Biomimetics*, 2016, pp. 667–672.

[4] M. Y. Jung, R. H. Taylor, and P. Kazanzides, “Safety design view: A conceptual framework for systematic understanding of safety features of medical robot systems,” in *Proc. IEEE Int. Conf. Robot. Autom.*, 2014, pp. 1883–1888.

[5] B. O. Mushage, J. C. Chedjou, and K. Kyamakya, “Fuzzy neural network and observer-based fault-tolerant adaptive nonlinear control of uncertain 5-dof upper-limb exoskeleton robot for passive rehabilitation,” *Nonlinear Dyn.*, vol. 87, no. 3, pp. 2021–2037, 2017.

[6] M. Vasic and A. Billard, “Safety issues in human-robot interactions,” in *Proc. 2013 IEEE Int. Conf. Robot. Autom.*, 2013, pp. 197–204.

[7] G. Muscio and F. Pierri, “A fault tolerant adaptive control for robot manipulators,” in *Proc. 2012 IEEE Int. Conf. Robot. Biomimetics*, 2012, pp. 1697–1702.

[8] Y. Kang, Z. Li, Y. Dong, and H. Xi, “Markovian-based fault-tolerant control for wheeled mobile manipulators,” *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 1, pp. 266–276, Jan. 2012.

[9] M. Van, S. S. Ge, and H. Ren, “Finite time fault tolerant control for robot manipulators using time delay estimation and continuous nonsingular fast terminal sliding mode control,” *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1681–1693, Jul. 2017.

[10] B. Zhao and Y. Li, “Local joint information based active fault tolerant control for reconfigurable manipulator,” *Nonlinear Dyn.*, vol. 77, no. 3, pp. 859–876, 2014.

[11] R. C. Hoover, R. G. Roberts, A. A. Maciejewski, P. S. Naik, and K. M. Ben-Gharbia, “Designing a failure-tolerant workspace for kinematically redundant robots,” *IEEE Trans. Autom. Sci. Eng.*, vol. 12, no. 4, pp. 1421–1432, Oct. 2015.

[12] R. G. Roberts and A. A. Maciejewski, “A local measure of fault tolerance for kinematically redundant manipulators,” *IEEE Trans. Robot. Autom.*, vol. 12, no. 4, pp. 543–552, Aug. 1996.

[13] C. J. Paredis and P. K. Khosla, “Designing fault-tolerant manipulators: How many degrees of freedom?” *Int. J. Robot. Res.*, vol. 15, no. 6, pp. 611–628, 1996.

[14] Q. Li and J. Zhao, “A universal approach for configuration synthesis of reconfigurable robots based on fault tolerant indices,” *Ind Robot: An Int. J.*, vol. 39, no. 1, pp. 69–78, 2012.

[15] I. D. Walker and J. R. Cavallaro, “The use of fault trees for the design of robots for hazardous environments,” in *Proc. Annu. Rel. Maintainability Symp.*, 1996, pp. 229–235.

[16] A. A. Maciejewski and R. G. Roberts, “On the existence of an optimally failure tolerant 7R manipulator Jacobian,” *Appl. Math. Comput. Sci.*, vol. 5, no. 2, pp. 343–357, 1995.

[17] K. M. Ben-Gharbia, A. A. Maciejewski, and R. Roberts, “An illustration of generating robots from optimal fault-tolerant Jacobians,” in *Proc. 15th IASTED Int. Conf. Robot. Appl.*, 2010, pp. 1–3.

[18] K. M. Ben-Gharbia, A. A. Maciejewski, and R. G. Roberts, “Kinematic design of redundant robotic manipulators for spatial positioning that are optimally fault tolerant,” *IEEE Trans. Robot.*, vol. 29, no. 5, pp. 1300–1307, Oct. 2013.

[19] K. M. Ben-Gharbia, A. A. Maciejewski, and R. Roberts, “Kinematic design of manipulators with seven revolute joints optimized for fault tolerance,” *IEEE Trans. Syst., Man, Cybern.: Syst.*, vol. 46, no. 10, pp. 1364–1373, Oct. 2016.

[20] B. Xie and A. A. Maciejewski, “Structure and performance analysis of the 7! robots generated from an optimally fault tolerant Jacobian,” *IEEE Robot. Autom. Lett.*, vol. 2, no. 4, pp. 1956–1963, Oct. 2017.

[21] C. S. Ukidve, J. E. McInroy, and F. Jafari, “Using redundancy to optimize manipulability of stewart platforms,” *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 4, pp. 475–479, Aug. 2008.

[22] D. L. Schneider, “Reliability and maintainability of modular robot systems: A roadmap for design,” Ph.D. dissertation, Dept. Mech. Eng., Univ. Texas at Austin, Austin, TX, USA, 1993.

[23] D. Nicholls, *System Reliability Toolkit*. Moscow, Russia: Riac, 2005.

[24] D. Mahar, W. Fields, and J. Reade, *Nonelectronic Parts Reliability Data*. Utica, NY, USA: Quanterion Solutions Inc., 2015.

[25] R. C. Thompson, “Principal submatrices ix: Interlacing inequalities for singular values of submatrices,” *Linear Algebra its Appl.*, vol. 5, no. 1, pp. 1–12, 1972.